Correlated Pseudorandomness

Achieving faster secure computation through pseudorandom correlation generators

Geoffroy Couteau

Start of the Story: Circa 2016

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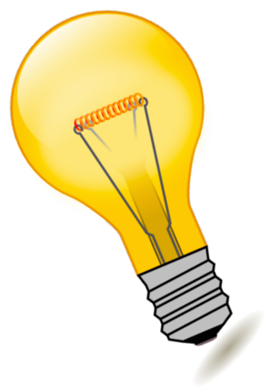

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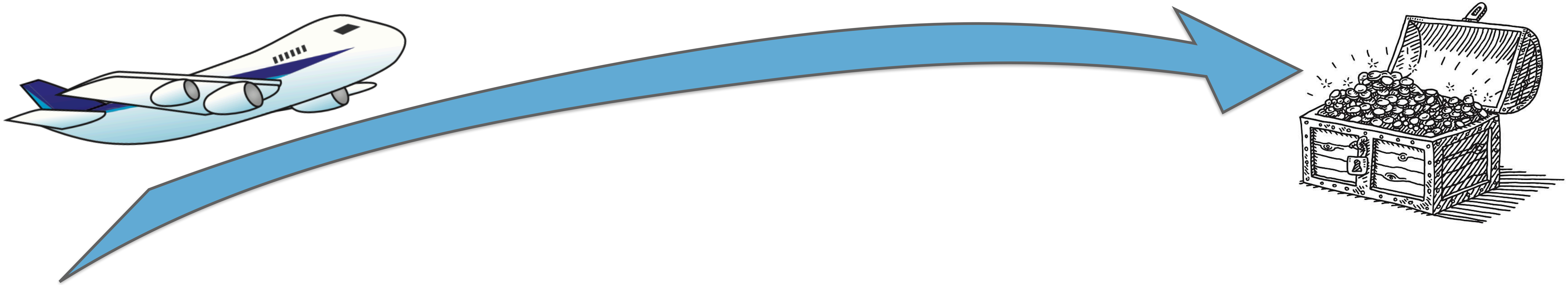
A few months and a CCS’17 paper later, we concluded that the answer was ‘not so much’

7 years later

The Journey through Correlated Pseudorandomess



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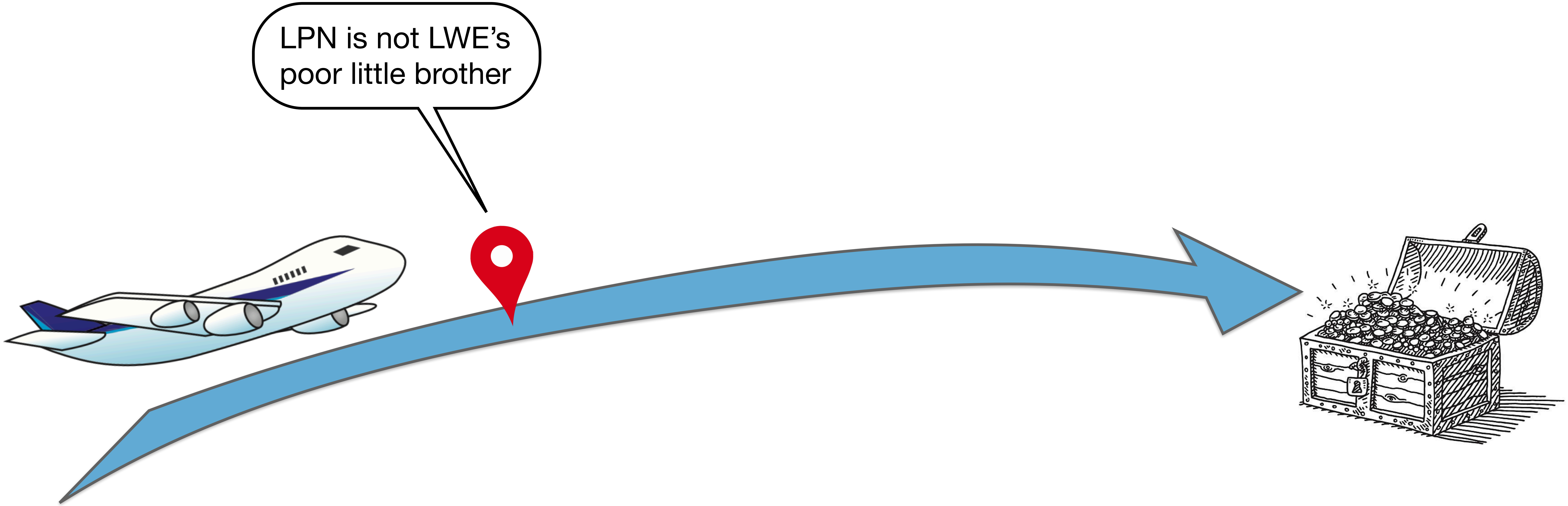

LPN is not LWE's poor little brother

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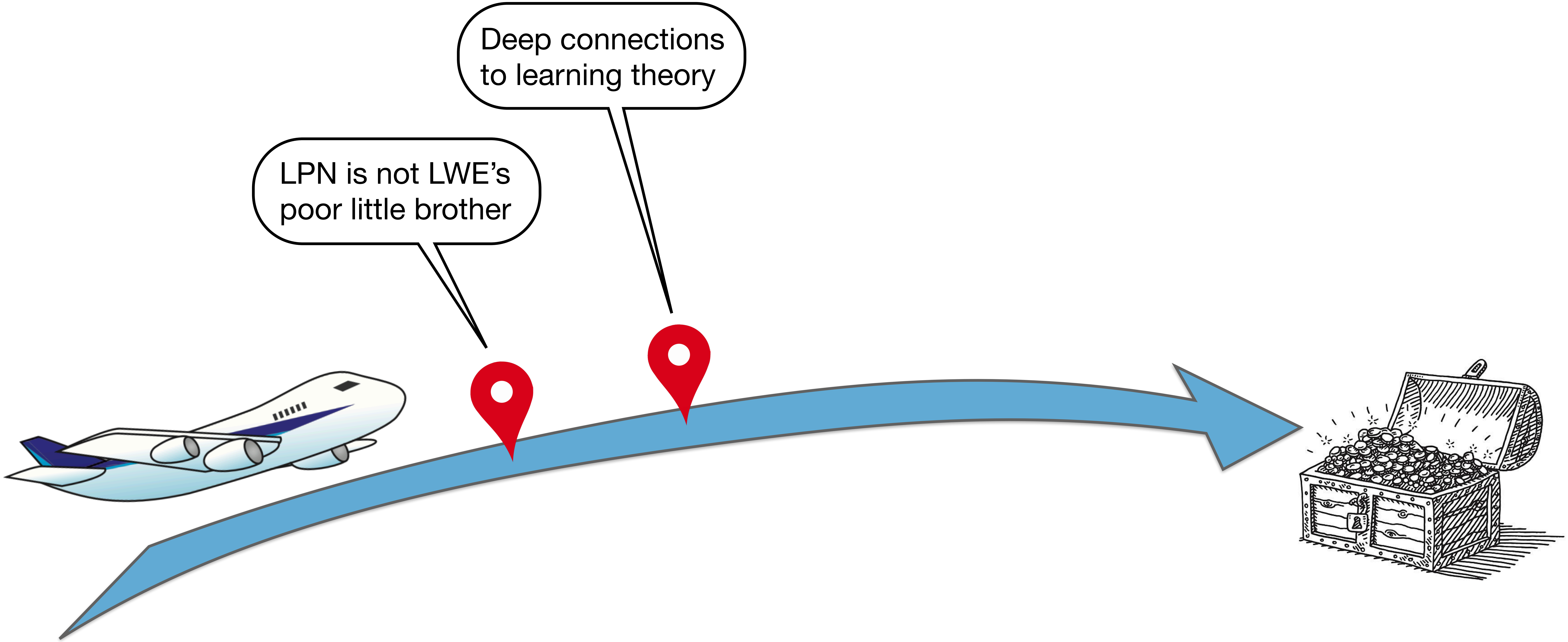

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Deep connections to learning theory

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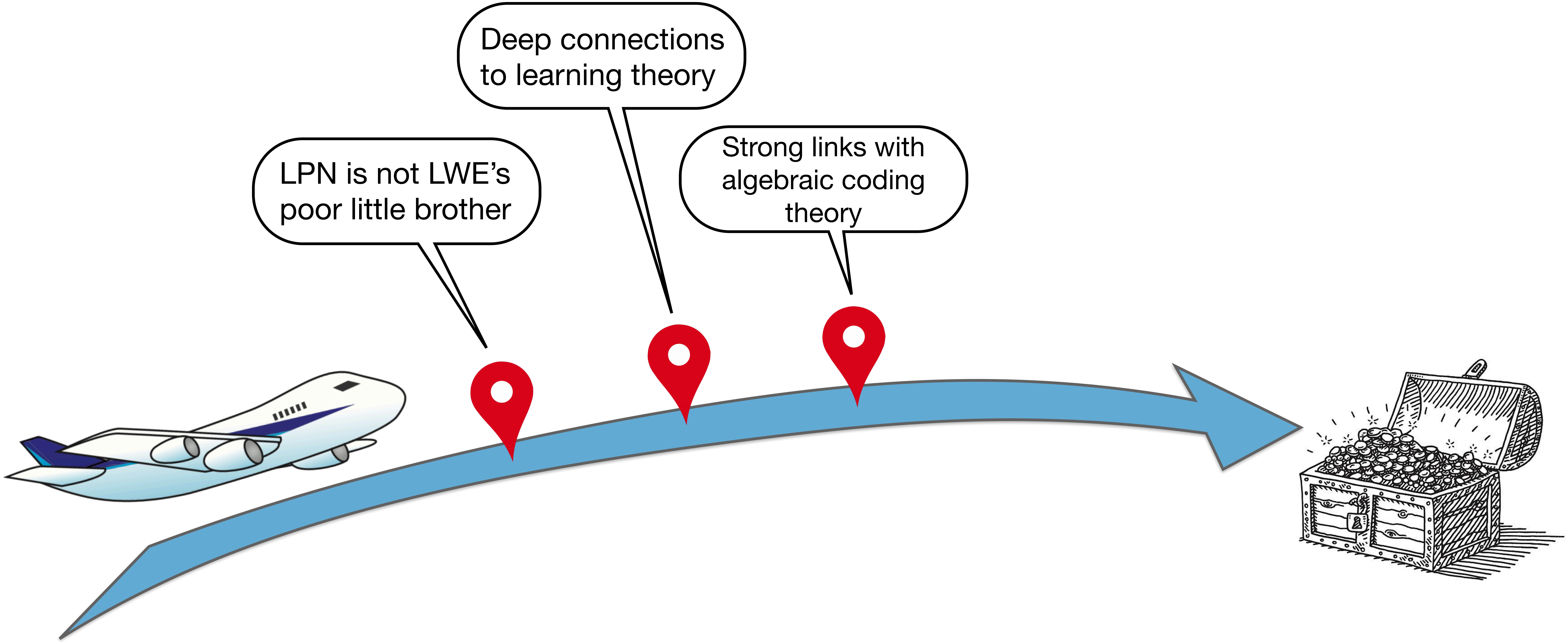

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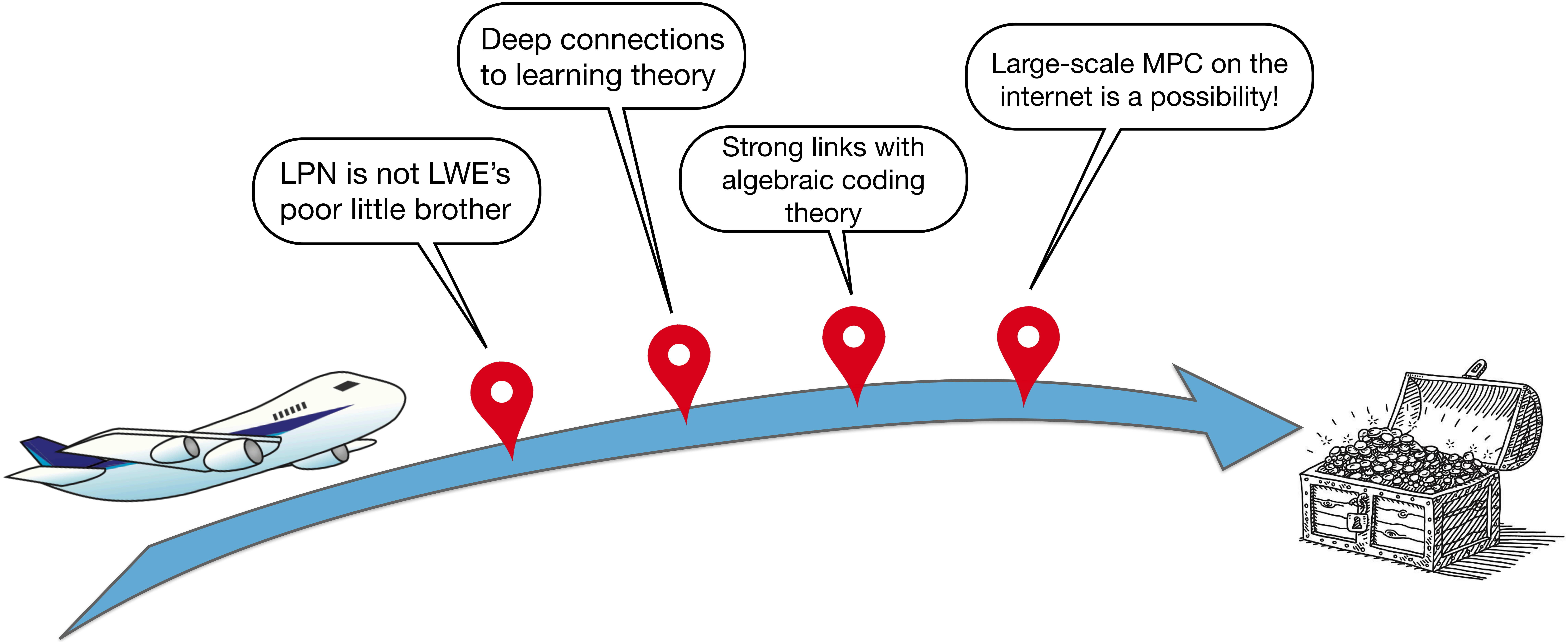

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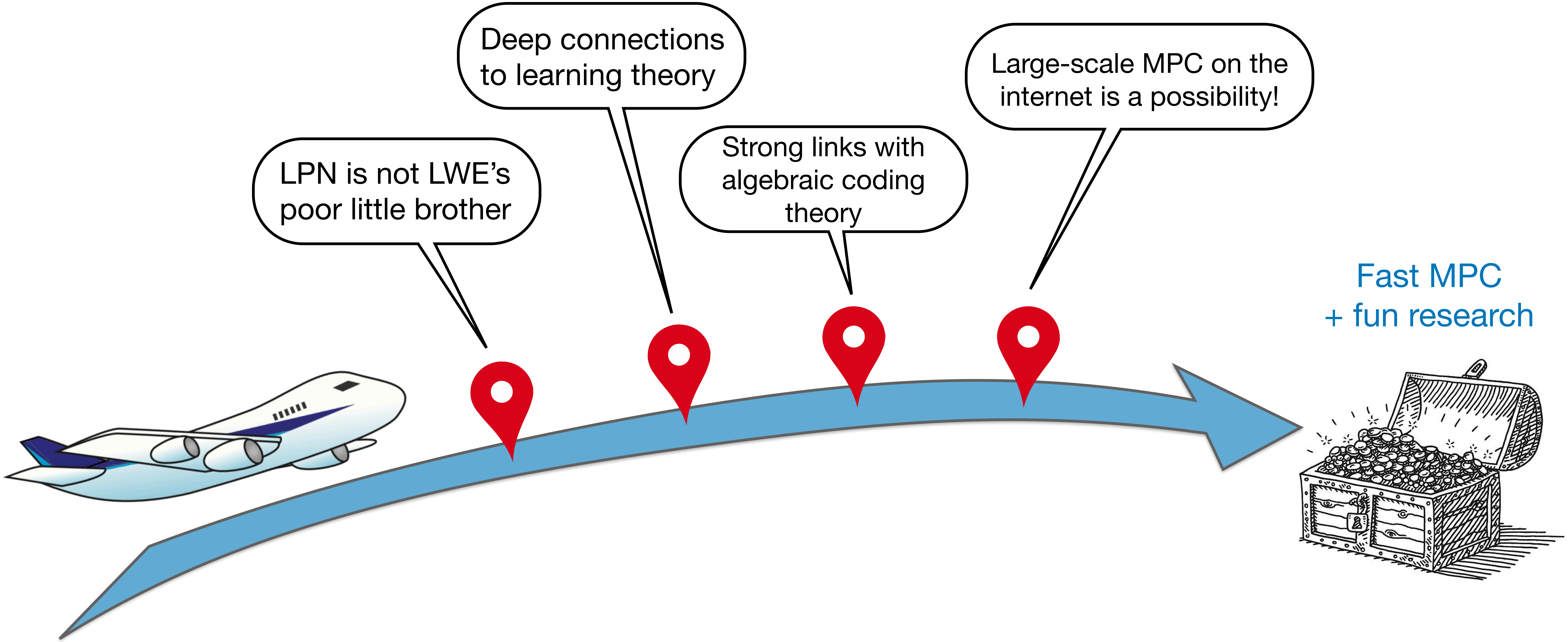
Large-scale MPC on the internet is a possibility!

The Journey through Correlated Pseudorandomess



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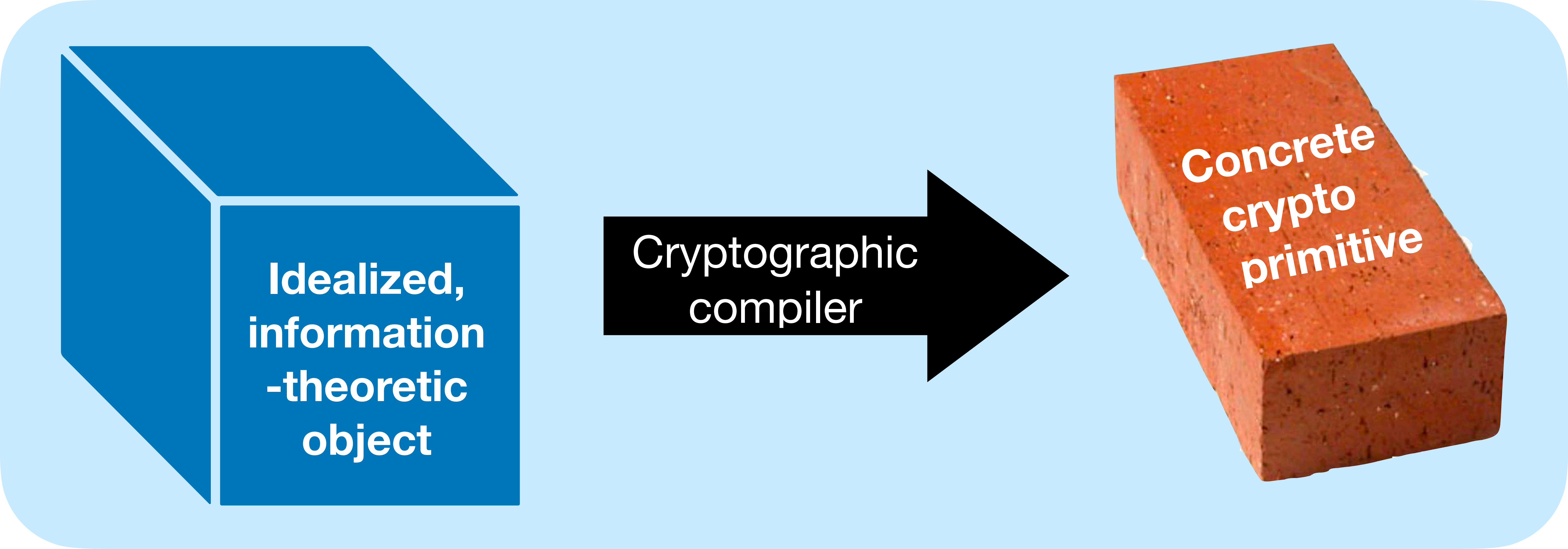
Fast MPC
+ fun research

A Standard Cryptographic Approach



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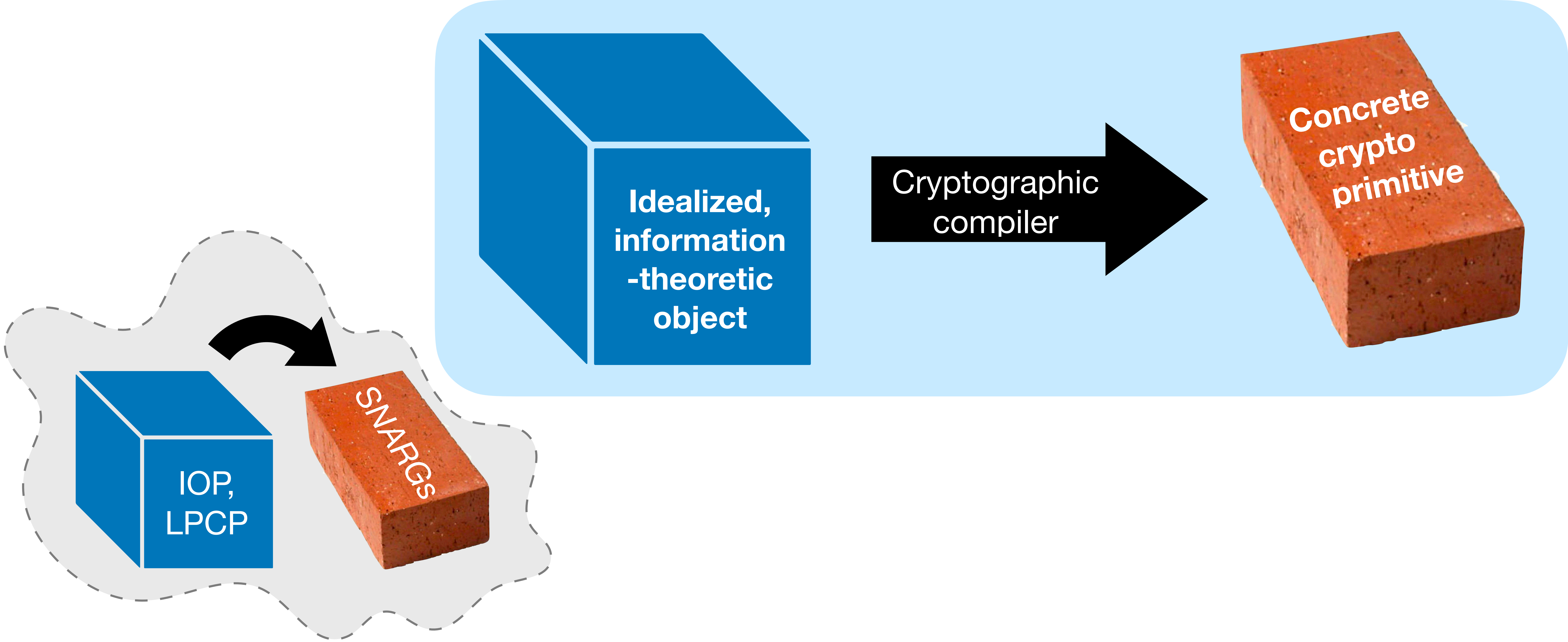

Idealized, information-theoretic object

Cryptographic compiler

Concrete crypto primitive

IOP, LPCP

SNARGs

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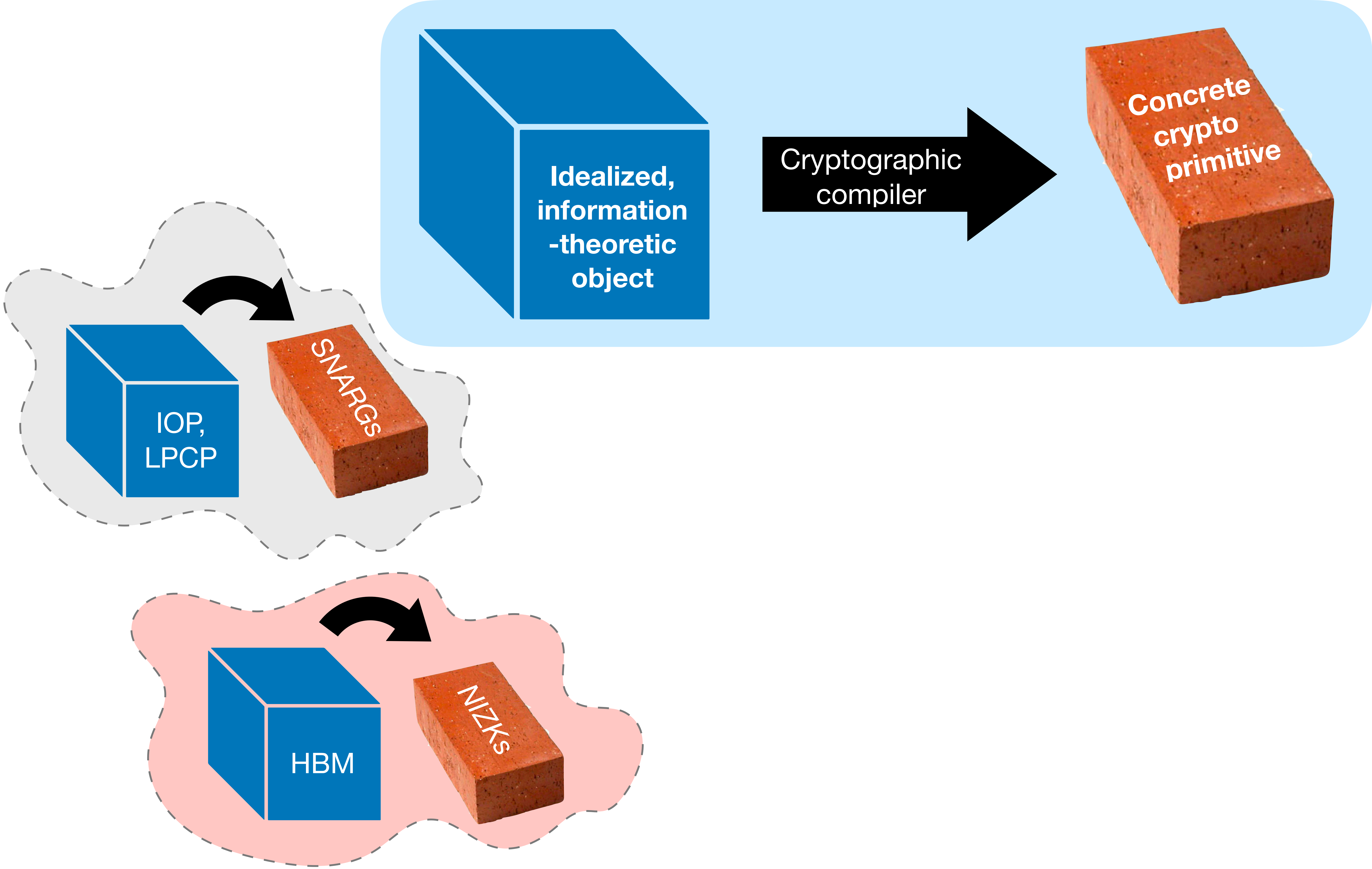

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Correlated
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MPC

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…

A Standard Cryptographic Approach

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Correlated randomness model

MPC

OTP

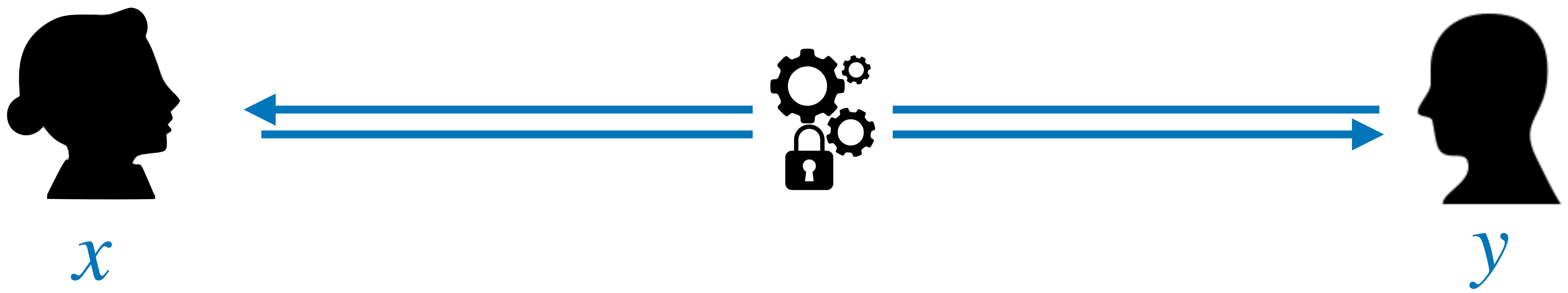
Encryption

• • •

Secure Computation…

- **Goal.** *Computing* a **public** function on secret inputs
- **Model.** n players, each with a private input x_i interacting through authenticated channels

$$\boxed{f:(x,y)\mapsto(z_A,z_B)}$$



- **Output:** Alice learns z_A and Bob learn z_B
- **Security:** Alice and Bob learn nothing else

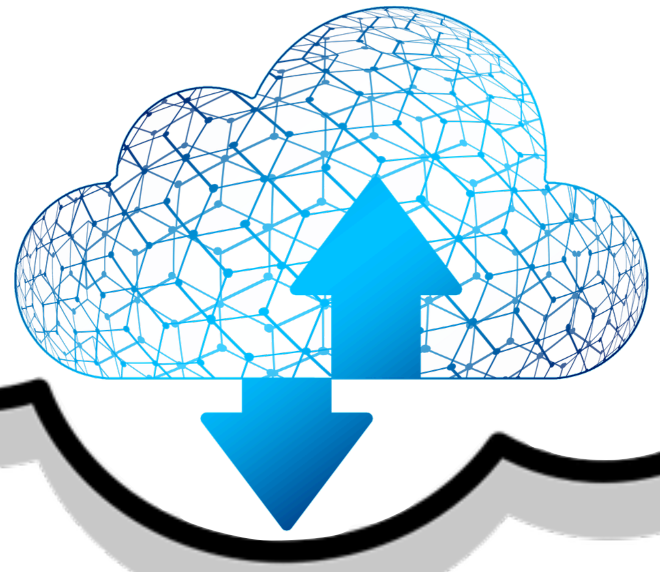
… Is a Practical Concern.

See a targeted advertising

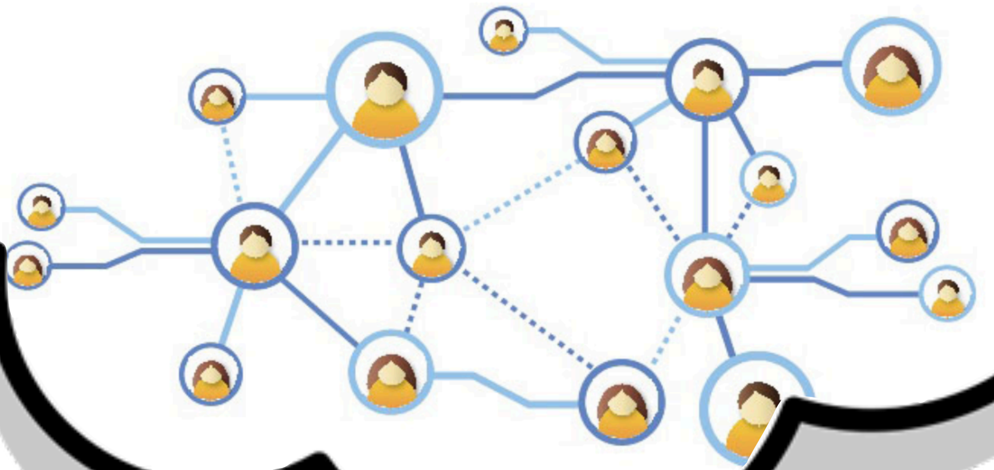
Yes—they're here! Those fashion-wise, comfort-wise Bates Originals—and PHI BATES, the smartest shoes on the campus, style-endorsed by the PHI BATES campus advisory board. Nationally advertised favorites, these shoes bring you an exclusive *extra* comfort—invisible extra width is built right in across the ball of the foot. They're "Slipper-Free Where Your Foot Bends." Drop in today and see the full Bates line.

Use a dating app

Search over our Cloud storage

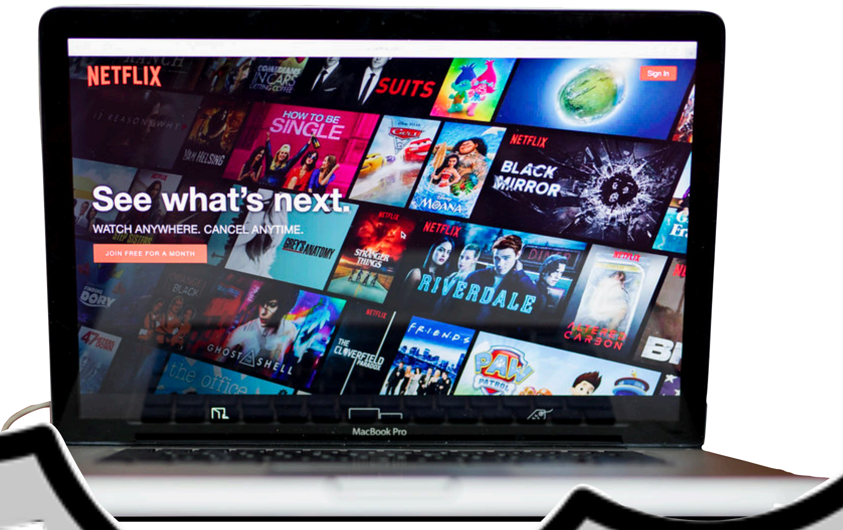


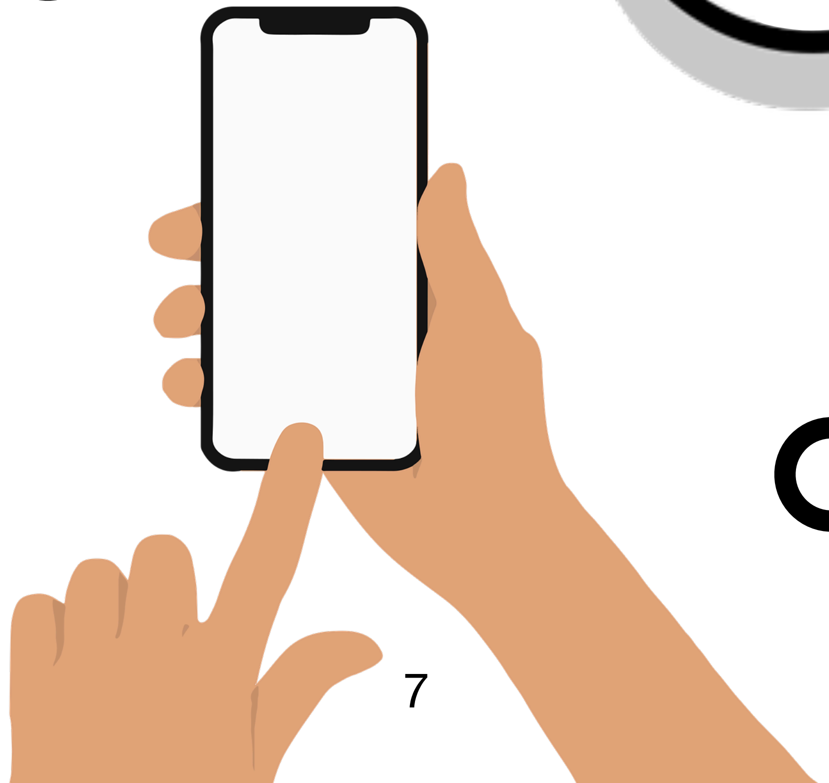
Use a social network



Use a healthcare app

Get a recommendation on a streaming platform





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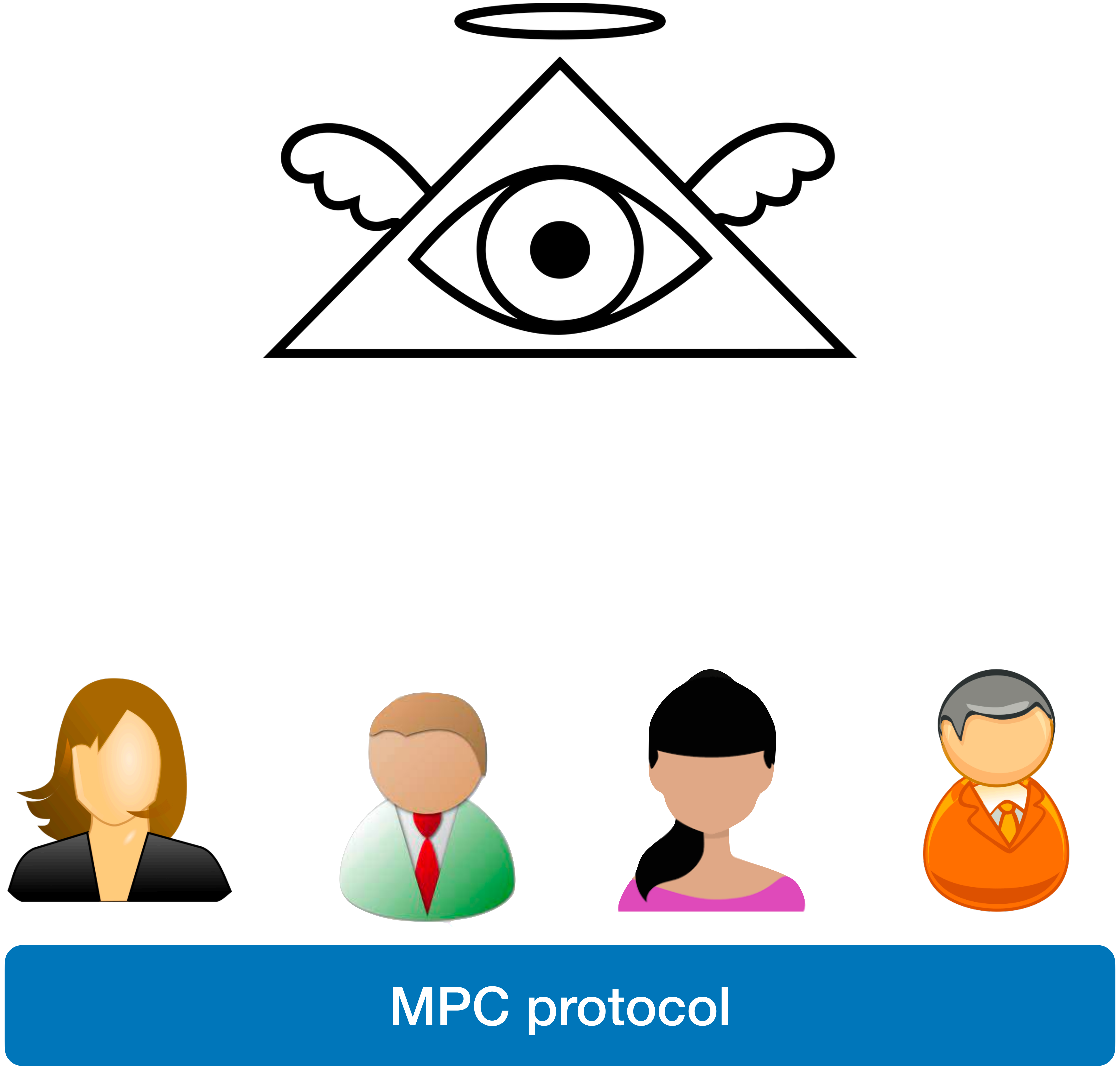
The Correlated Randomness Model



MPC protocol

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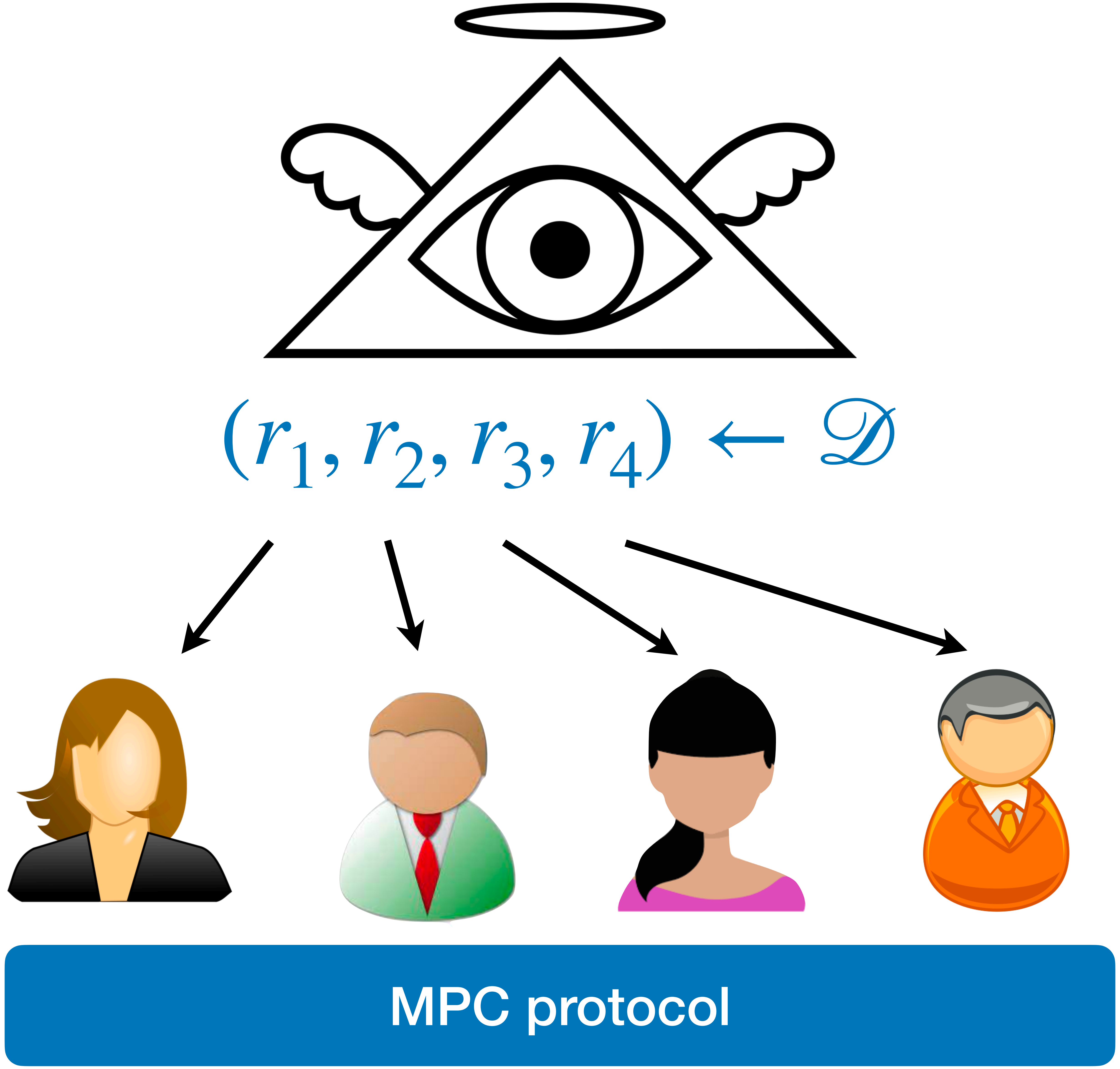

$(r_1, r_2, r_3, r_4) \leftarrow \mathscr{D}$

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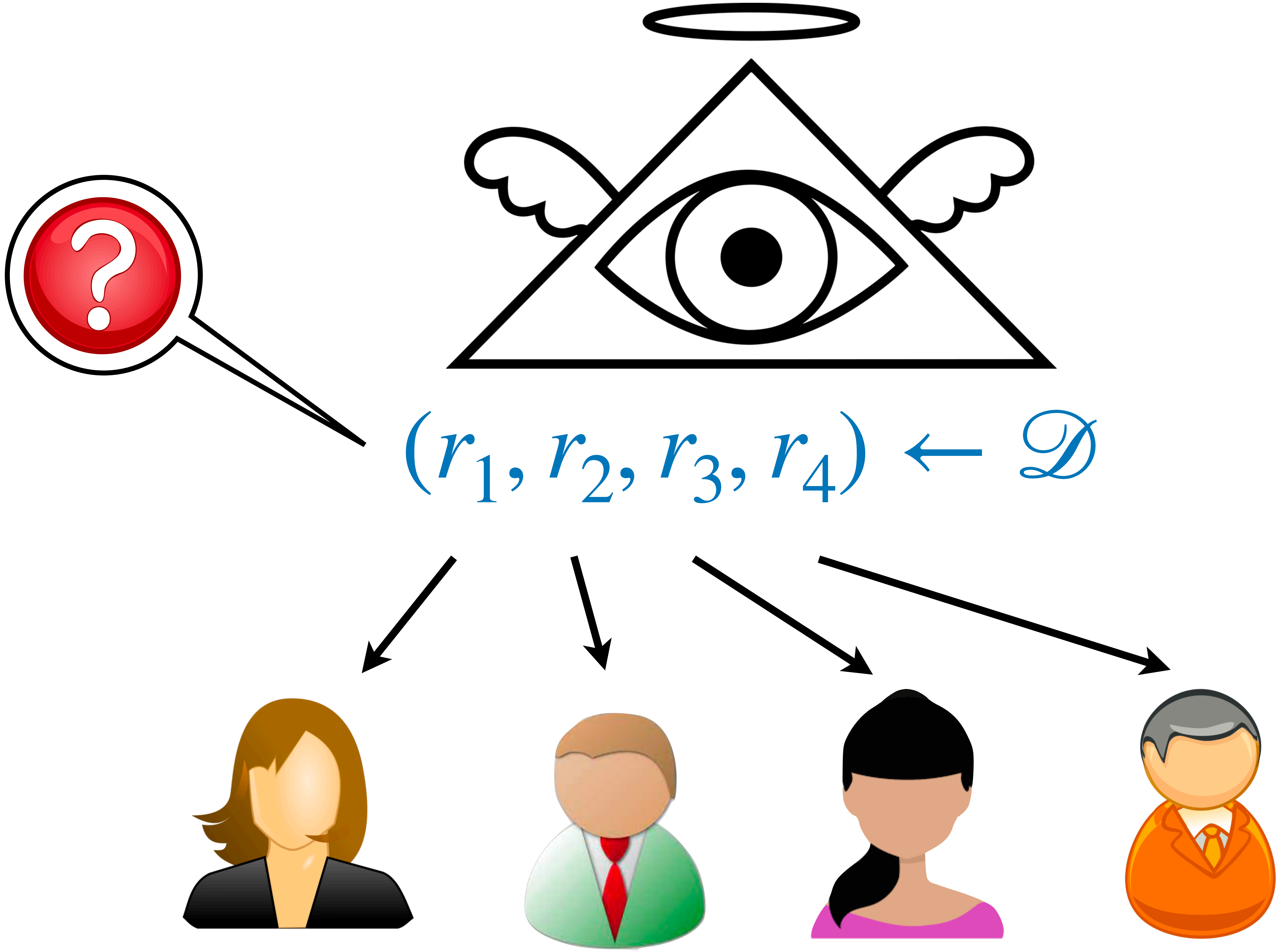

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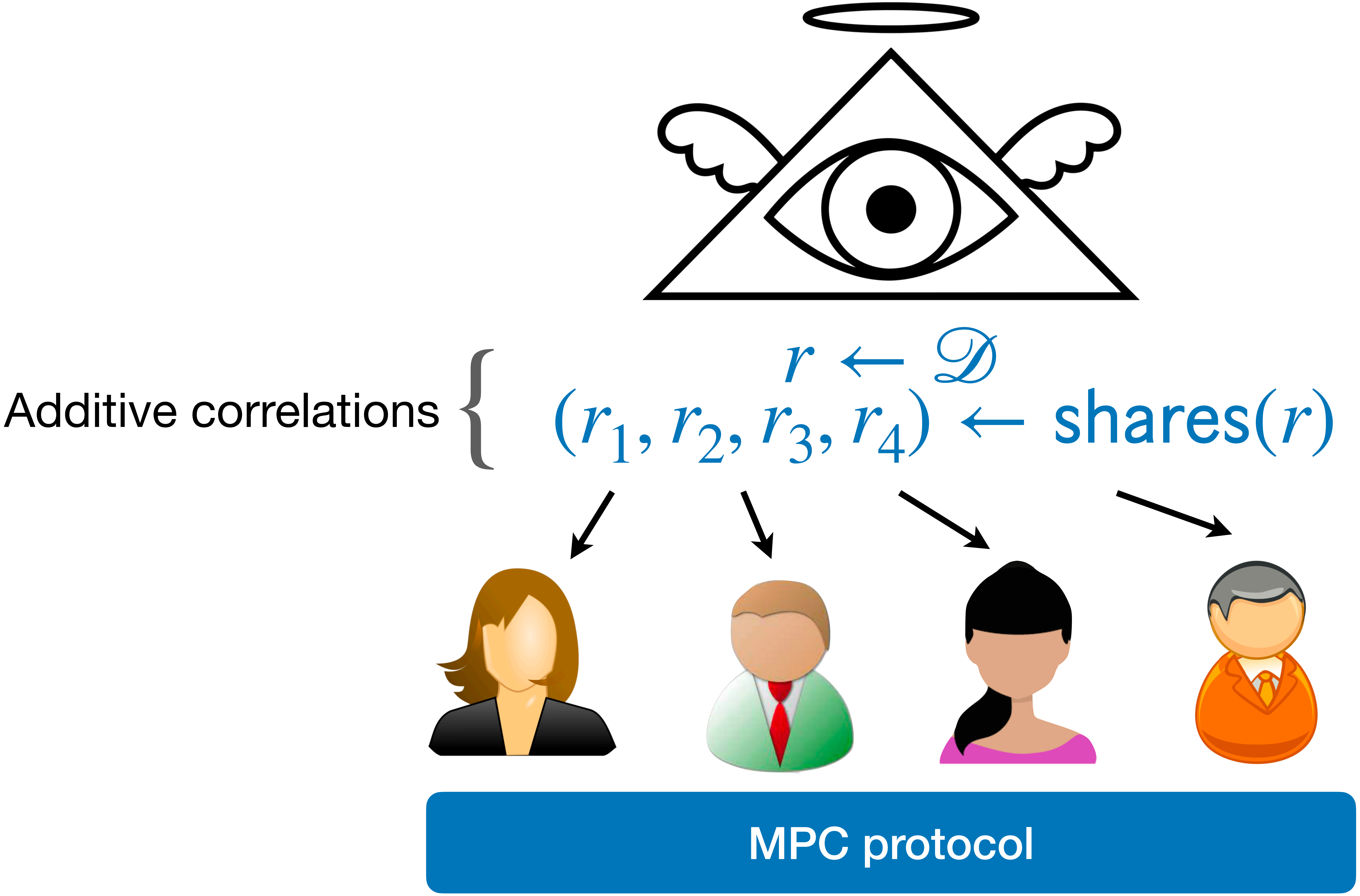

$$r \leftarrow \mathscr{D}$$

Additive correlations $\left\{ \quad (r_1, r_2, r_3, r_4) \leftarrow \mathsf{shares}(r) \right.$

MPC protocol

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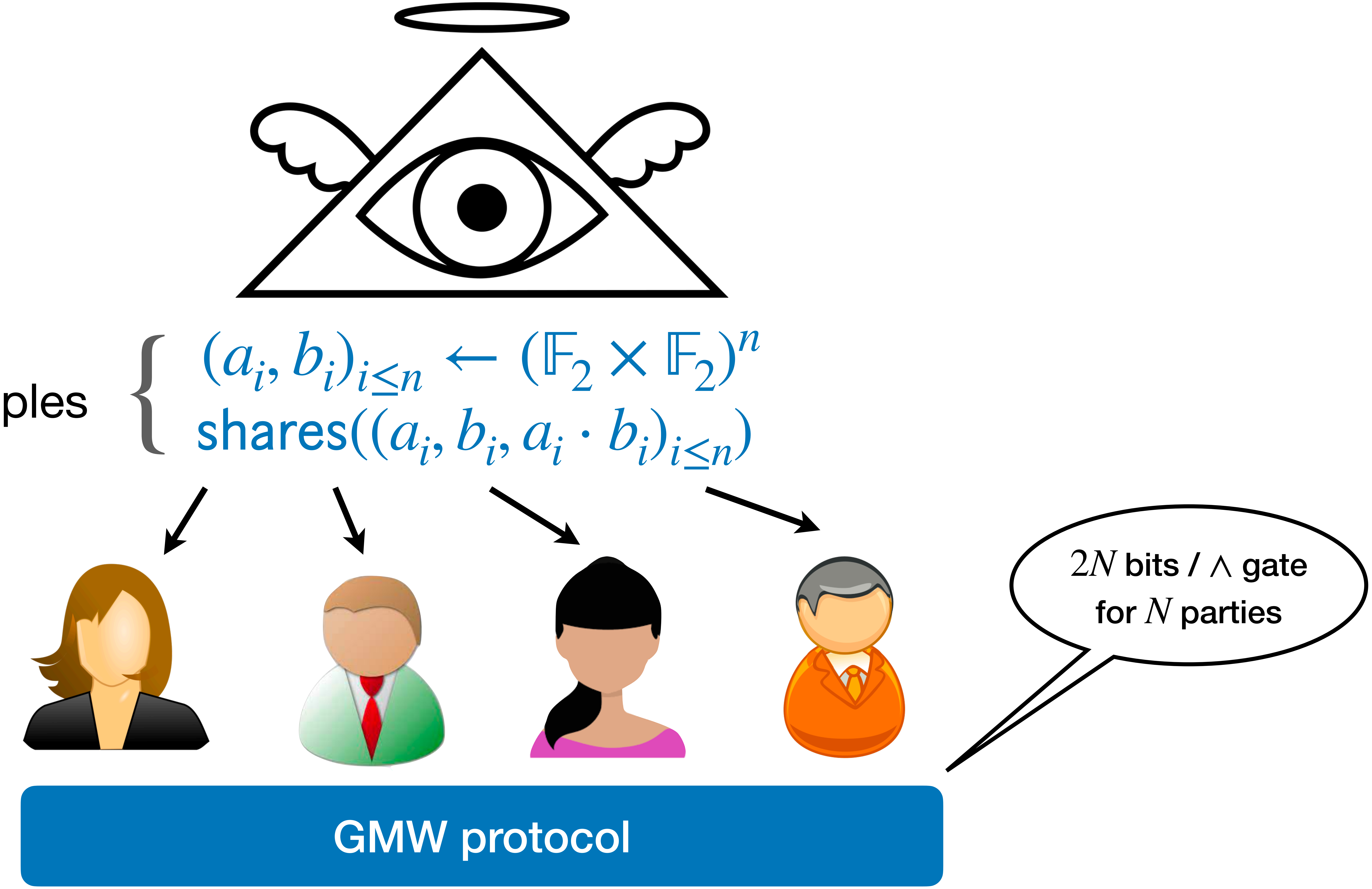

Example: Beaver triples $\left\{ \begin{array}{l} (a_i, b_i)_{i\leq n} \leftarrow (\mathbb{F}_2 \times \mathbb{F}_2)^n \\ \text{shares}((a_i, b_i, a_i \cdot b_i)_{i\leq n}) \end{array} \right.$

GMW protocol

$2N$ bits / $\wedge$ gate
for N parties

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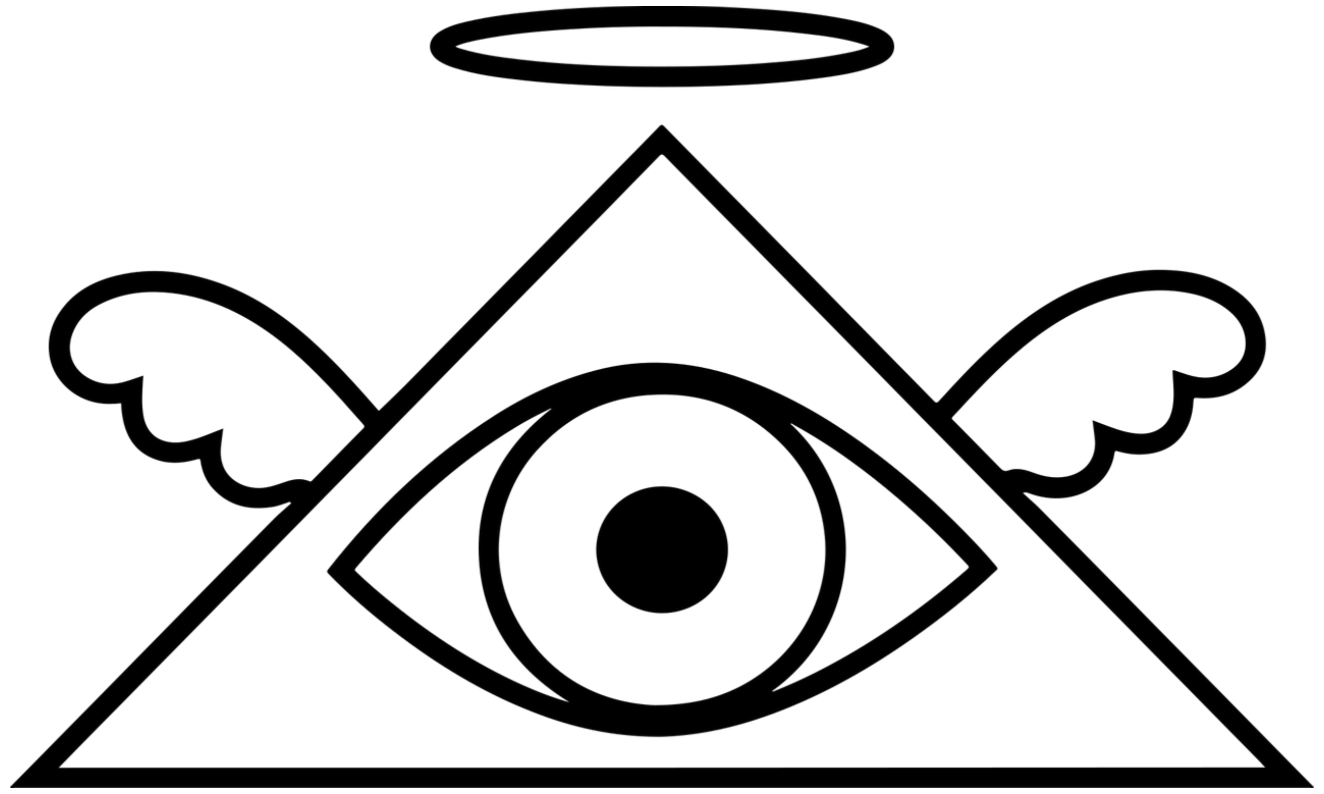

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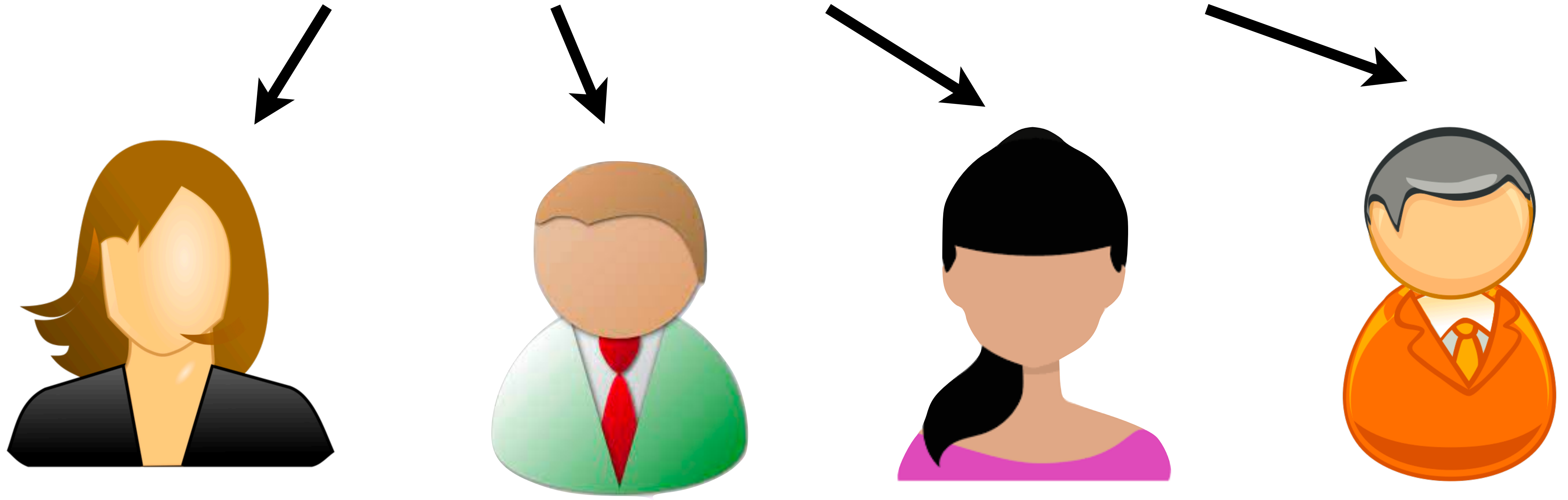

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Very practical*

A Template to Instantiate *Efficiently* the Correlated Randomness Model

Given a correlation C, the dealer distributes shares of $C(r)$

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Pseudorandom correlation generator

$\mathsf{Gen}(1^\lambda) \rightarrow (\mathsf{seed}_A, \mathsf{seed}_B)$ such that

(1) $(\mathsf{Expand}(A, \mathsf{seed}_A), \mathsf{Expand}(B, \mathsf{seed}_B))$ looks like n samples from the target correlation, and

(2) $\mathsf{Expand}(A, \mathsf{seed}_A)$ looks ‘random conditioned on satisfying the correlation with $\mathsf{Expand}(B, \mathsf{seed}_B)$’ to Bob (similar property w.r.t. Alice).

A Template to Instantiate *Efficiently* the Correlated Randomness Model: MPC with silent preprocessing

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Preprocessing phase

Online phase

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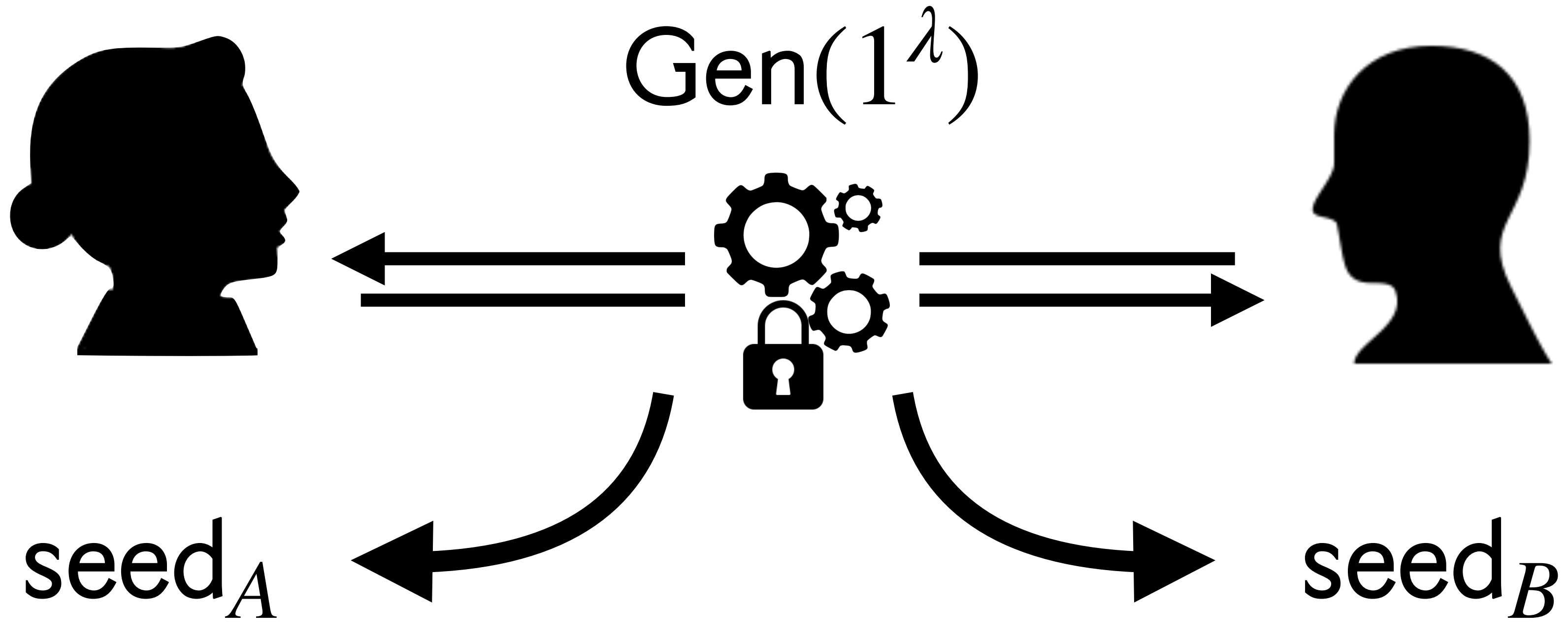
One-time short interaction



Interactive protocol with short communication and computation; Alice and Bob store a small seed afterwards.

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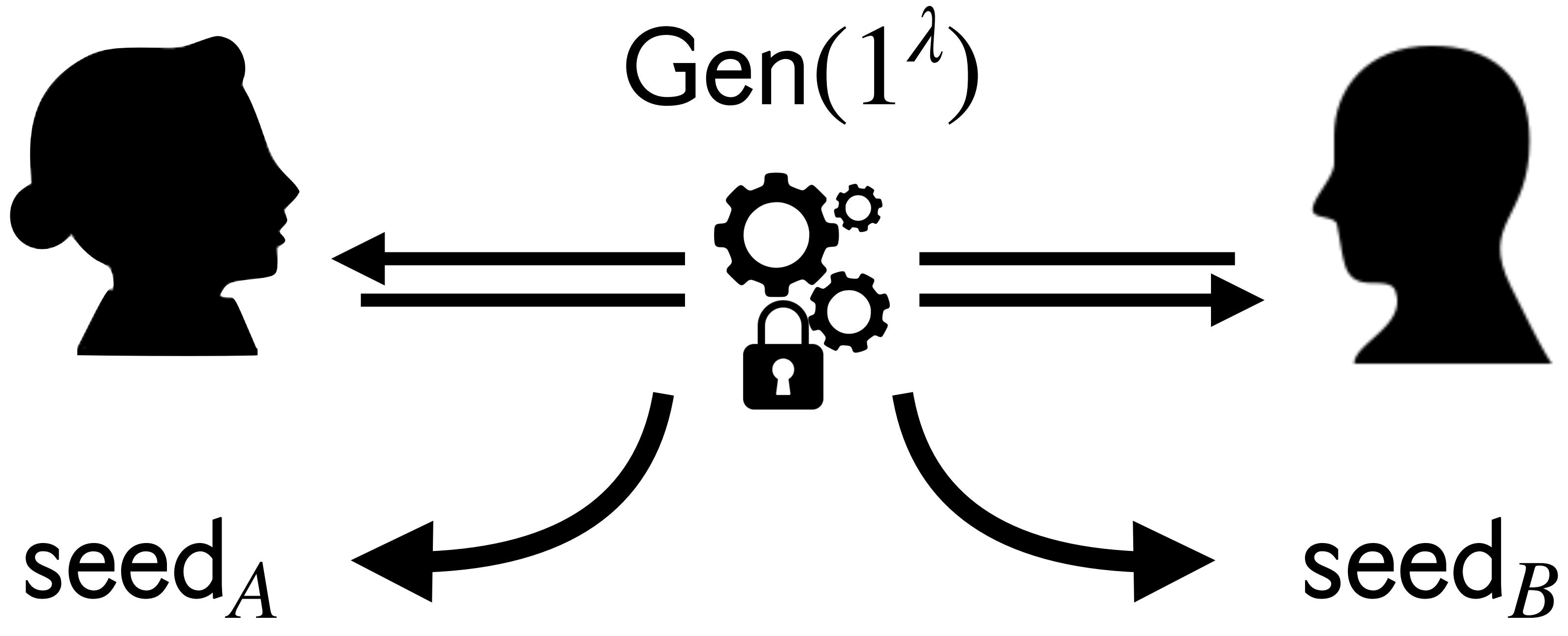

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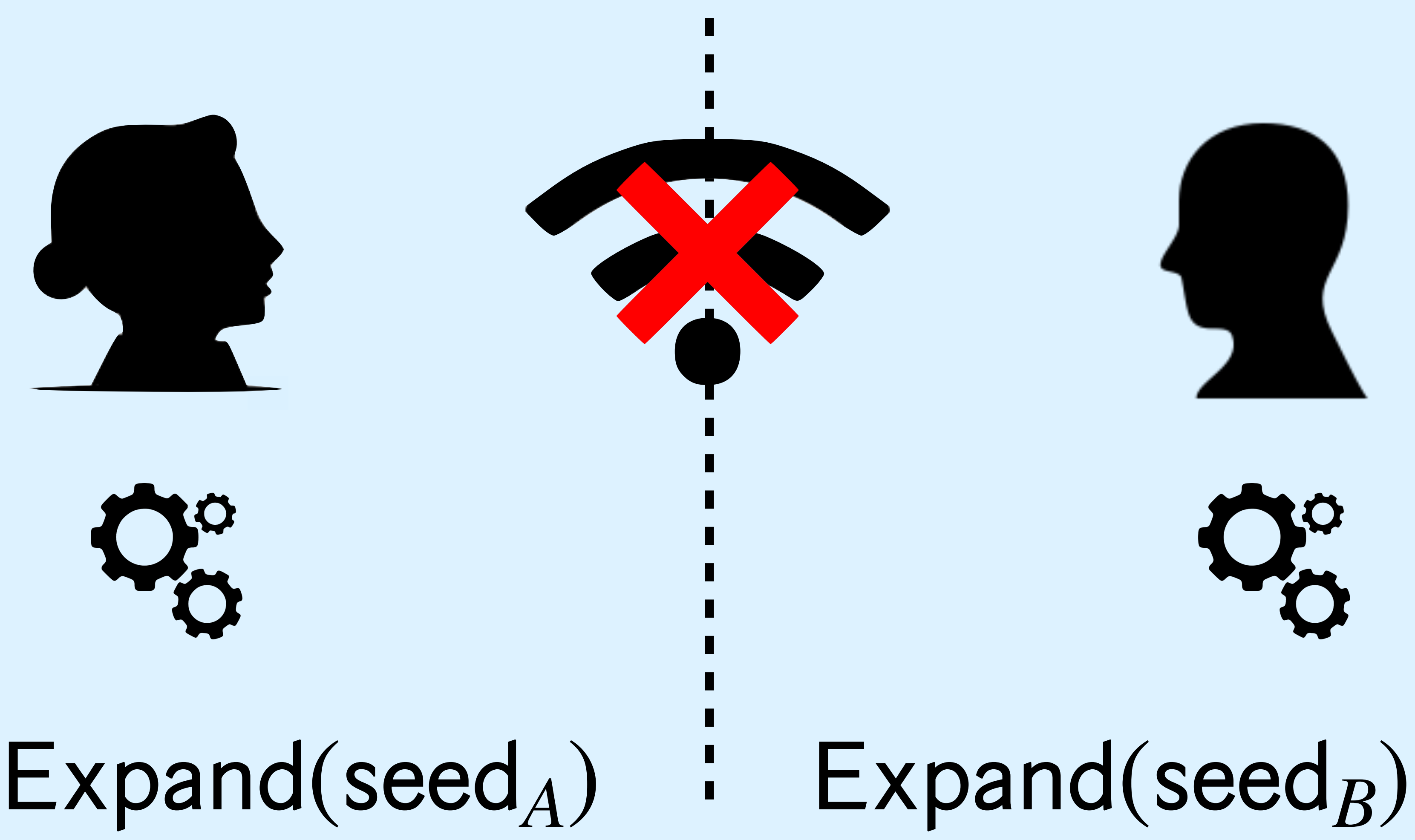

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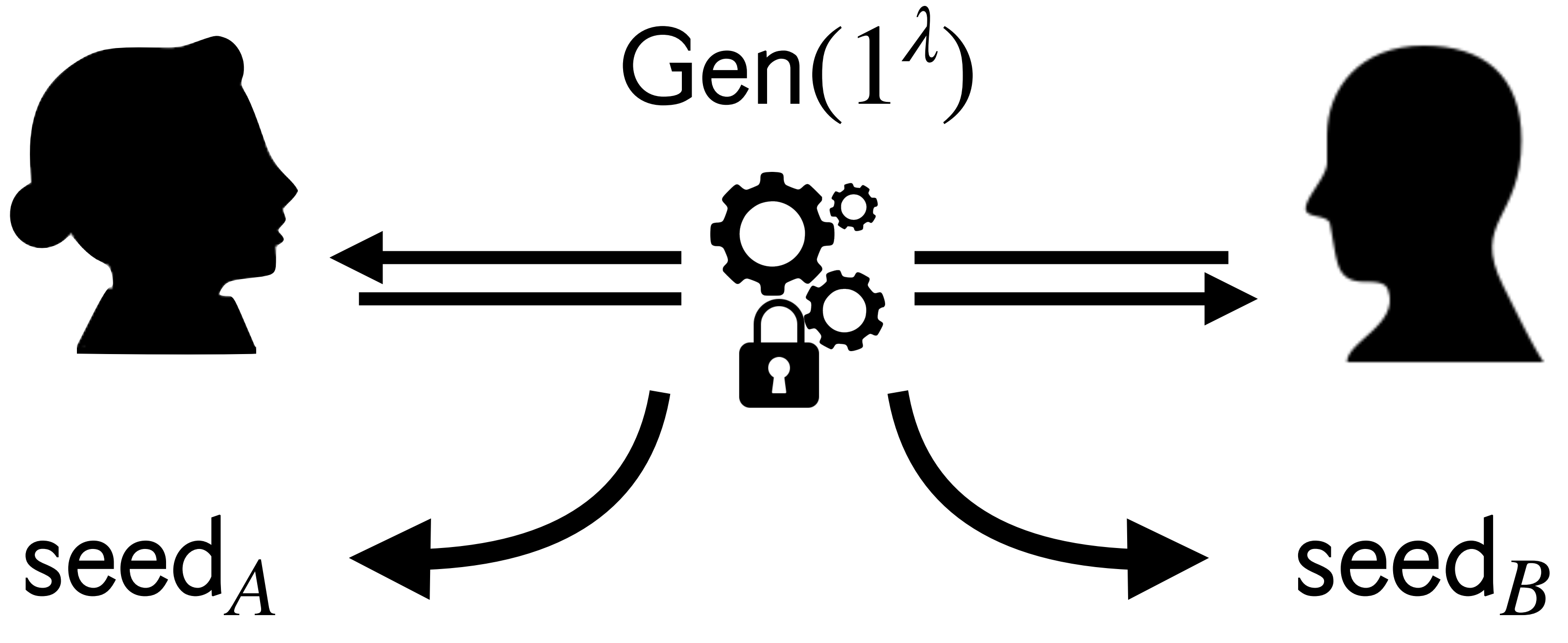

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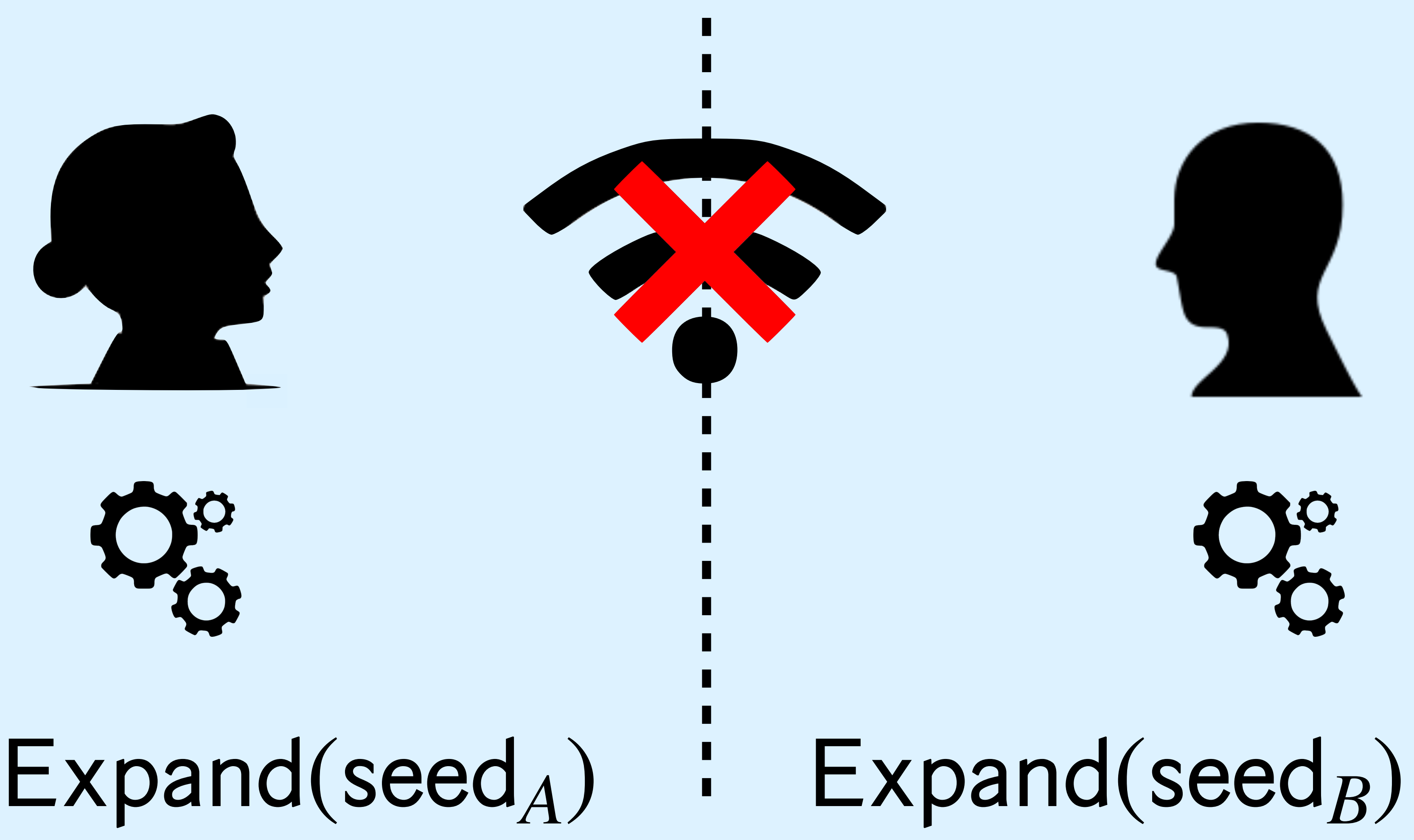

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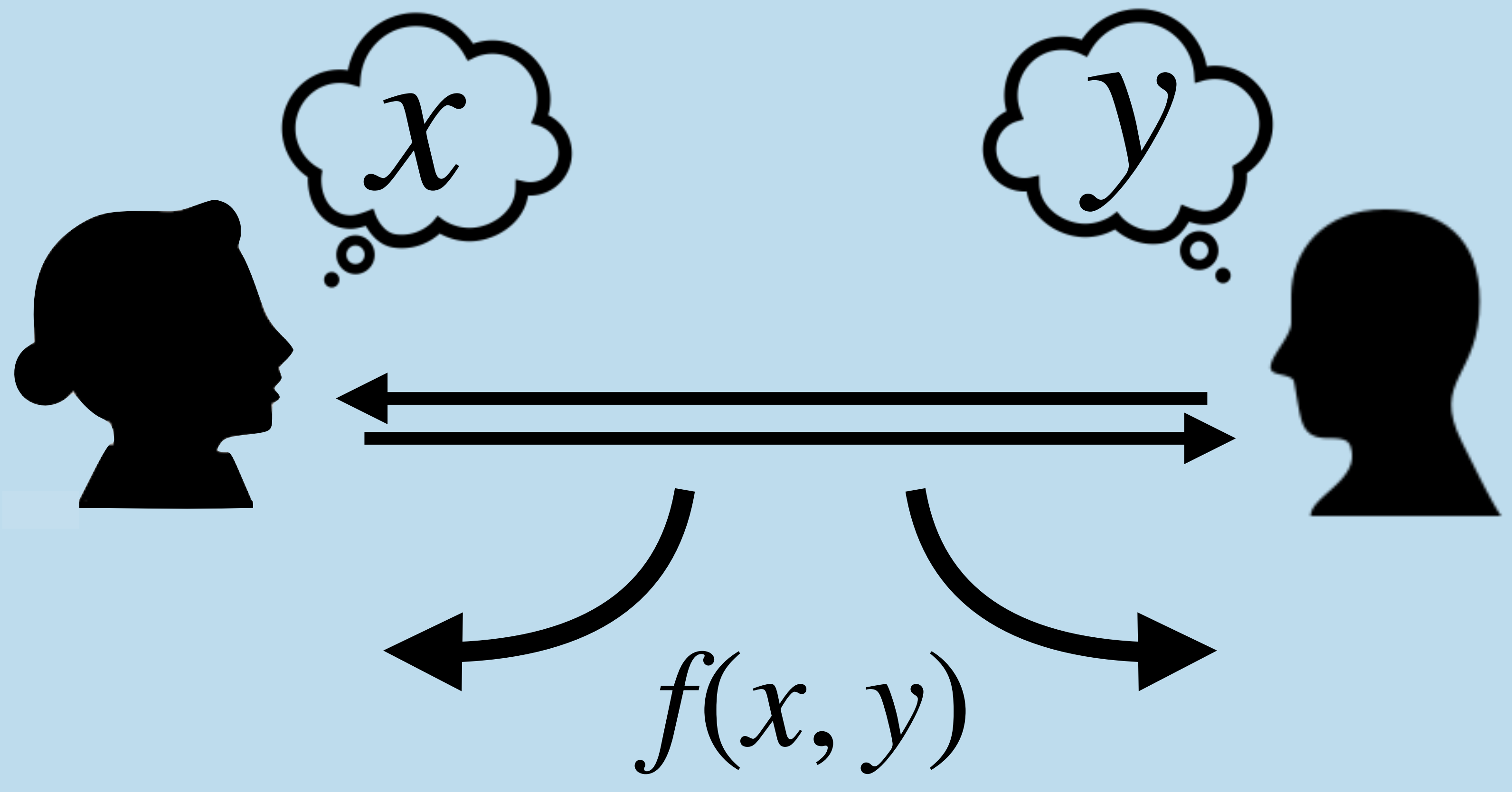
Preprocessing phase

Non-cryptographic



Alice and Bob consume the preprocessing material in a fast, non-cryptographic online phase.

Online phase

Q: What Correlations C do we Consider?

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R: It depends! What MPC Protocol do you Want?

One does not simply build some MPC protocol

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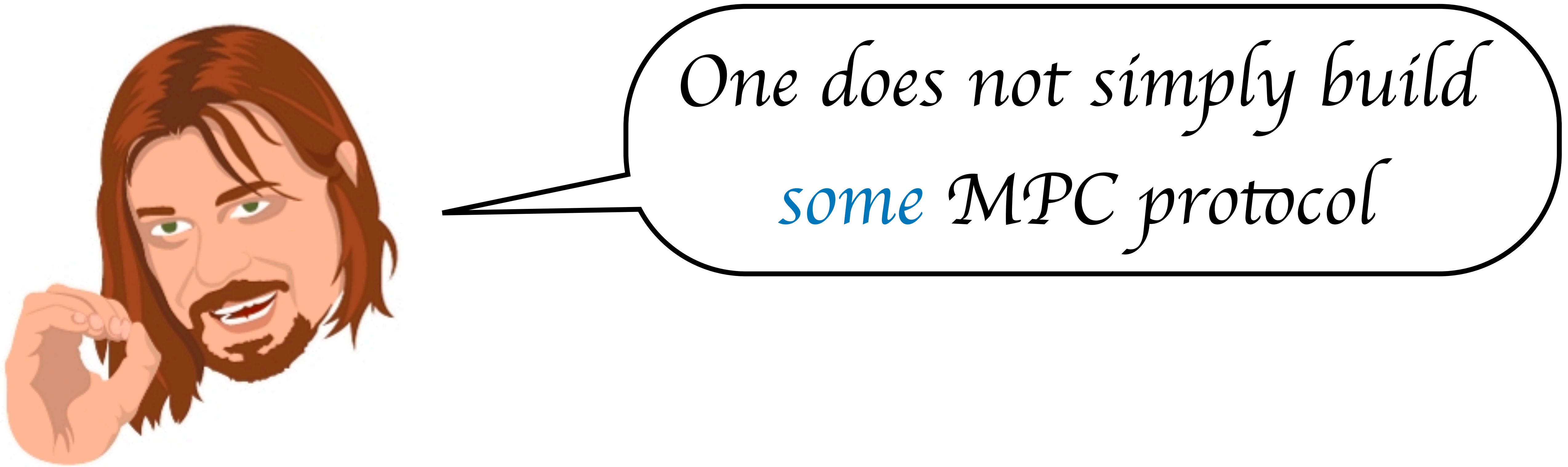

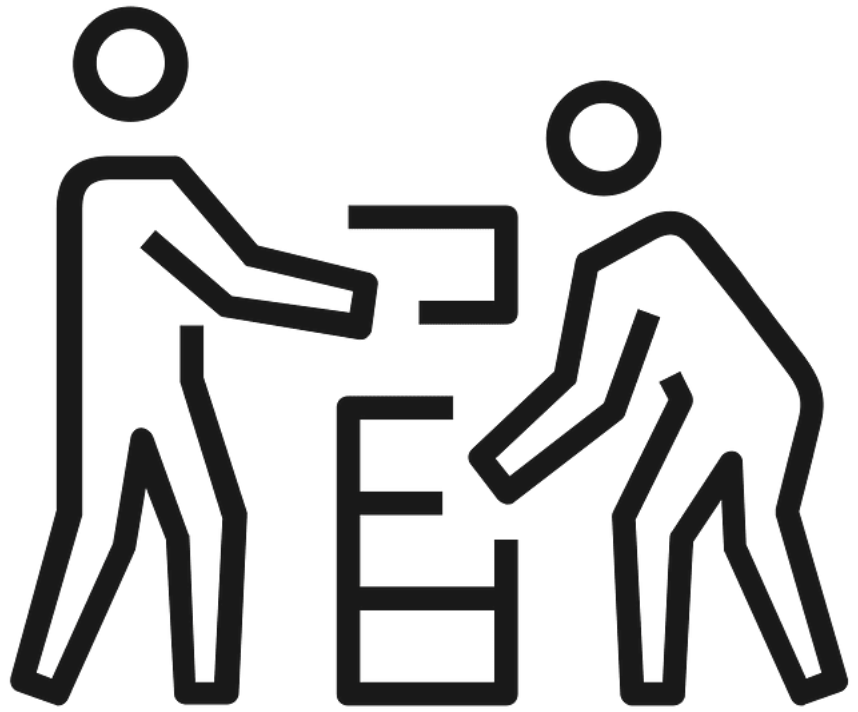

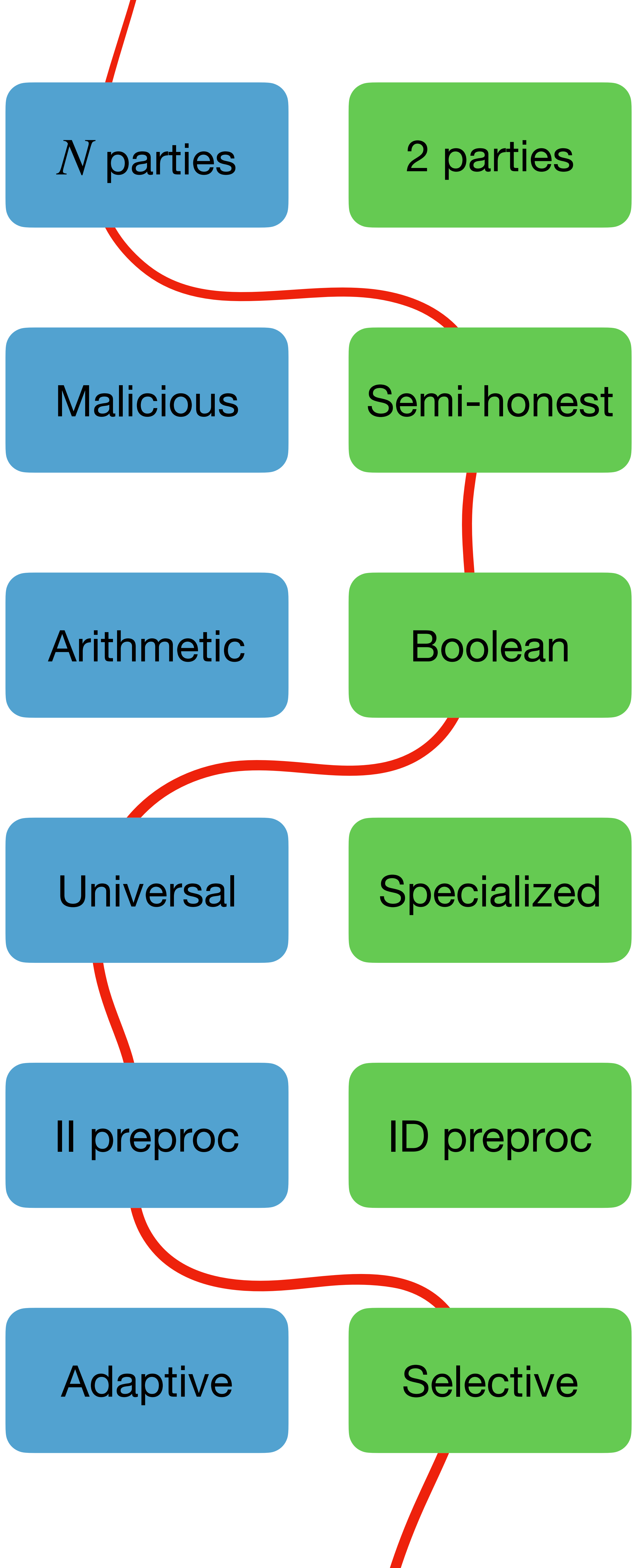

N parties	2 parties
Malicious	Semi-honest
Arithmetic	Boolean
Universal	Specialized
II preproc	ID preproc
Adaptive	Selective

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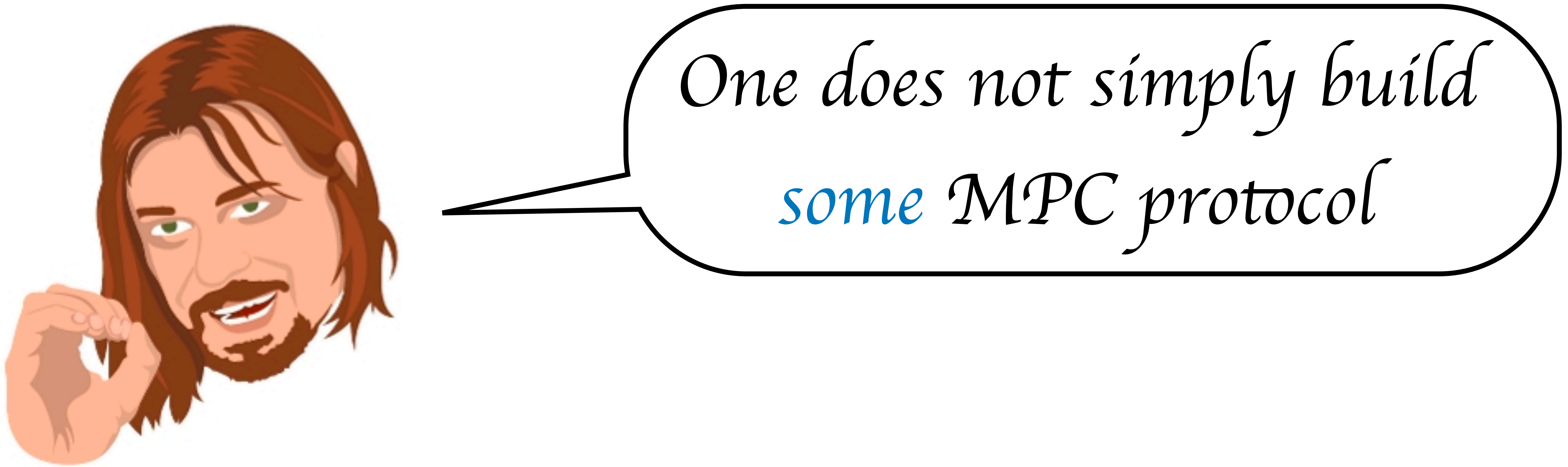
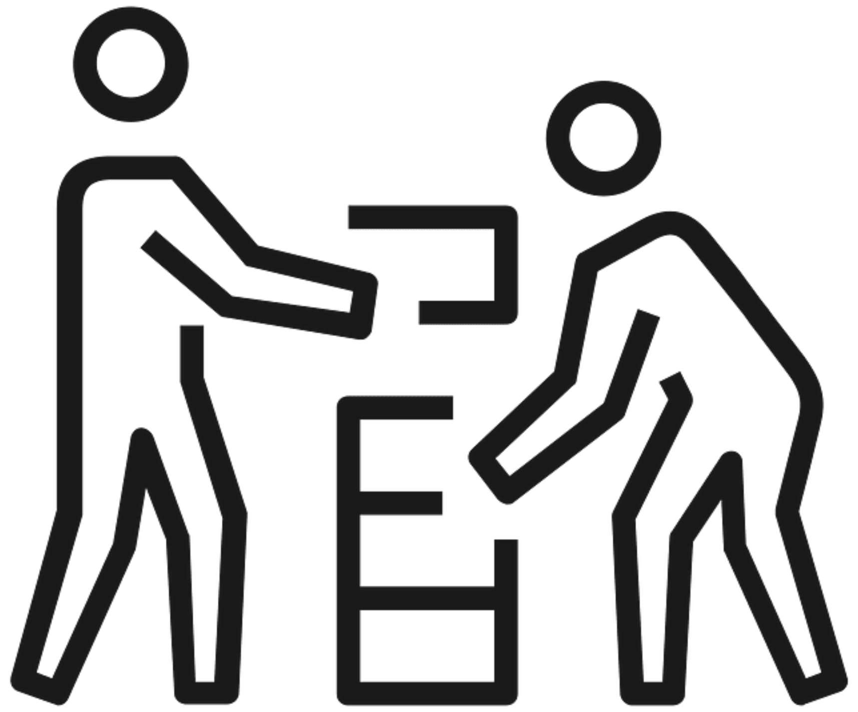
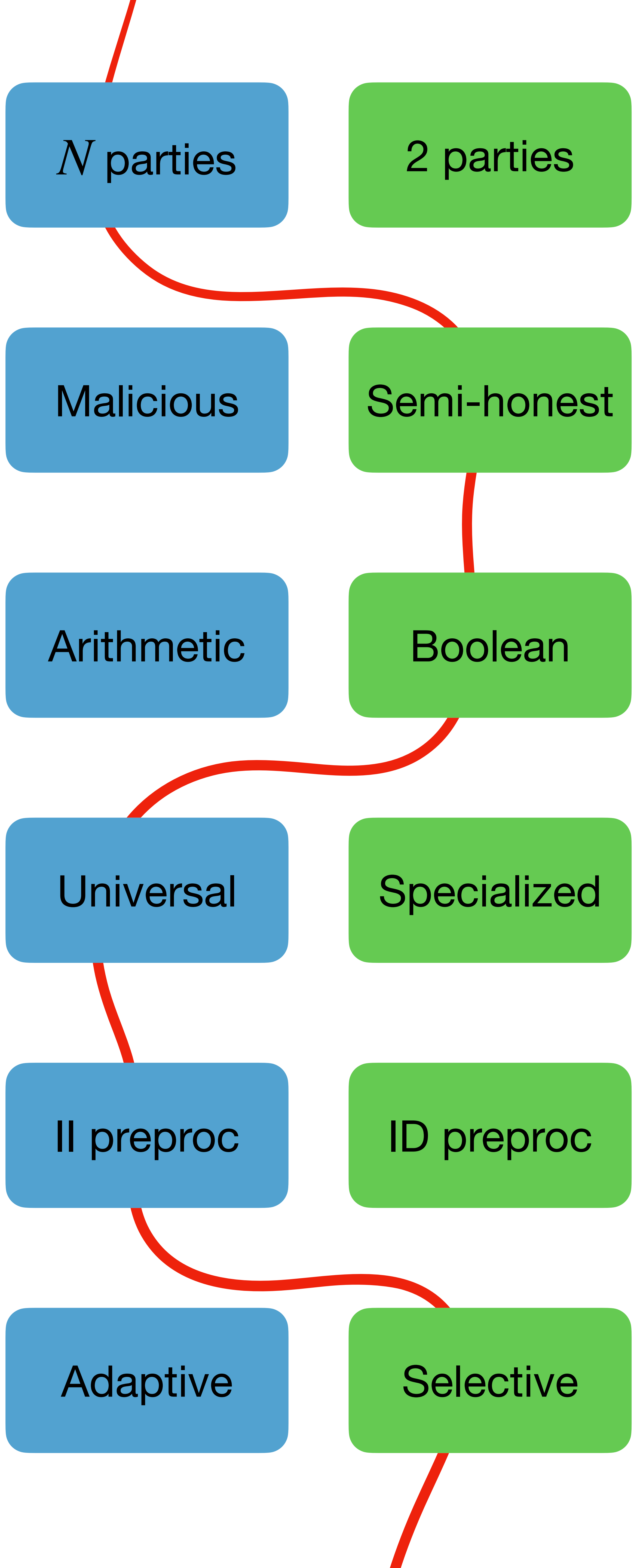


Depending on the application, you'll want:

VOLE, OT, OLE, bilinear correlations, Beaver triples, authenticated Beaver triples, daBits, circuit-dependent correlations, polynomial correlations, matrix triples, OTTT…

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Distributing $\langle C(r) \rangle$ *succinctly*

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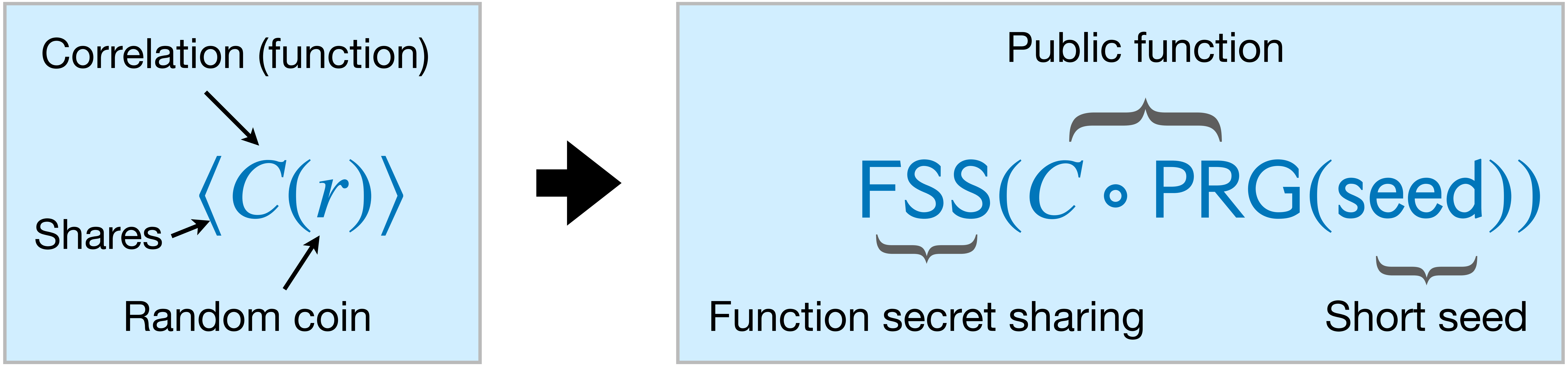

Correlation (function)

Shares

$$\langle C(r) \rangle$$

Random coin

$$\underbrace{\mathsf{FSS}}_{\text{Function secret sharing}}(\overbrace{C \circ \mathsf{PRG}}^{\text{Public function}}(\underbrace{\mathsf{seed}}_{\text{Short seed}}))$$

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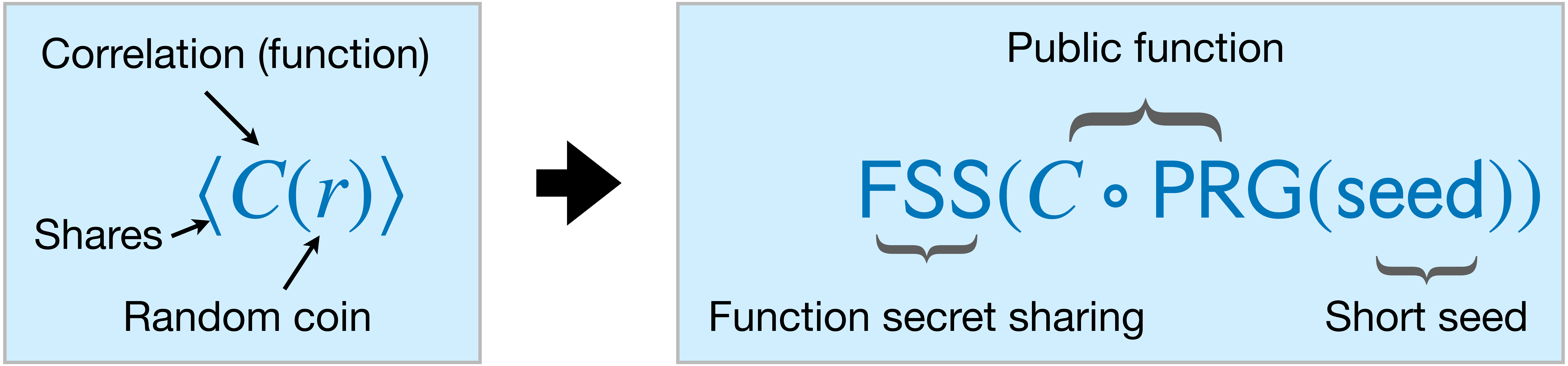

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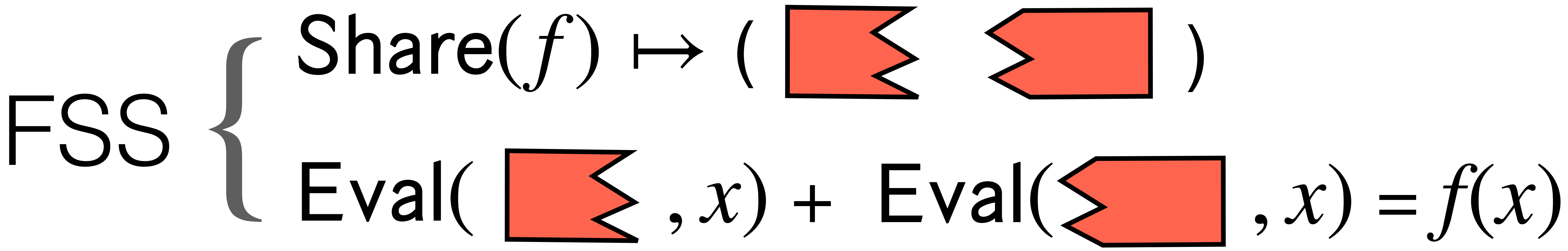
Function Secret Sharing

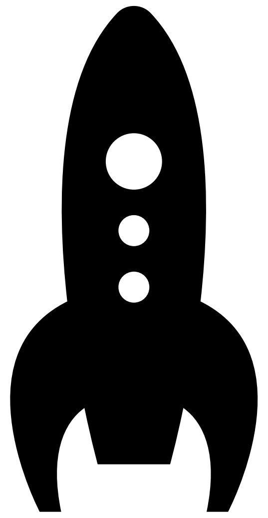


$$\text{FSS}\begin{cases}\text{Share}(f)\mapsto(\quad\quad)\\ \text{Eval}(\quad,x)+\text{Eval}(\quad,x)=f(x)\end{cases}$$

Function Secret Sharing

$$\text{FSS}\begin{cases}\text{Share}(f) \mapsto (\blacksquare \quad \blacksquare) \\ \text{Eval}(\blacksquare, x) + \text{Eval}(\blacksquare, x) = f(x)\end{cases}$$

Low end



OWF

Point functions

IT

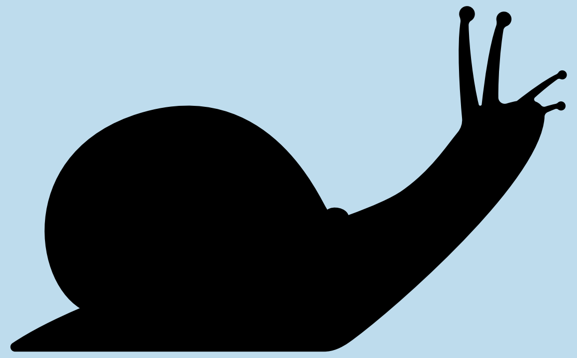
Lin. comb.

Mid end

DDH, DCR, class groups

NC^1

High end



iO, Multi-key threshold FHE

All polytime functions

Function Secret Sharing

FSS
- Share(f) $\mapsto$ (🟥 🟥)
- Eval(🟥 , x) + Eval(🟥 , x) = $f(x)$

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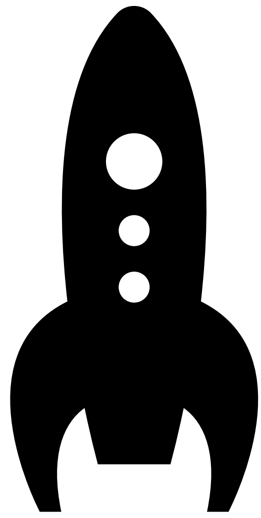

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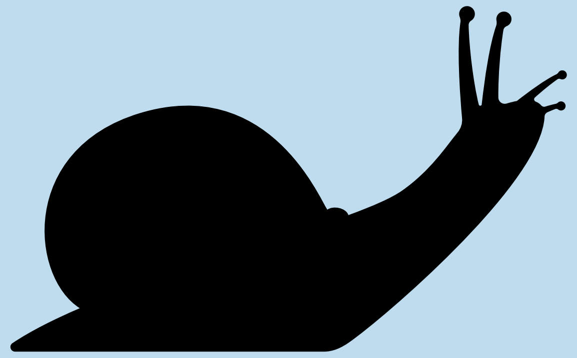

iO, Multi-key threshold FHE

All polytime functions

What Functions can be Shared?

Sharing an arbitrary function:

f

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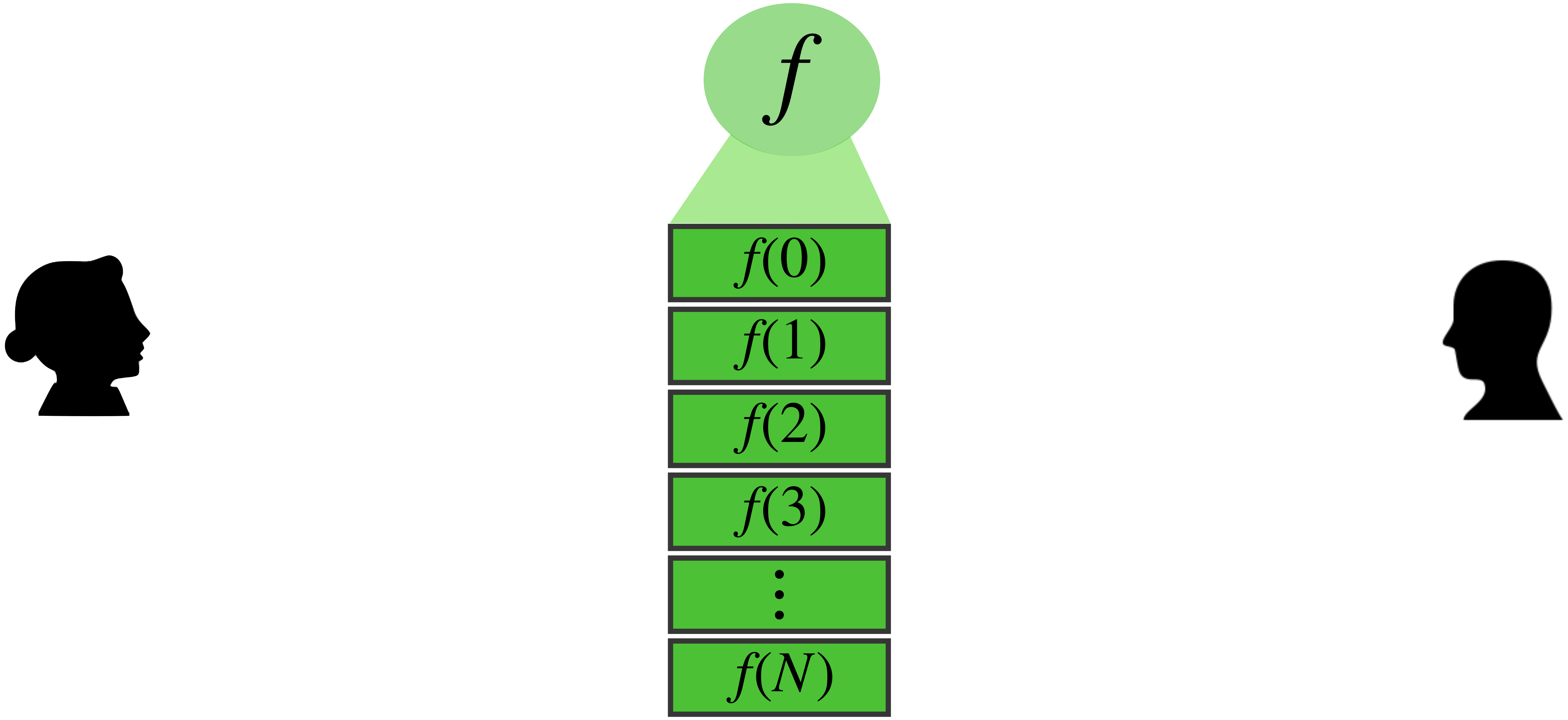


f

$f(0)$

$f(1)$

$f(2)$

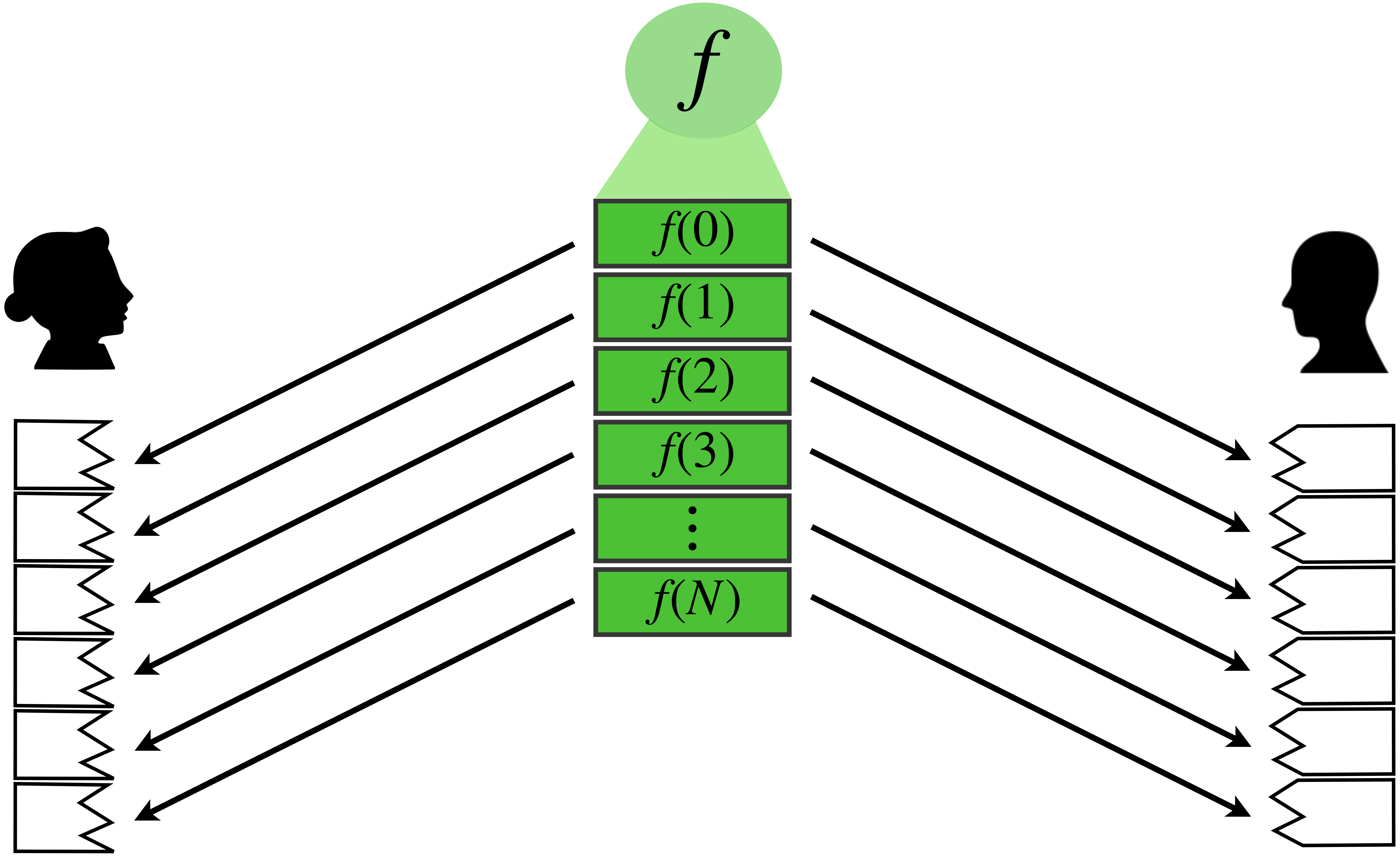
$f(3)$

$\vdots$

$f(N)$

What Functions can be Shared?

Sharing an arbitrary function:



What Functions can be Shared?

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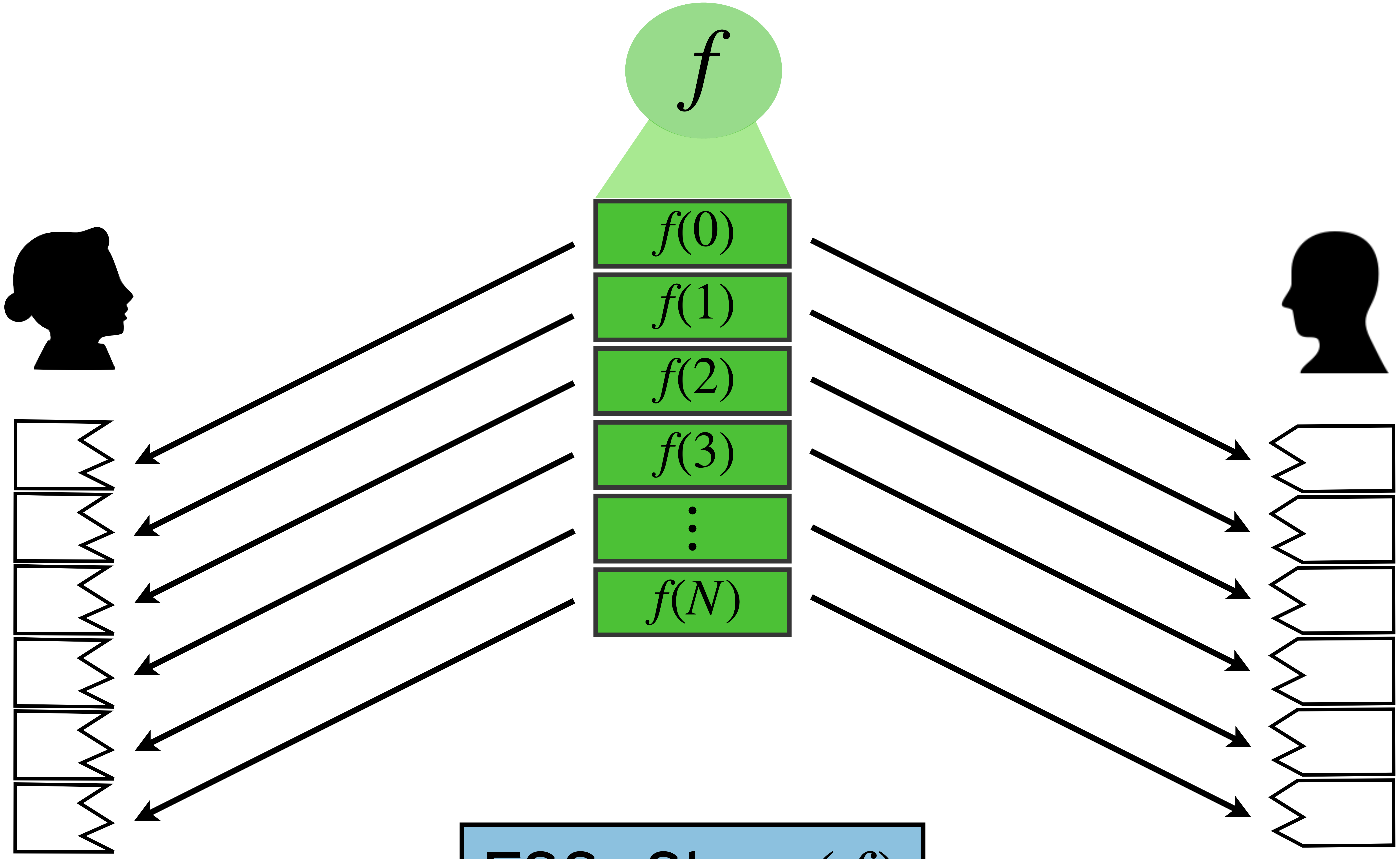

FSS . Share(f)

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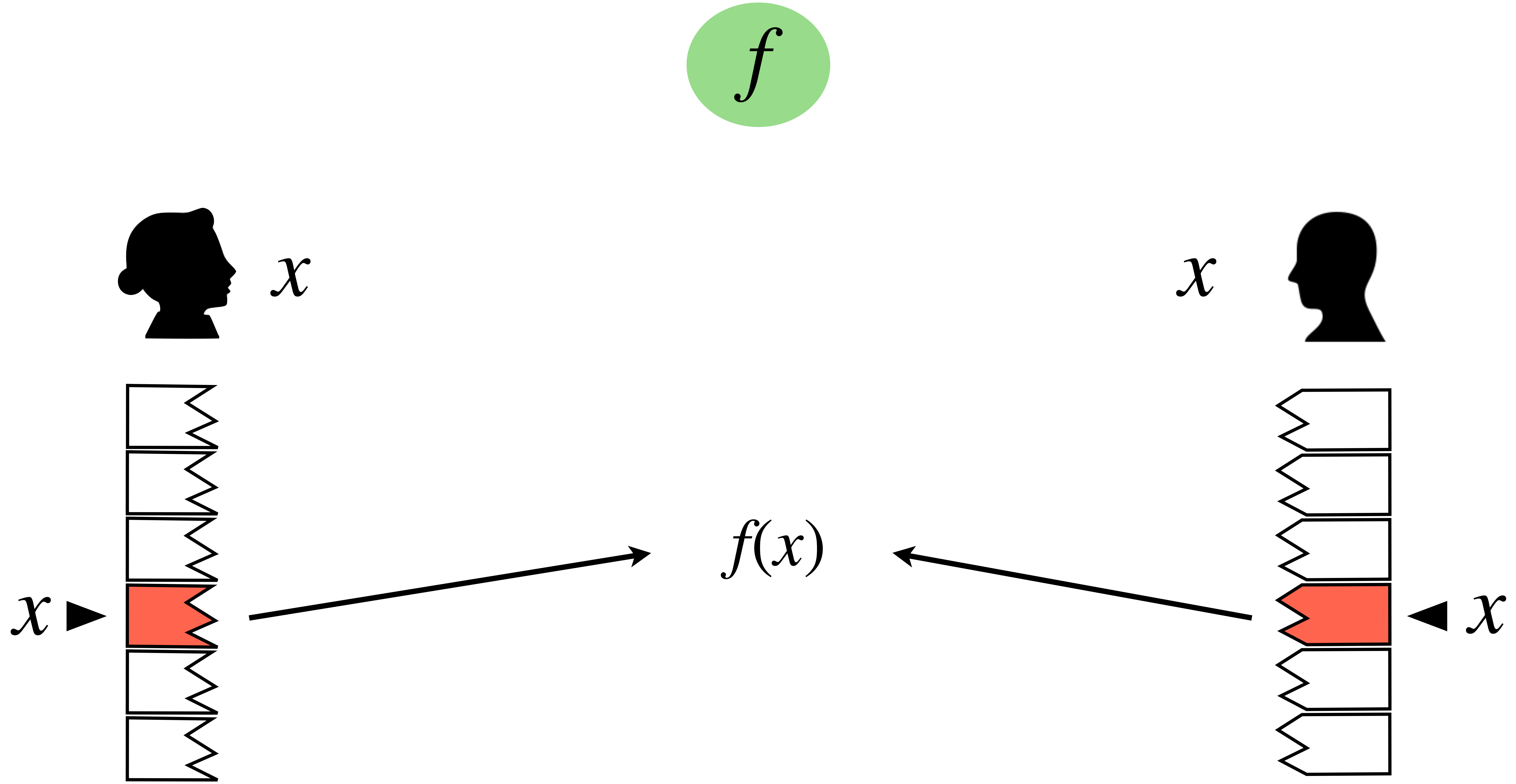

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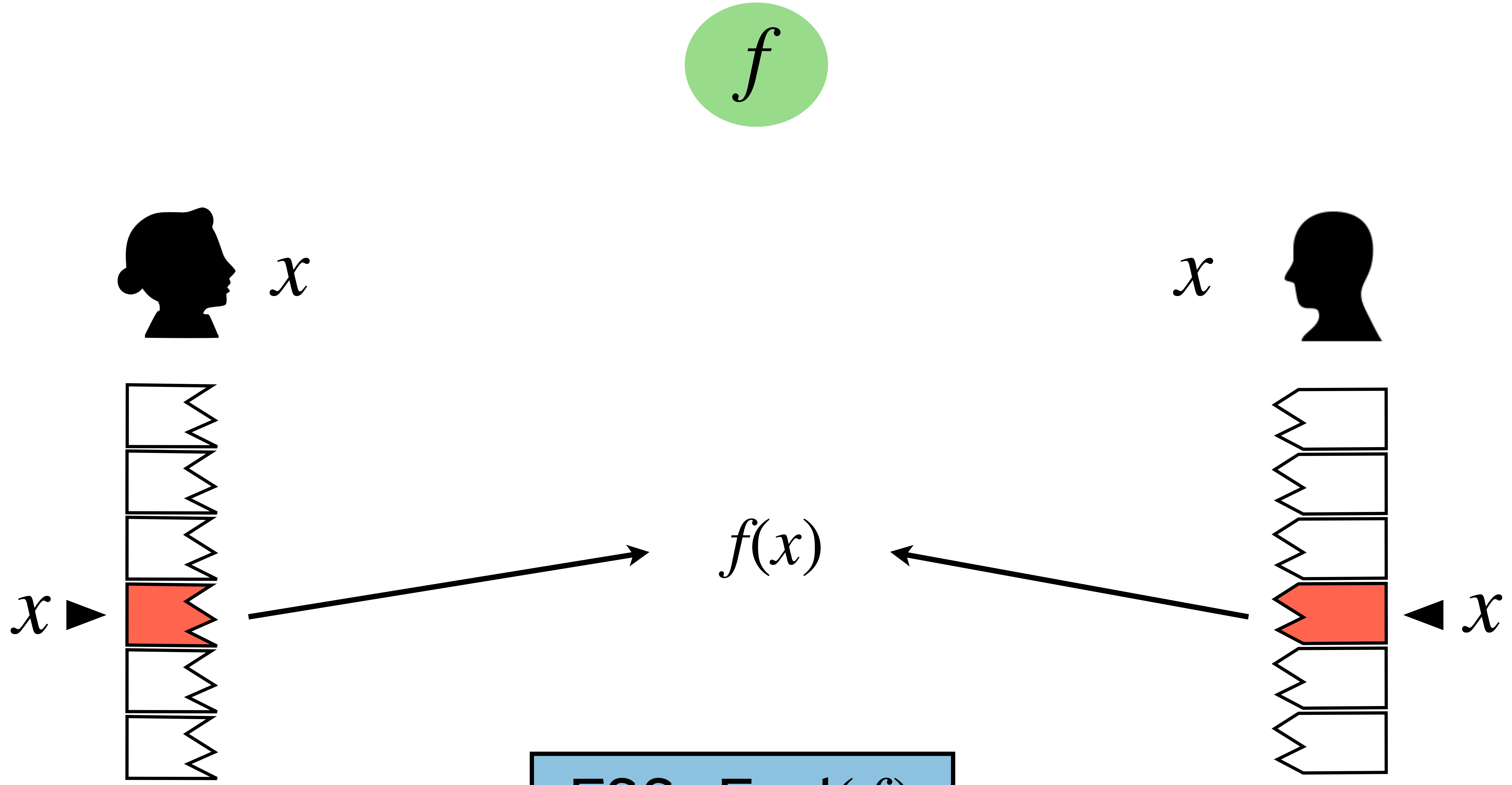

FSS . Eval(f)

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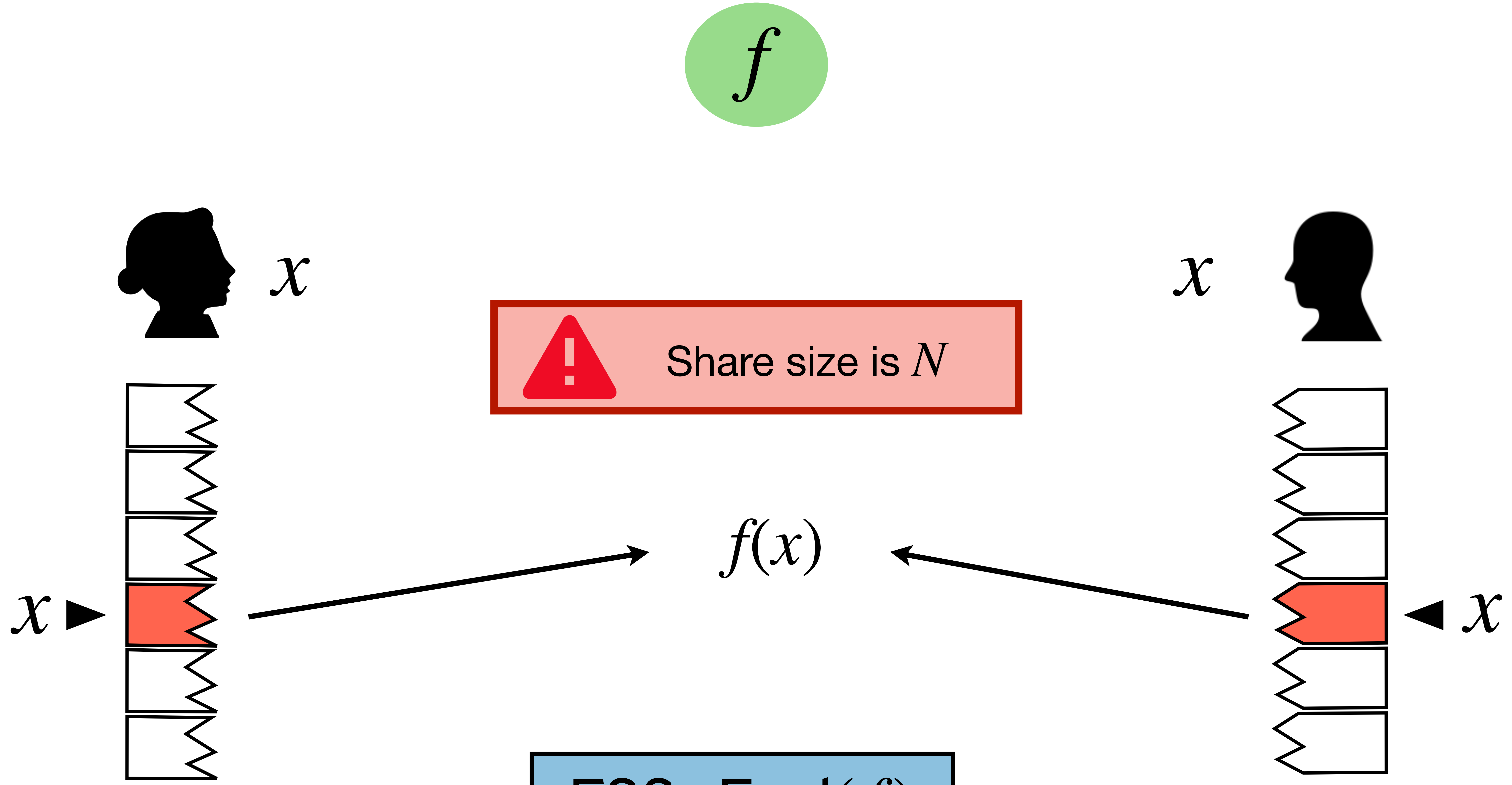

Share size is N

FSS . Eval(f)

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succinctly

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$$\forall x, f(x) = 0$$

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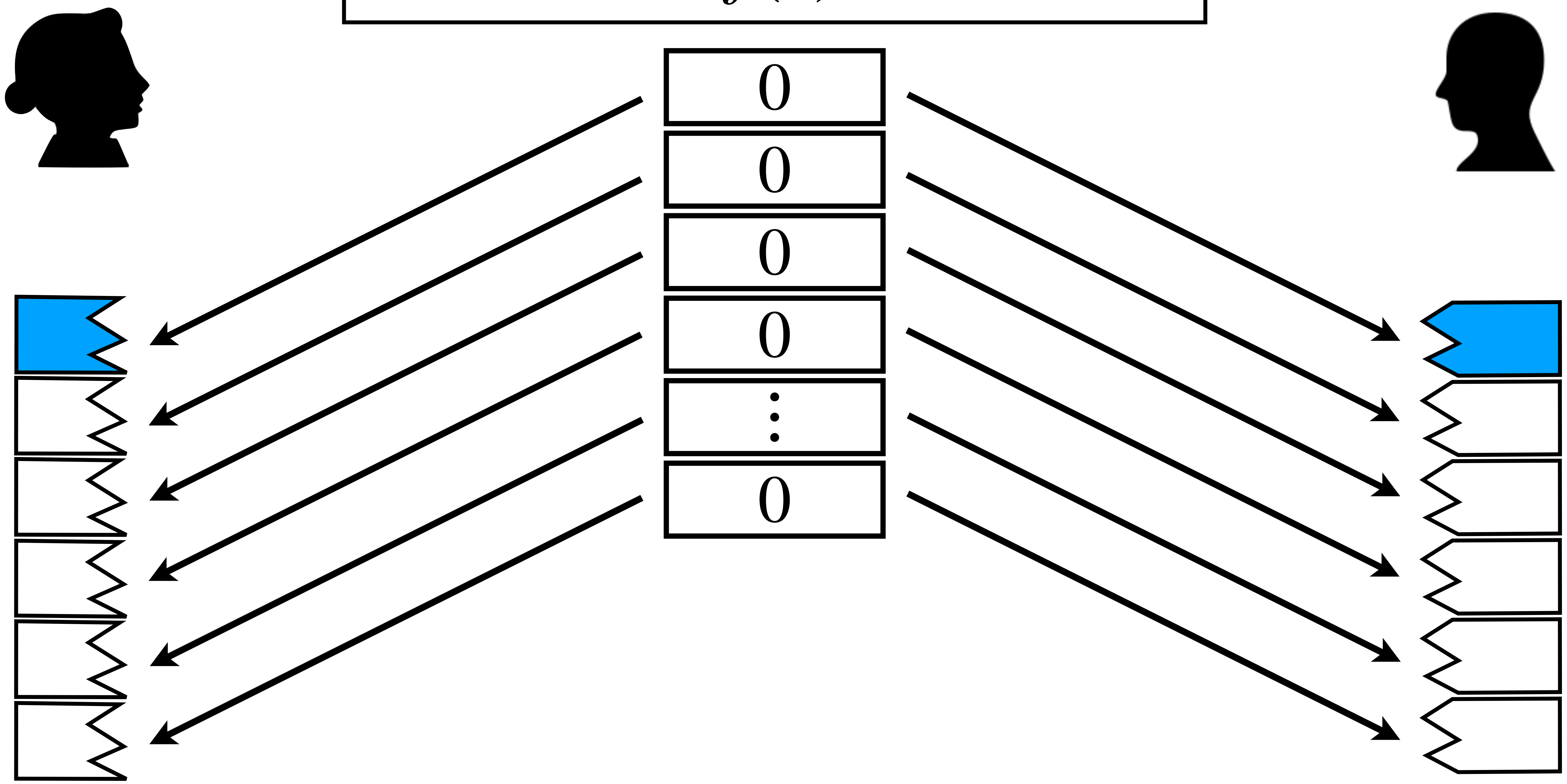

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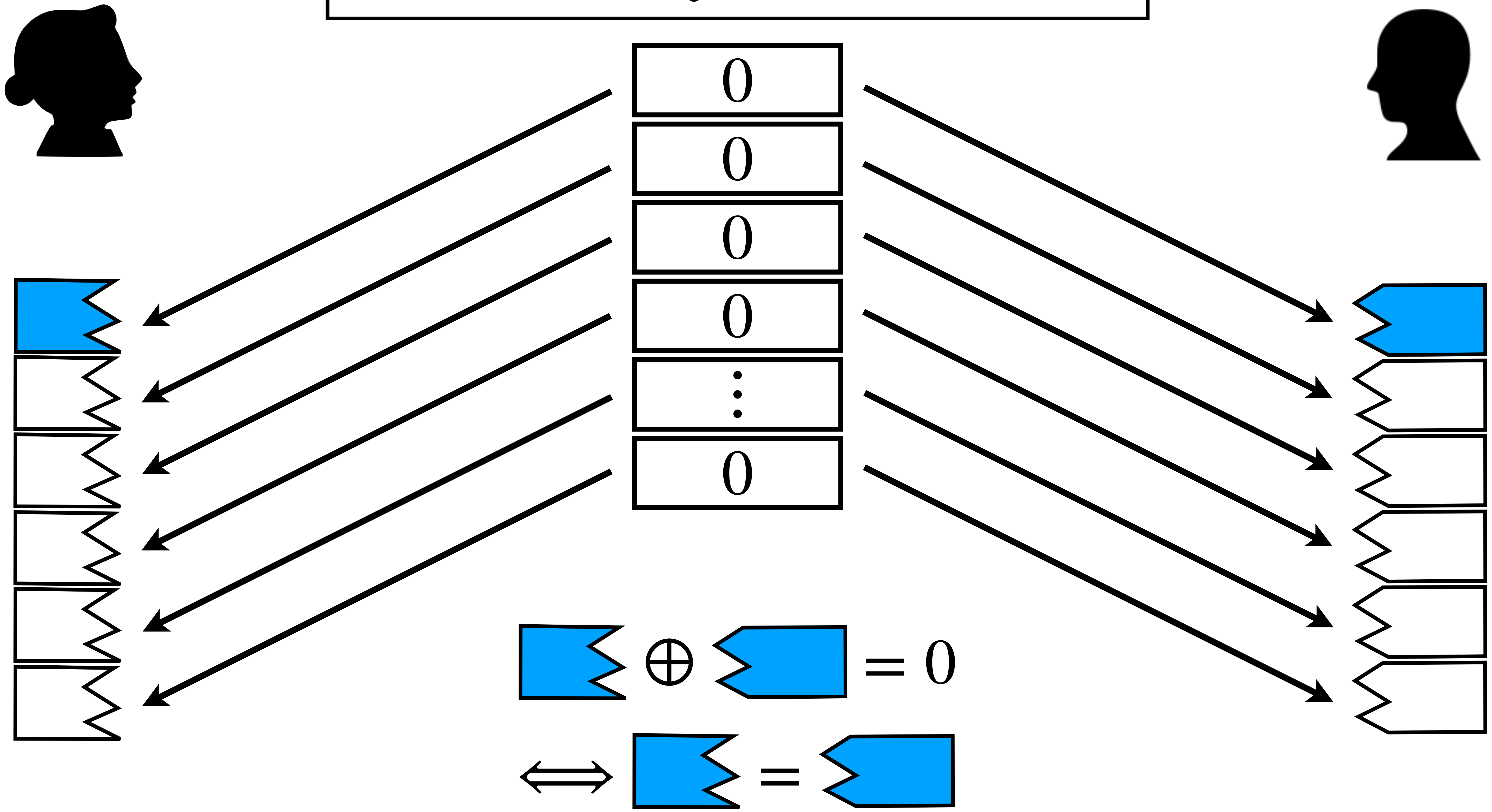

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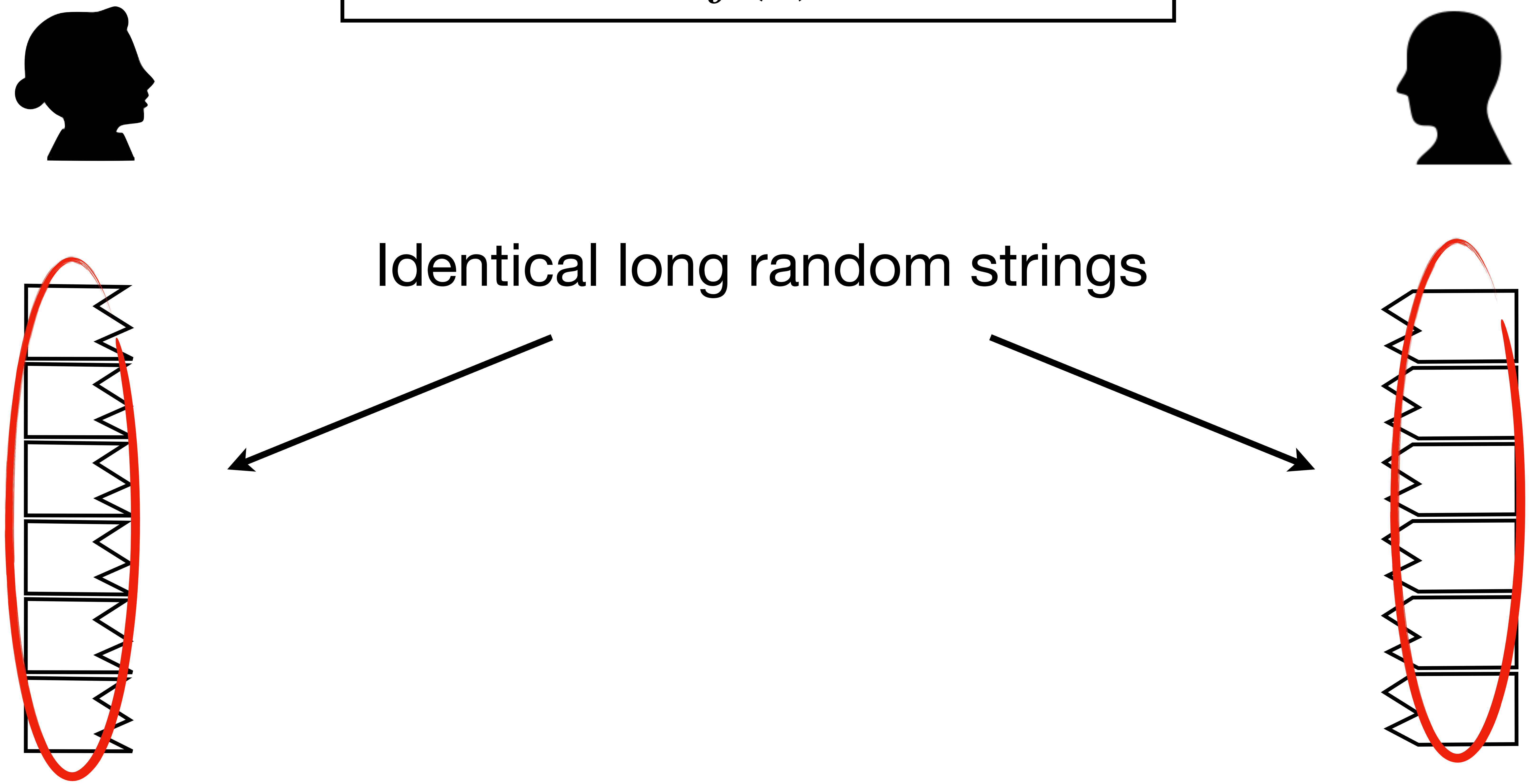

Identical long random strings

What Functions can be Shared?

succinctly

Sharing the all zero function:

$$\forall x, f(x) = 0$$

We just need a PRG!

What Functions can be Shared?

succinctly

Sharing a *point function*

$$f_{\alpha,\beta}(x \neq \alpha) = 0, f(\alpha) = \beta$$

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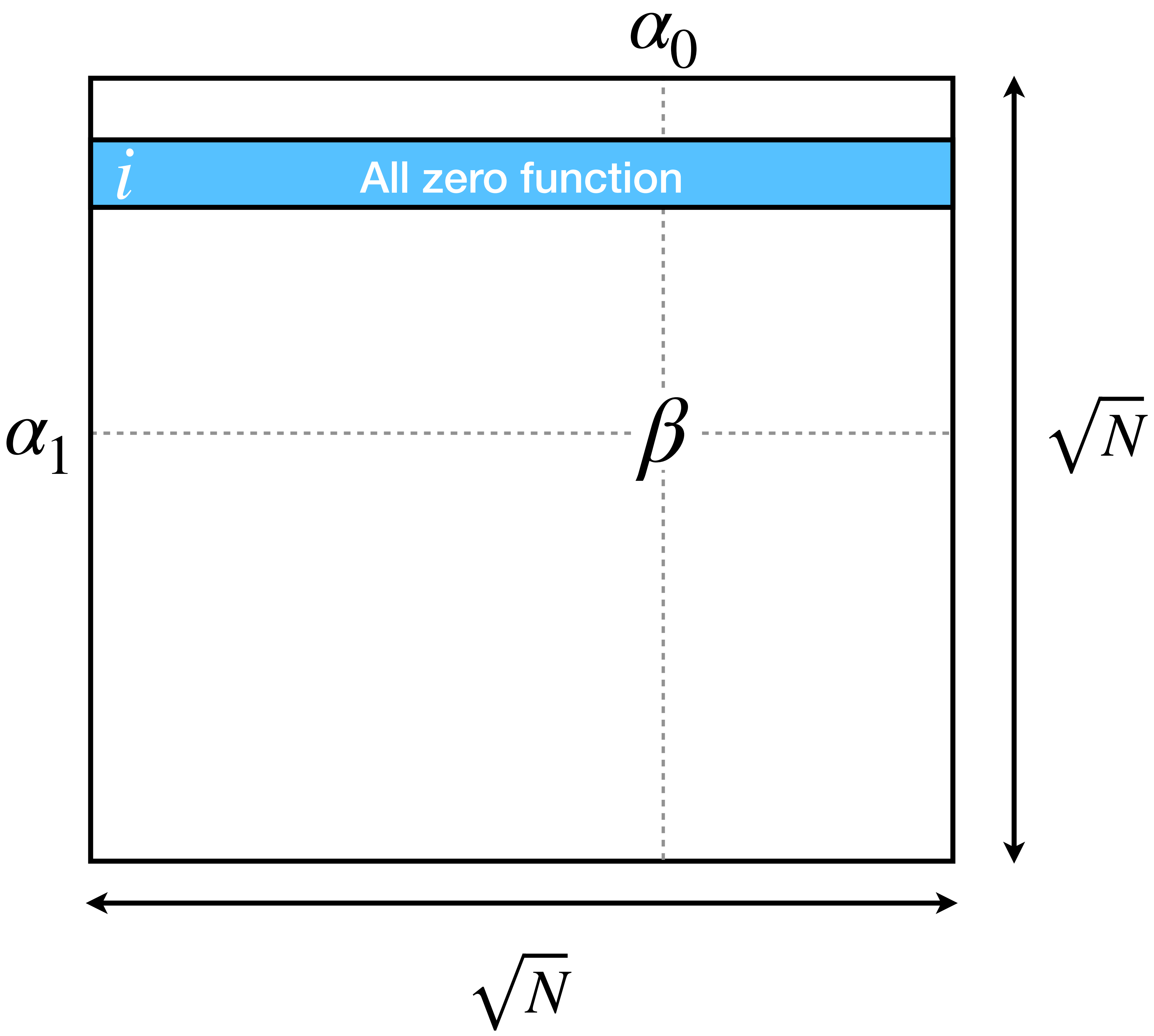

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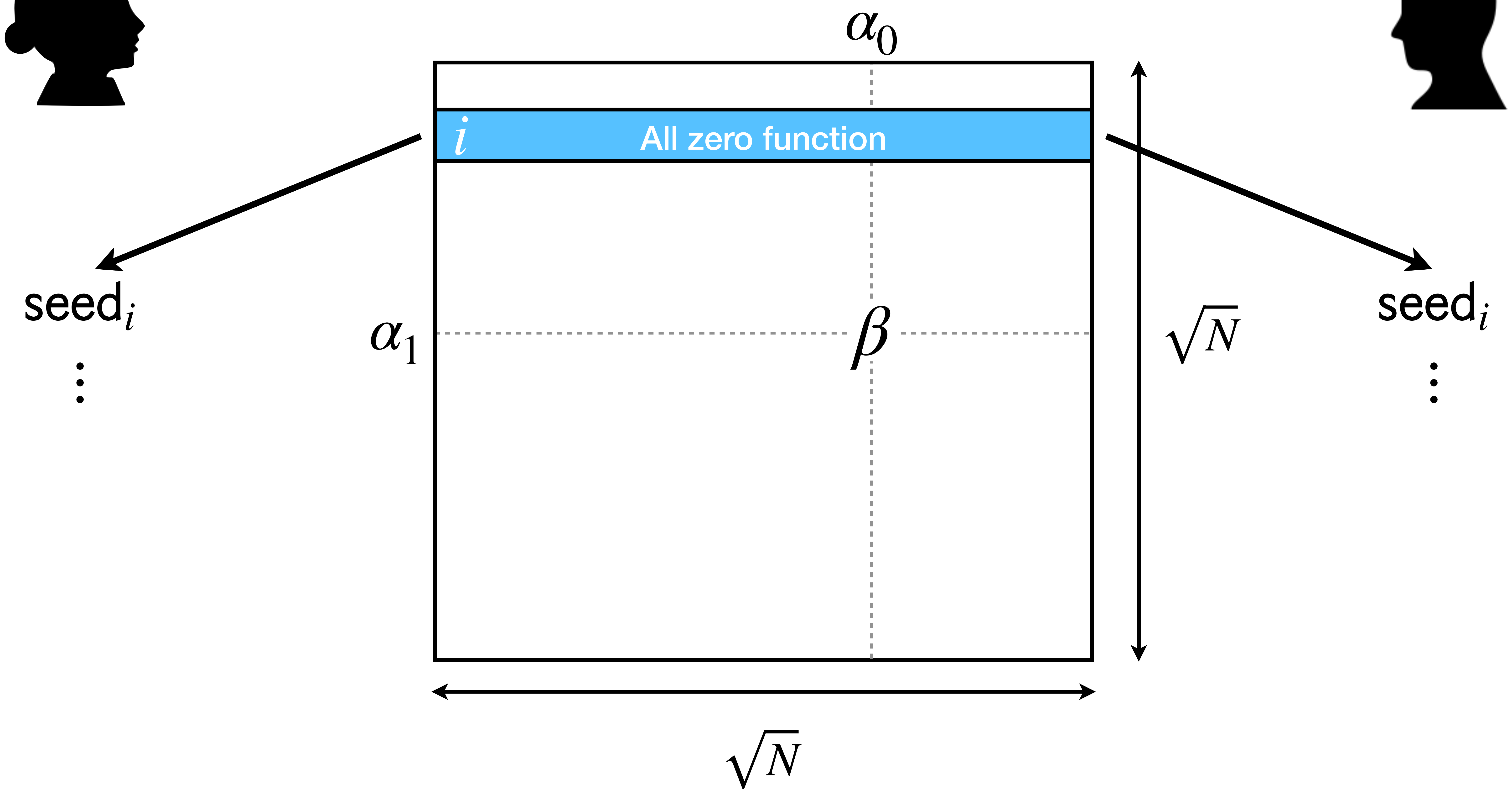

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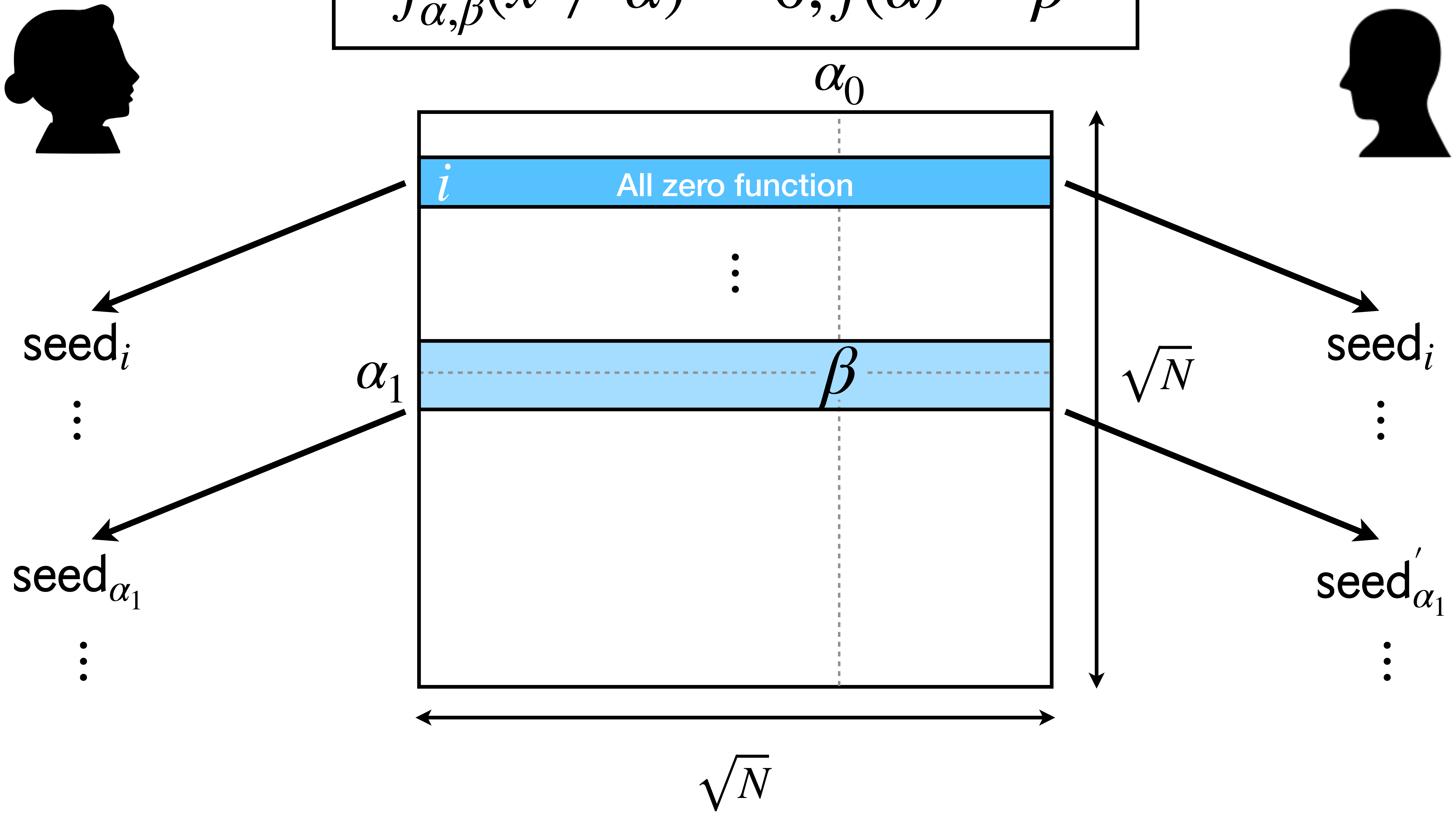
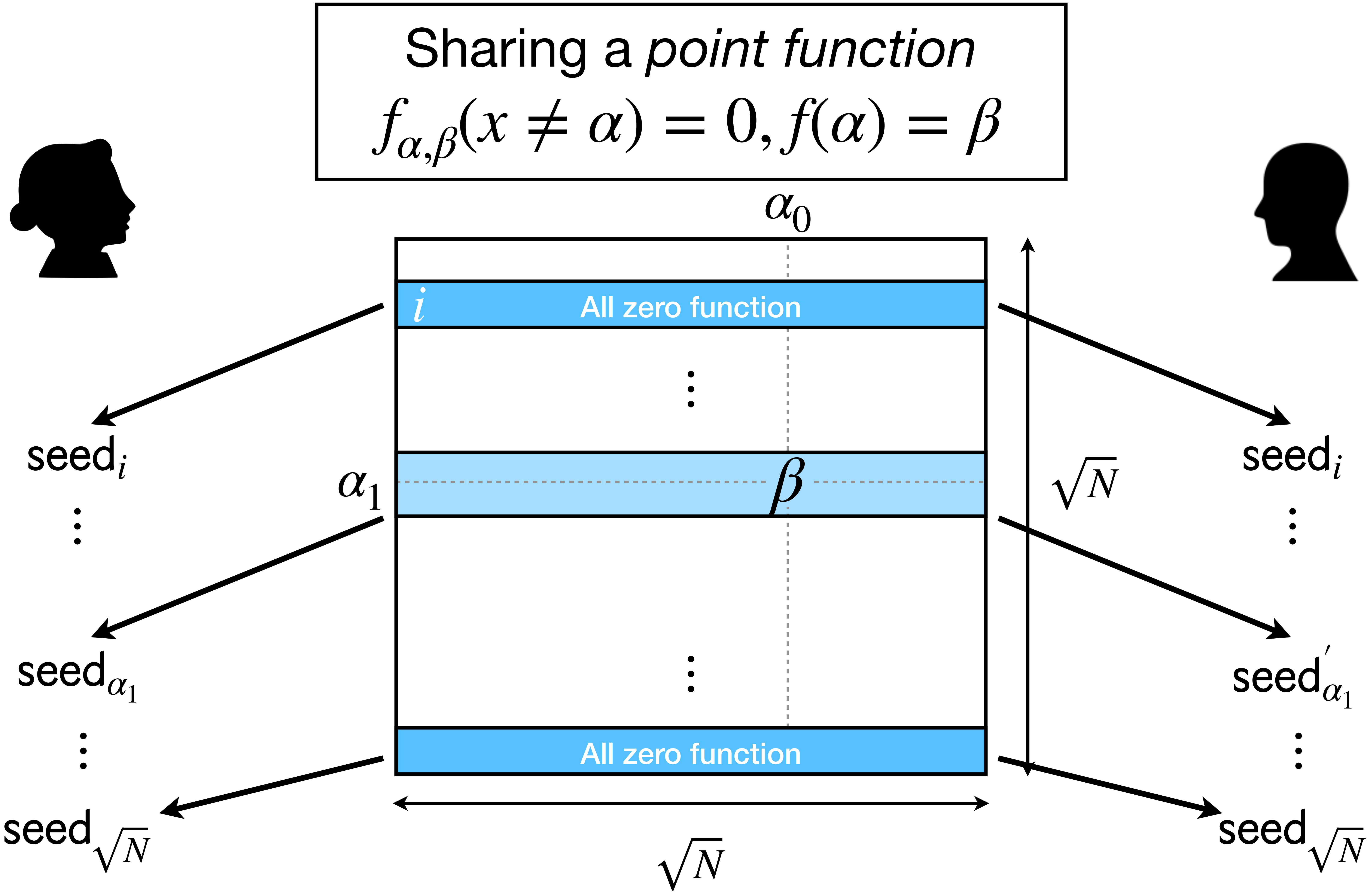


What Functions can be $\underset{\textbf{succinctly}}{\wedge}$ Shared?

succinctly



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seed$_i$

⋮

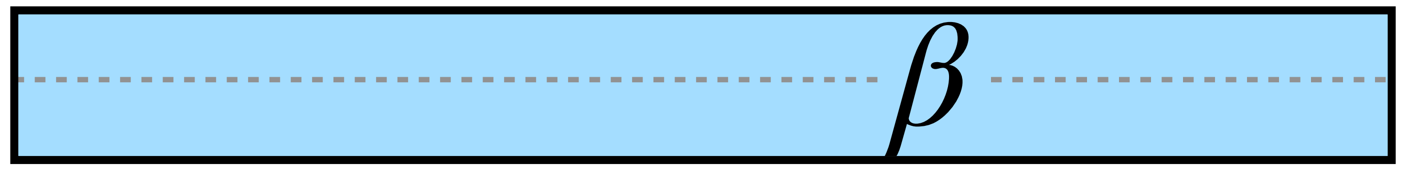
seed$_{\alpha_1}$

⋮

seed$_{\sqrt{N}}$

Public

$\Delta =$



$\oplus$ PRG(seed$_{\alpha_1}$) $\oplus$ PRG(seed$'_{\alpha_1}$)

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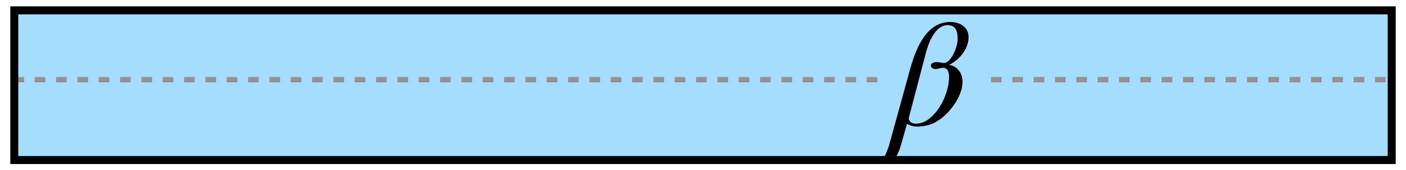

$\oplus \mathrm{PRG}(\mathrm{seed}_{\alpha_1}) \oplus \mathrm{PRG}(\mathrm{seed}'_{\alpha_1})$

b_i seed$_i$

$\vdots$

b_{α_1} seed$_{\alpha_1}$

$\vdots$

$b_{\sqrt{N}}$ seed$_{\sqrt{N}}$

seed$_i$ b_i

$\vdots$

seed$'_{\alpha_1}$ $1 - b_{\alpha_1}$

$\vdots$

seed$_{\sqrt{N}}$ $b_{\sqrt{N}}$

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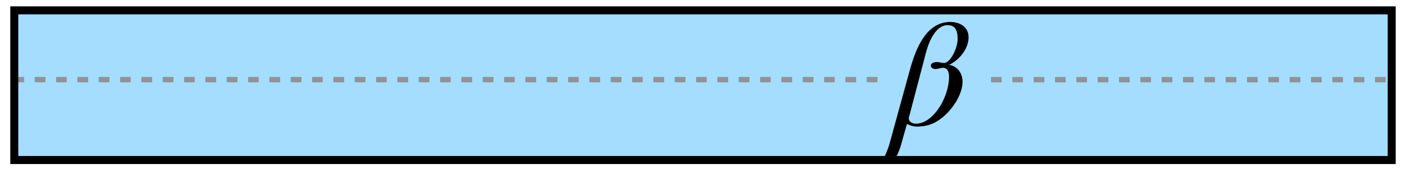

$\oplus$ PRG(seed$_{\alpha_1}$) $\oplus$ PRG(seed$'_{\alpha_1}$)

Eval :

PRG(seed$_j$) $\oplus \Delta \cdot b_j$

seed$_i$ b_i

$\vdots$

seed$'_{\alpha_1}$ $1 - b_{\alpha_1}$

$\vdots$

seed$_{\sqrt{N}}$ $b_{\sqrt{N}}$

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$$f_{\alpha,\beta}(x \neq \alpha) = 0, f(\alpha) = \beta$$

Recursing:

b_i seed$_i$

$\vdots$

b_{α_1} seed$_{\alpha_1}$

$\vdots$

$b_{\sqrt{N}}$ seed$_{\sqrt{N}}$

Giving Δ and sharing the b_i's are both essentially sharing a $\sqrt{N}$-size point function again: we can recurse the process!

seed$_i$ b_i

$\vdots$

seed$'_{\alpha_1}$ $1 - b_{\alpha_1}$

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This + later improvements [BGI16]:
FSS for point functions with keys of size
$O(\lambda \cdot \log N)$

Back to the PCG Template

$$\underbrace{\mathsf{FSS}}_{\text{Function secret sharing}}(\overbrace{C \circ \mathsf{PRG}}^{\text{Public function}}(\underbrace{\mathsf{seed}}_{\text{Short seed}}))$$

We can succinctly share point functions

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Secret sharing is additively homomorphic!

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Linear combinations of

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Linear combinations of

Are there any PRGs in this class?

LPN to the Rescue

The LPN assumption - primal

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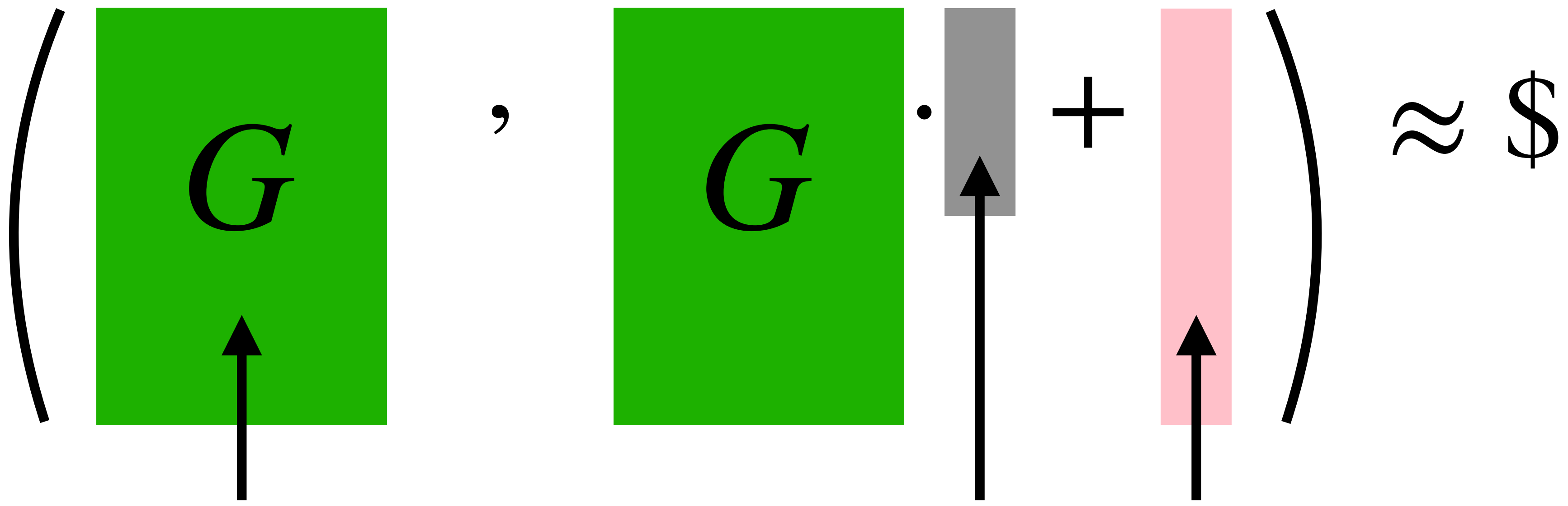


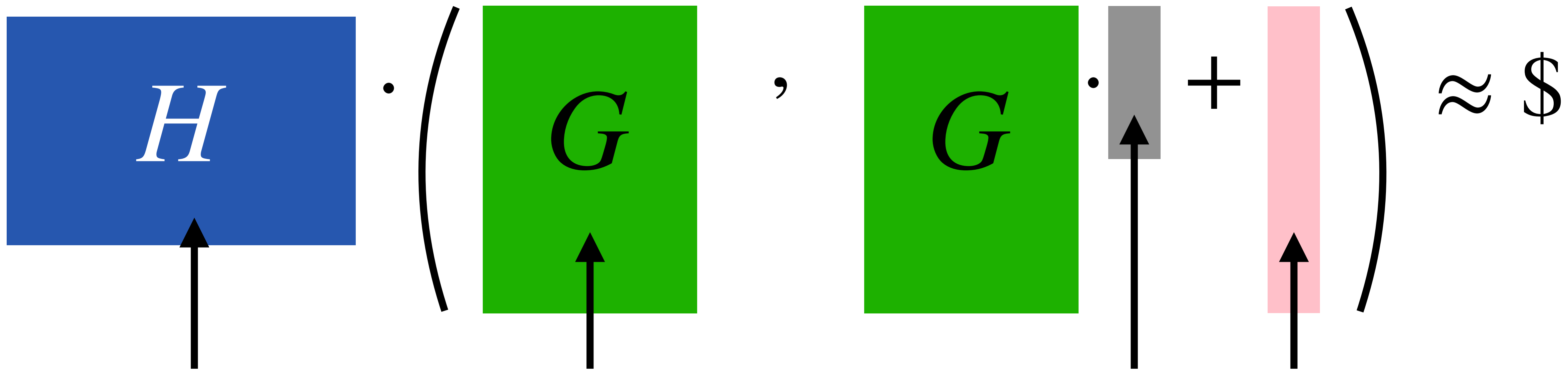
Random matrix

Short secret

Sparse noise

LPN to the Rescue

The LPN assumption - primal



Parity-check matrix of G

Random matrix

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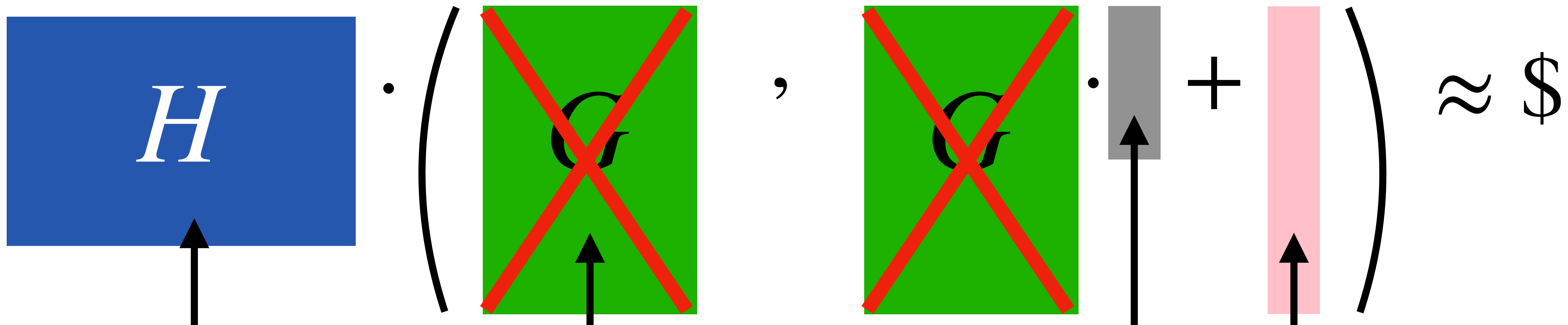

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$\text{PRG} : (\alpha_i)_{i\leq t} \mapsto H \cdot \sum_{i=1}^{t} \vec{u}_{\alpha_i}$, where $\vec{u}_{\alpha_i}$ is the unit vector with a 1 at α_i

LPN to the Rescue

The LPN assumption - dual

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PRG : $(\alpha_i)_{i\leq t} \mapsto \underbrace{H \cdot \sum_{i=1}^{t}}_{\text{Linear combination}} \overrightarrow{u}_{\alpha_i}$, where $\overrightarrow{u}_{\alpha_i}$ is the unit vector with a 1 at α_i

LPN to the Rescue

The LPN assumption - dual

LPN yields a simple PRG in the class:

$$\text{PRG} : (\alpha_i)_{i \leq t} \mapsto H \cdot \underbrace{\sum_{i=1}^{t}}_{\text{Linear combination}} \underbrace{\vec{u}_{\alpha_i}}_{\text{(Truth table of) point functions}}, \text{ where } \vec{u}_{\alpha_i} \text{ is the unit vector with a 1 at } \alpha_i$$

Back to the PCG Template Again

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We have FSS for a class that contains a PRG

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The heavy lifting in the many subsequent works boils down to:

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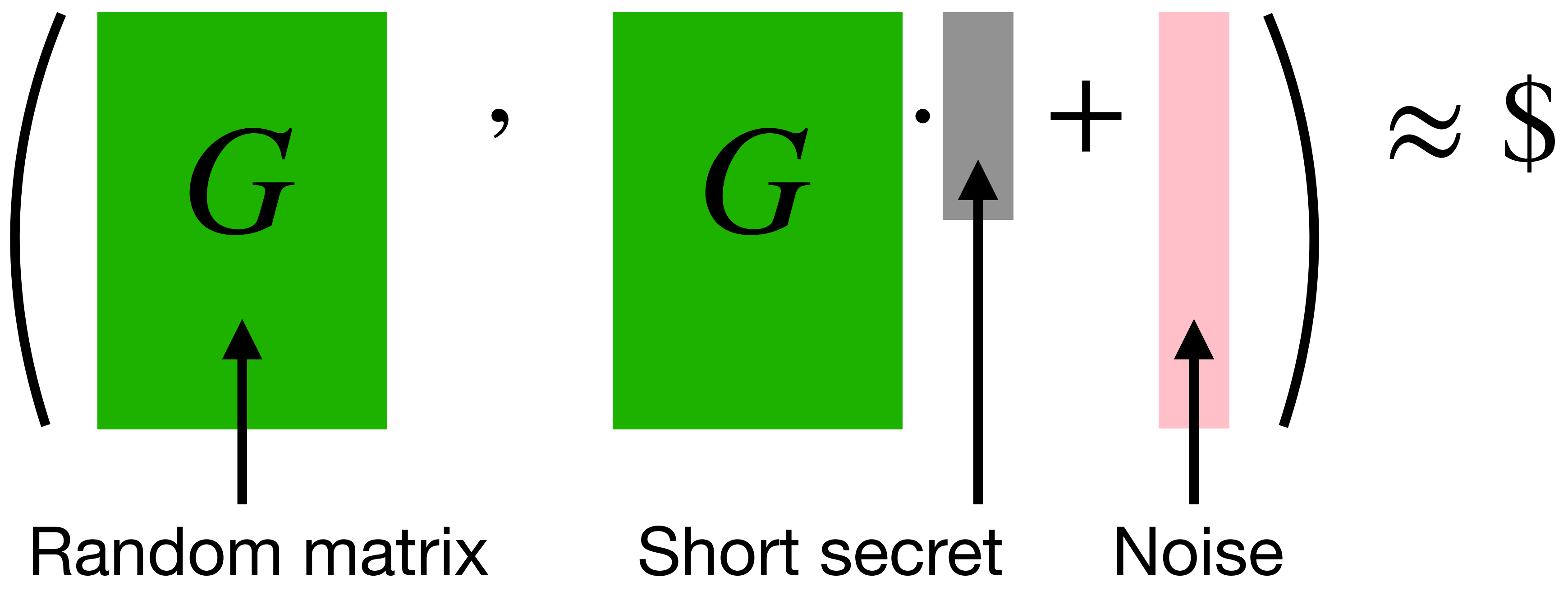
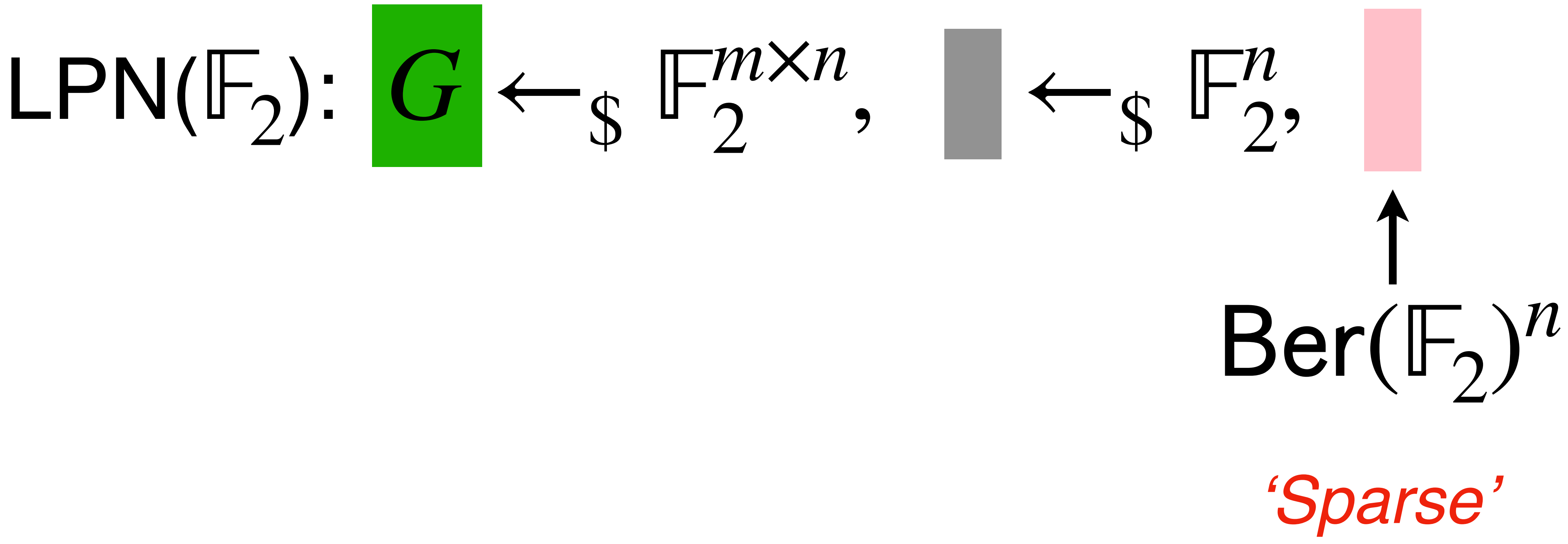
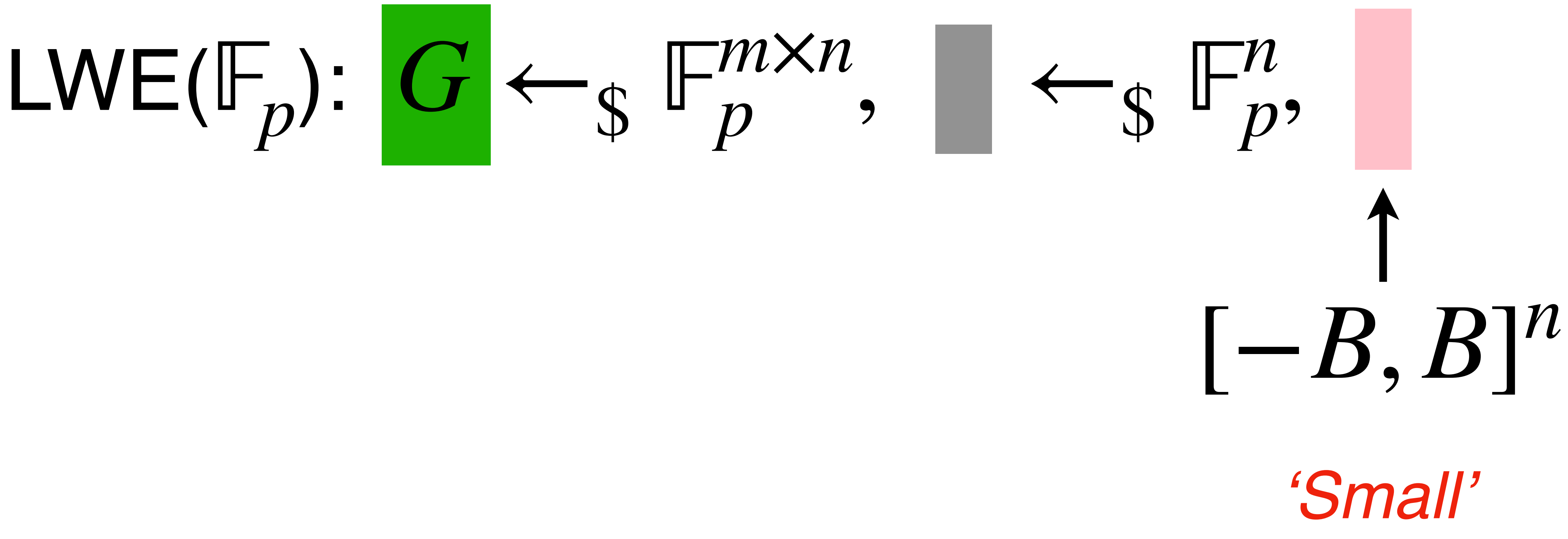
Digression: LPN versus LWE

LPN and LWE







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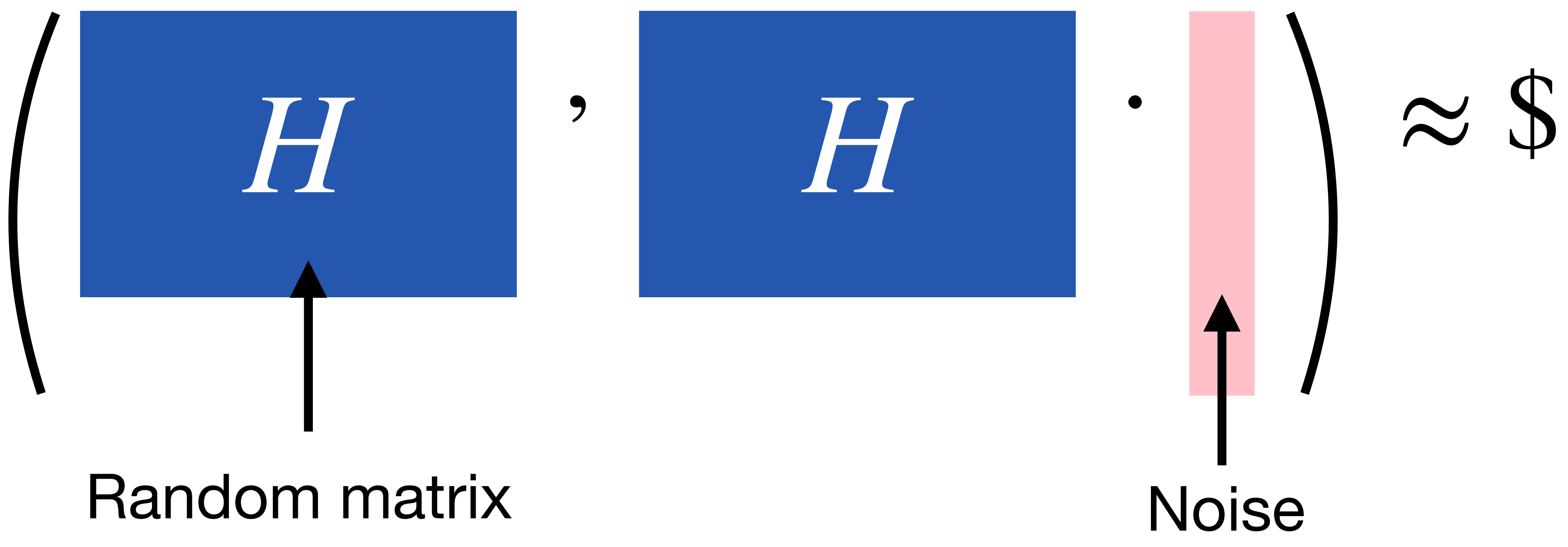
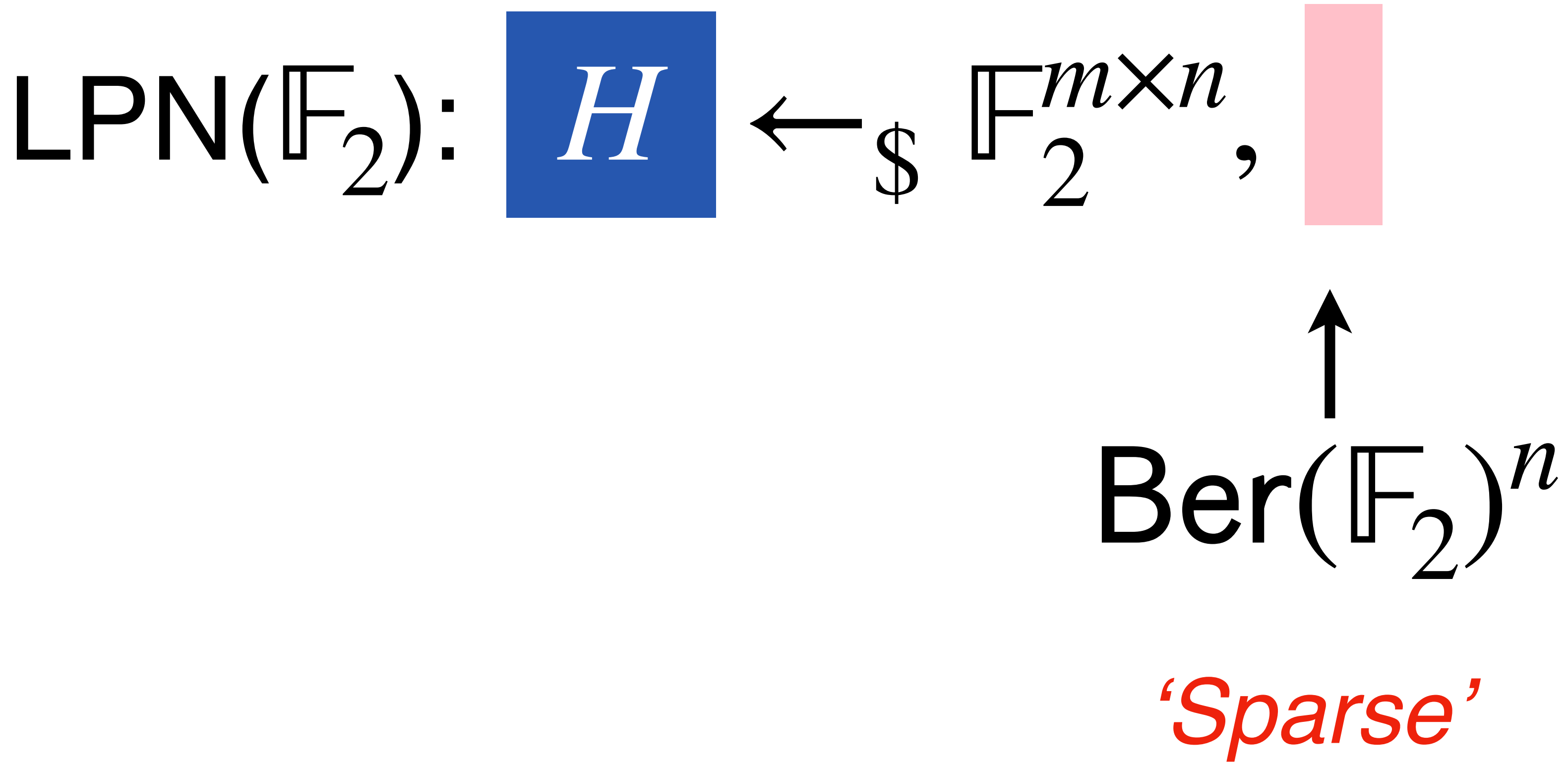
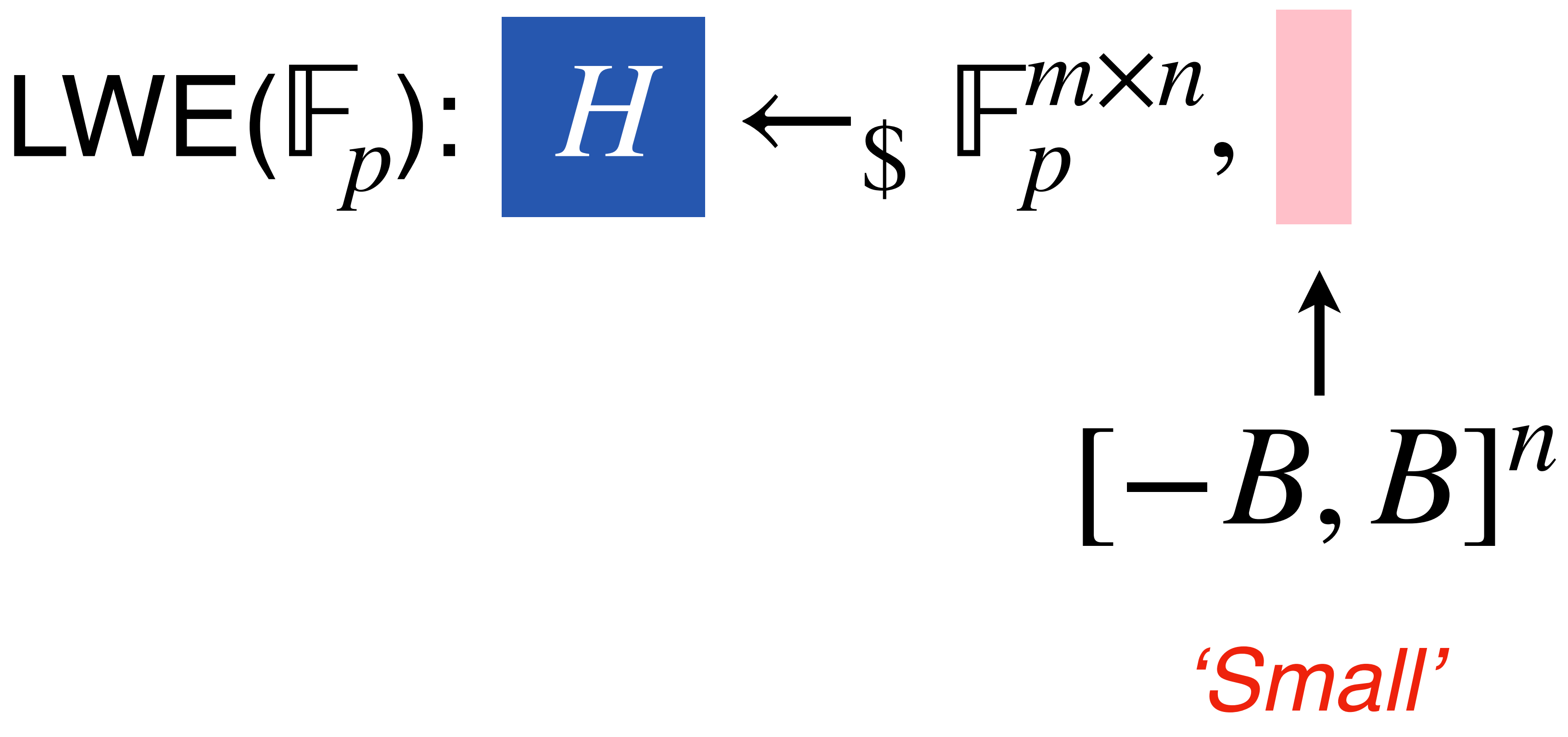




'Sparse'



'Small'

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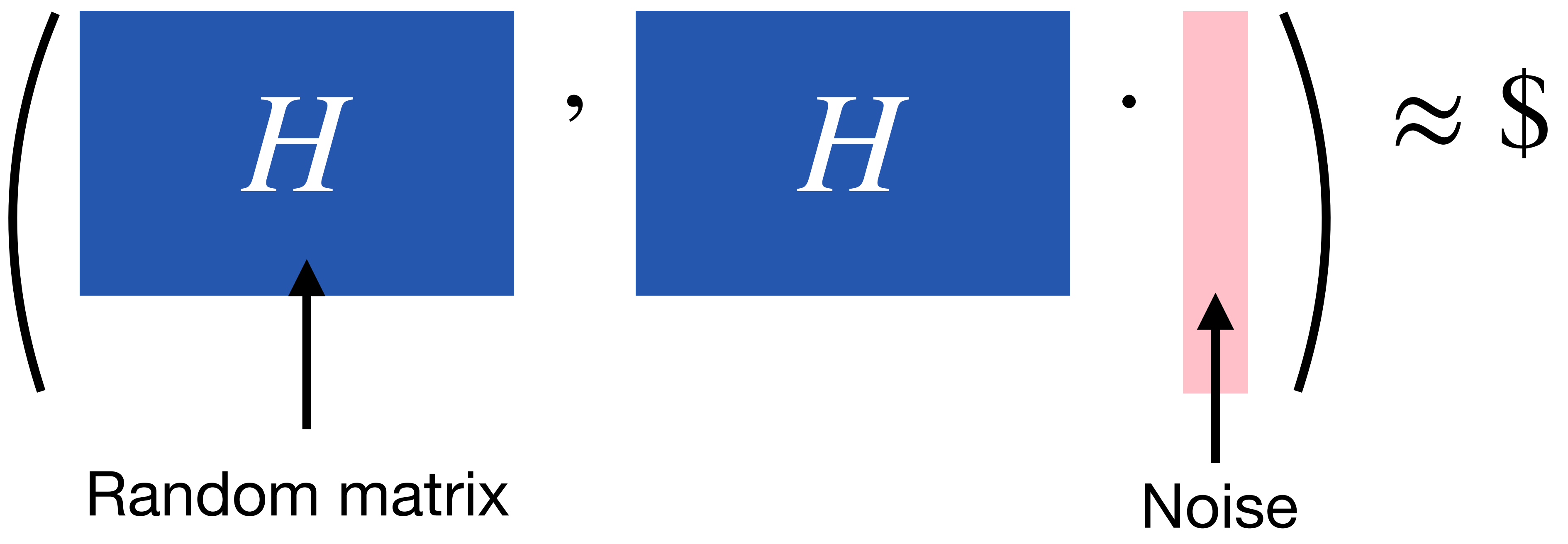

LPN($\mathbb{F}_2$): $H \leftarrow_\$ \mathbb{F}_2^{m\times n}$,

Ber($\mathbb{F}_2$)^n

'Sparse'

Statistical security

LWE($\mathbb{F}_p$): $H \leftarrow_\$ \mathbb{F}_p^{m\times n}$,

$[-B, B]^n$

'Small'

$O(n)$ entropy in the noise $\Longrightarrow$ LHL, statistical security, lattice trapdoors, lossiness…

Digression: LPN versus LWE

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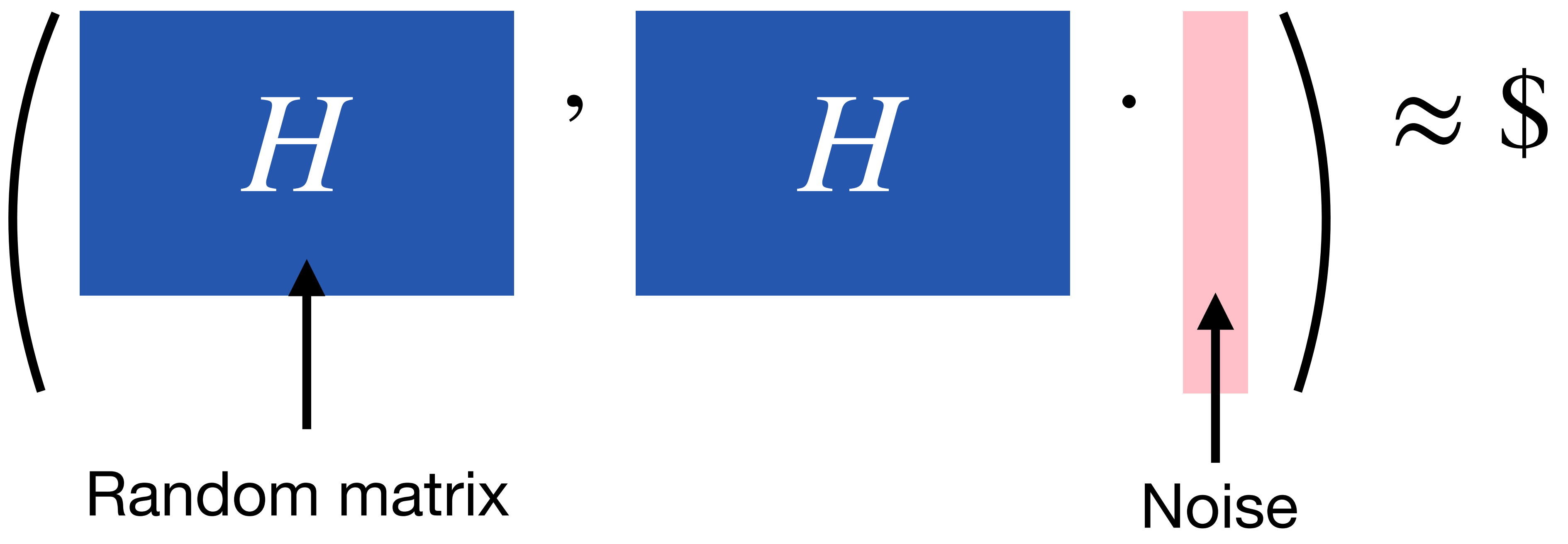
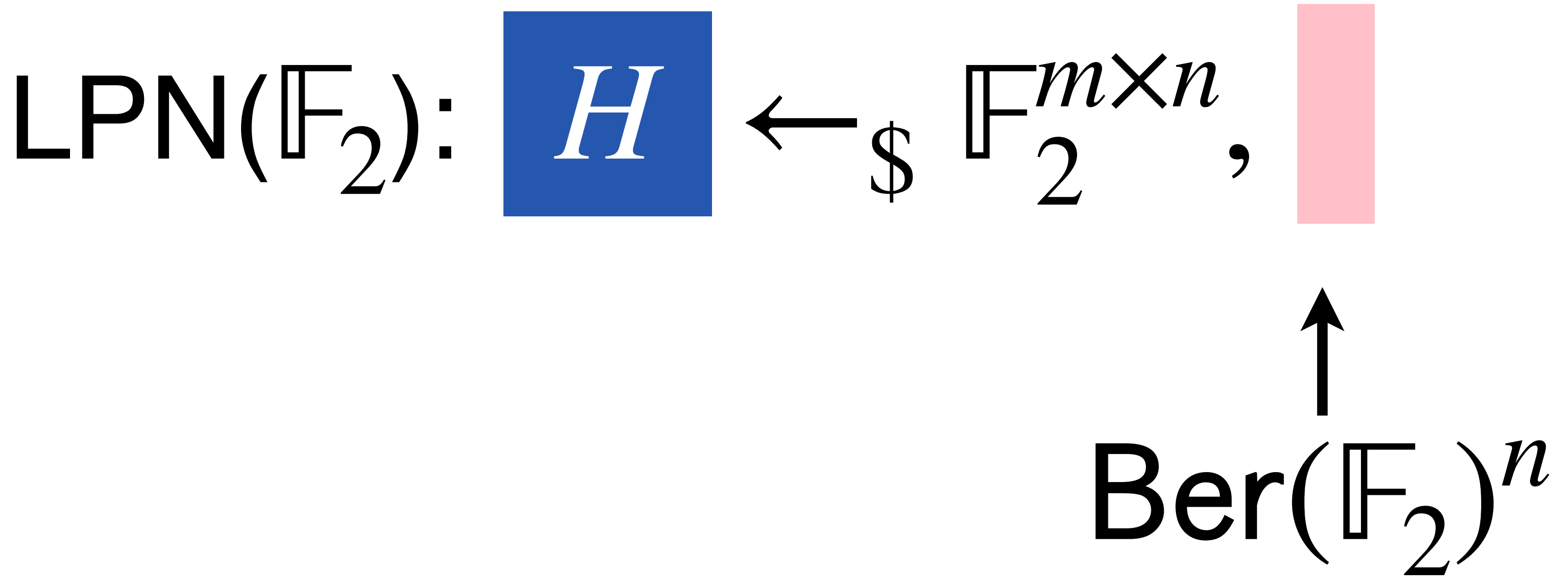


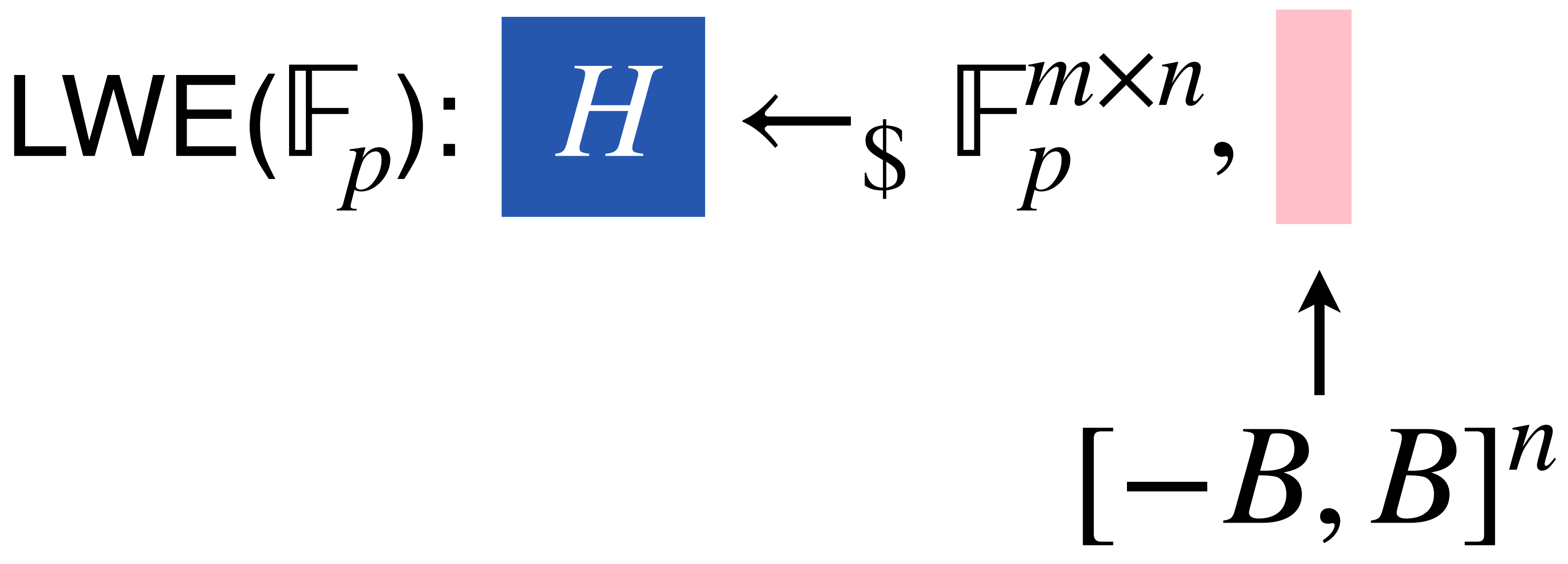
Compressibility



'Sparse'

$t \cdot \log n \ll n$ entropy in the noise $\Longrightarrow$ compressibility! Crucially used in recent results: PCGs, but also iO and batch OT.

Statistical security



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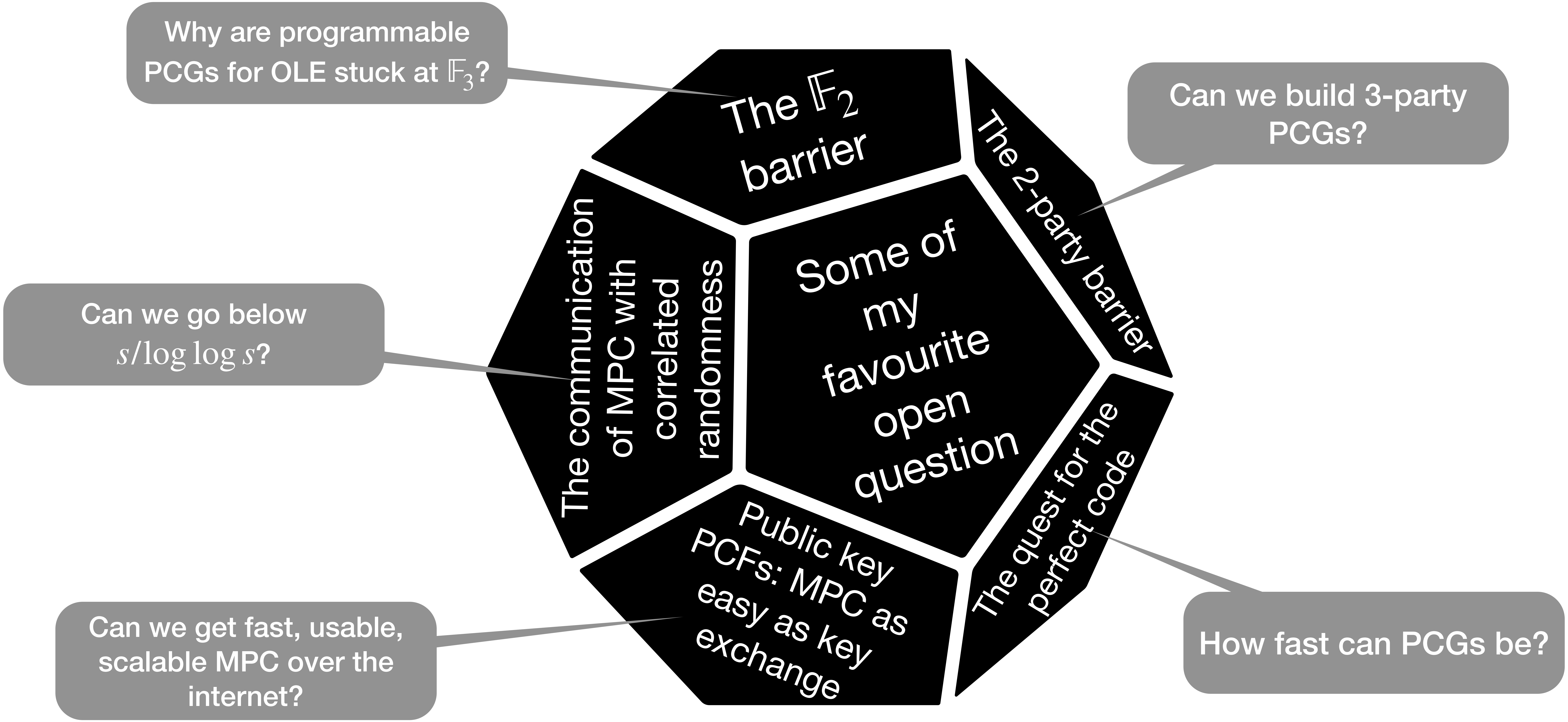
Some of my Favourite Open Questions



Some of my favourite open question

The $\mathbb{F}_2$ barrier

Why are programmable PCGs for OLE stuck at $\mathbb{F}_3$?

The 2-party barrier

Can we build 3-party PCGs?

The communication of MPC with correlated randomness

Can we go below $s/\log\log s$?

Public key PCFs: MPC as easy as key exchange

Can we get fast, usable, scalable MPC over the internet?

The quest for the perfect code

How fast can PCGs be?

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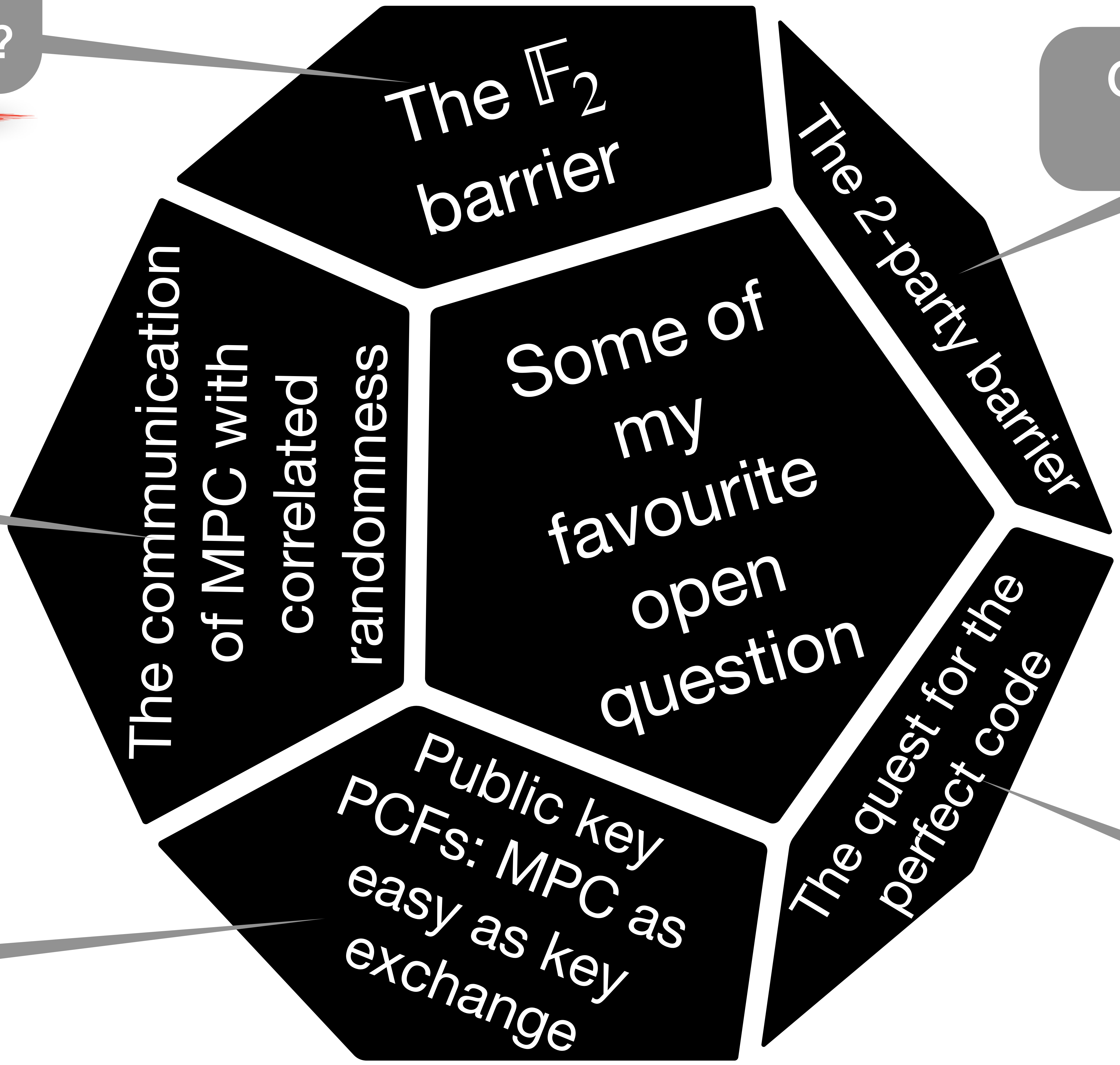

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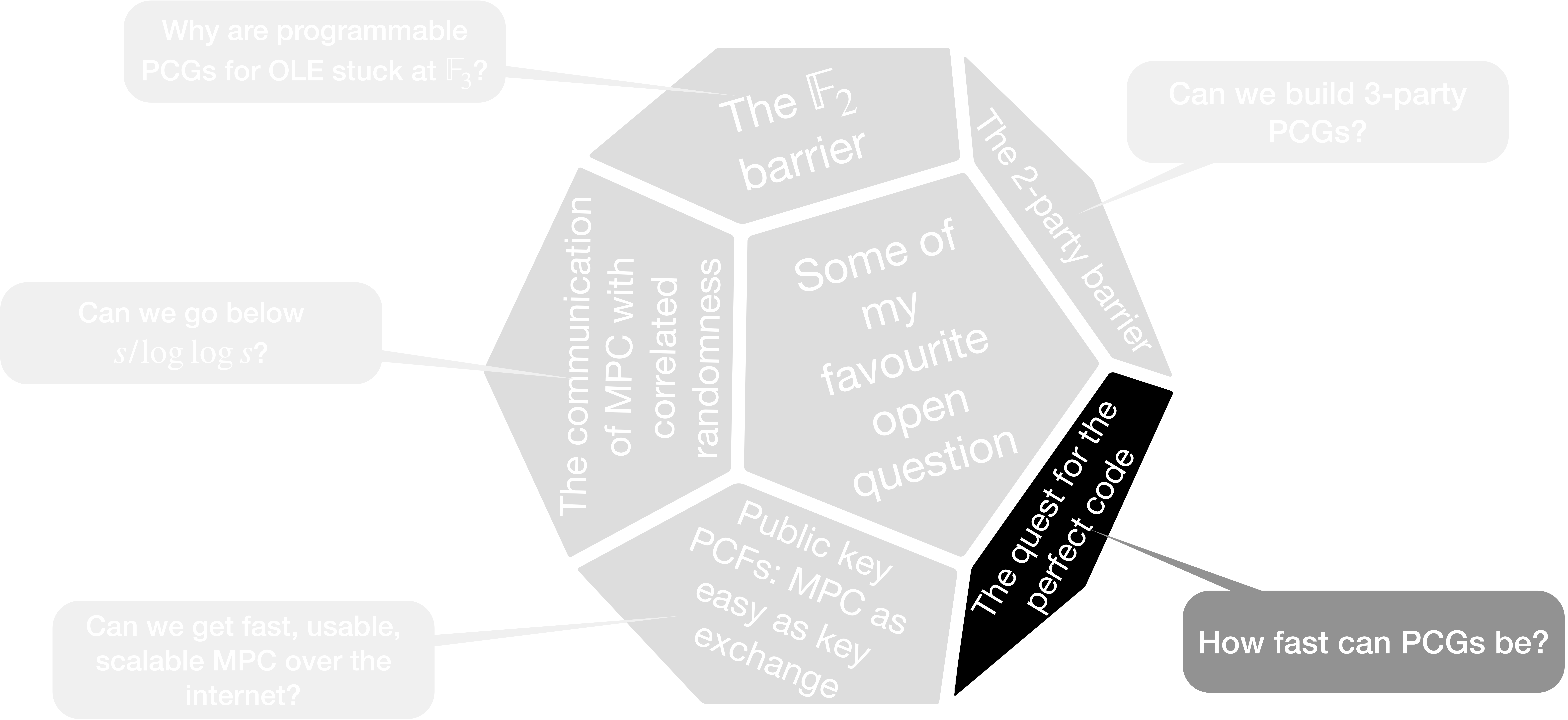

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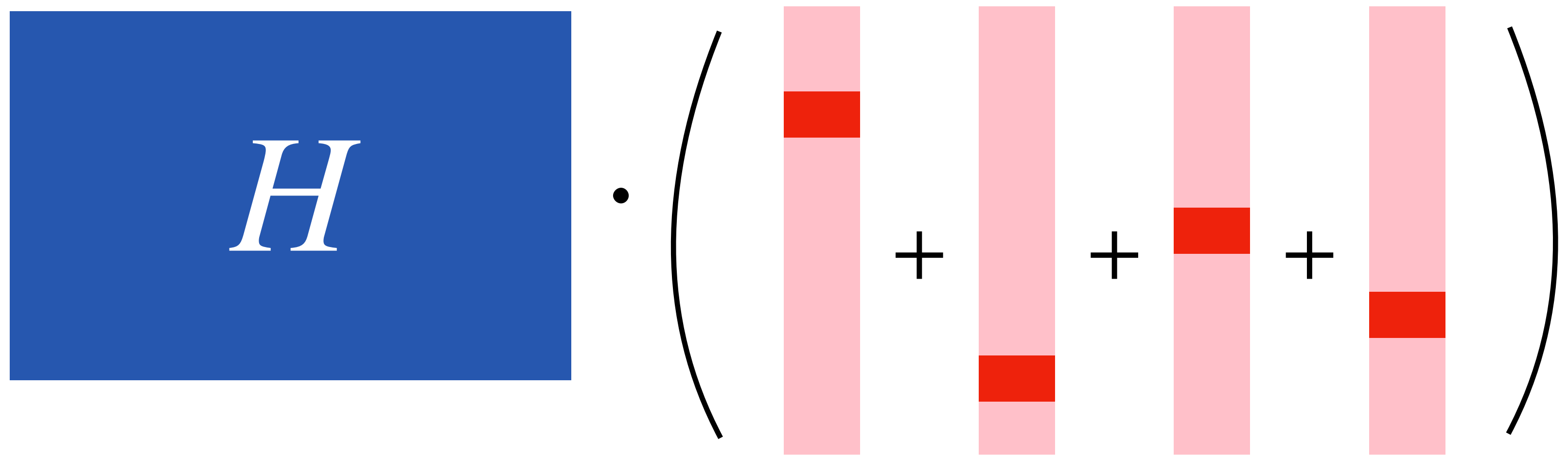
$\mathsf{PRG} : (\alpha_i)_{i\le t} \mapsto$



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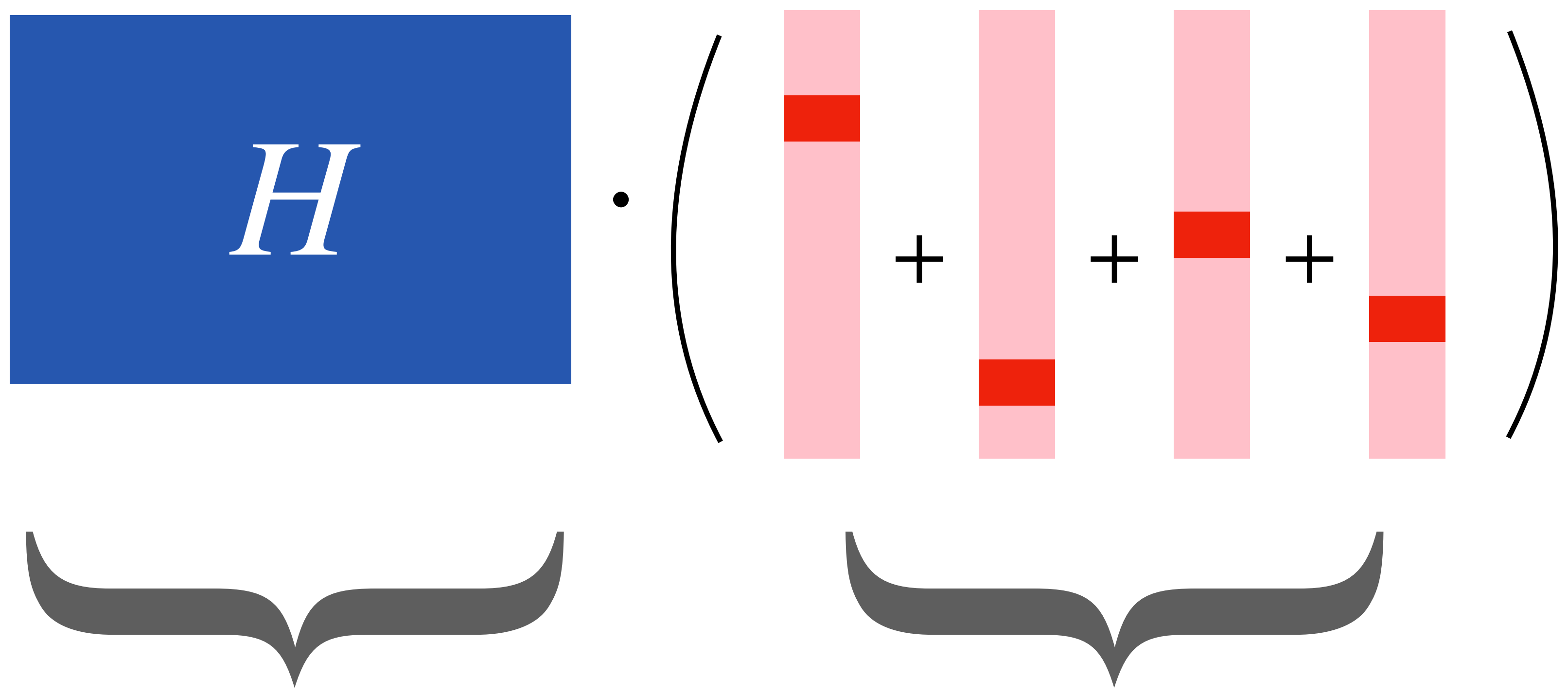

Multiplying by a random matrix of size $\Omega(n^2)$

Generating and summing unit vectors

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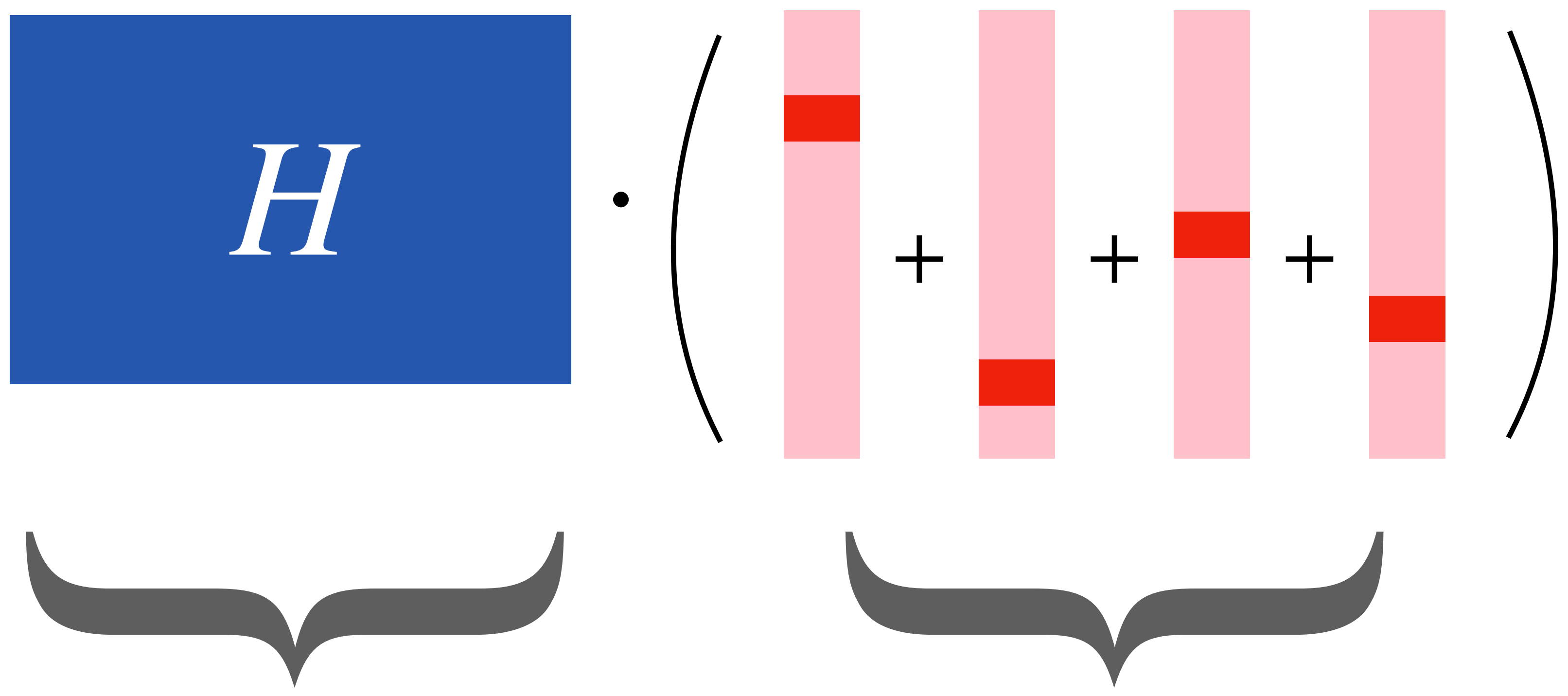

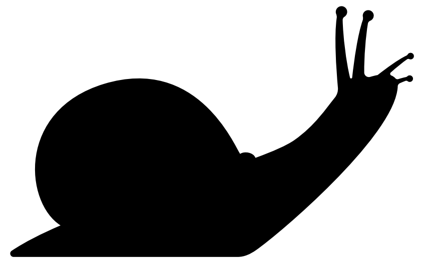

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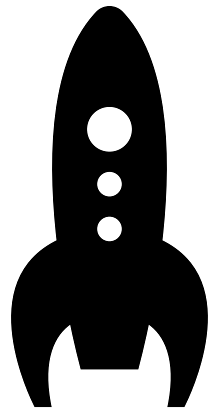
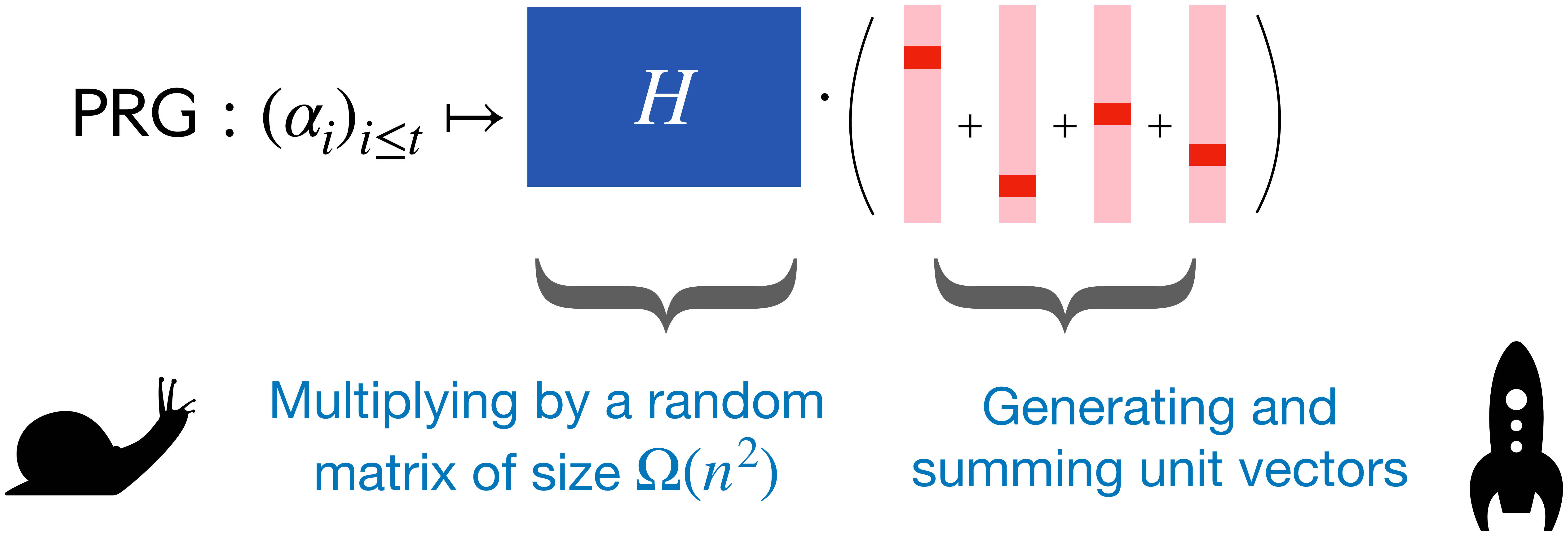


⚠ n is the total amount of correlated randomness we want to generate! (Think: $n \sim 2^{30}$)

Making the PRG more Efficient



PRG : $(\alpha_i)_{i\leq t} \mapsto H \cdot \left(+ + + \right)$

Multiplying by a random matrix of size $\Omega(n^2)$

Generating and summing unit vectors

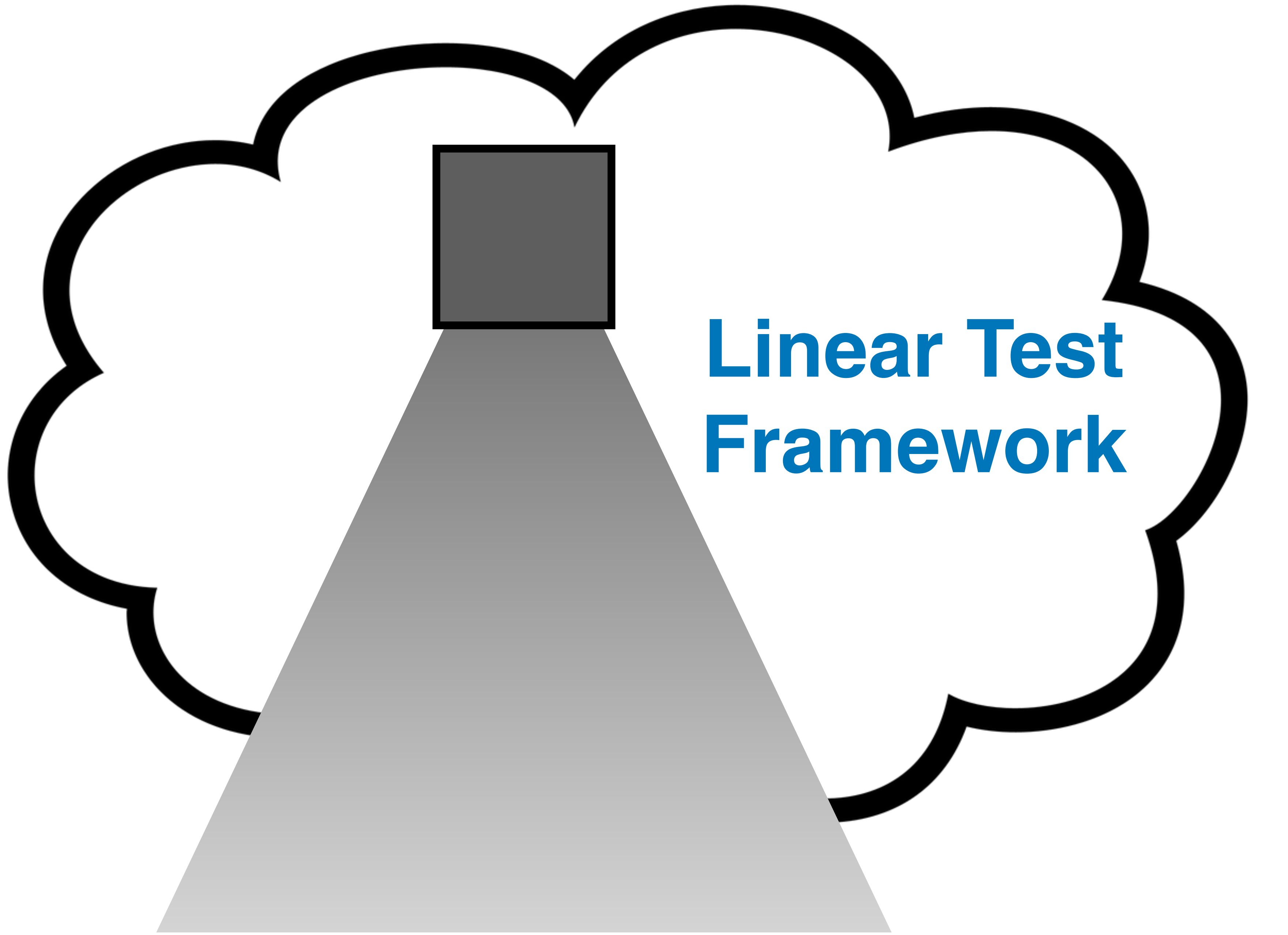
⚠ n is the total amount of correlated randomness we want to generate! (Think: $n \sim 2^{30}$)

Can we replace H with a matrix that allows for fast matrix-vector product?

We need a rule of thumb to know which matrices will yield *plausible* variants of LPN

Security of (variants of) LPN - Linear Tests

A tremendous number of attacks on LPN have been published…

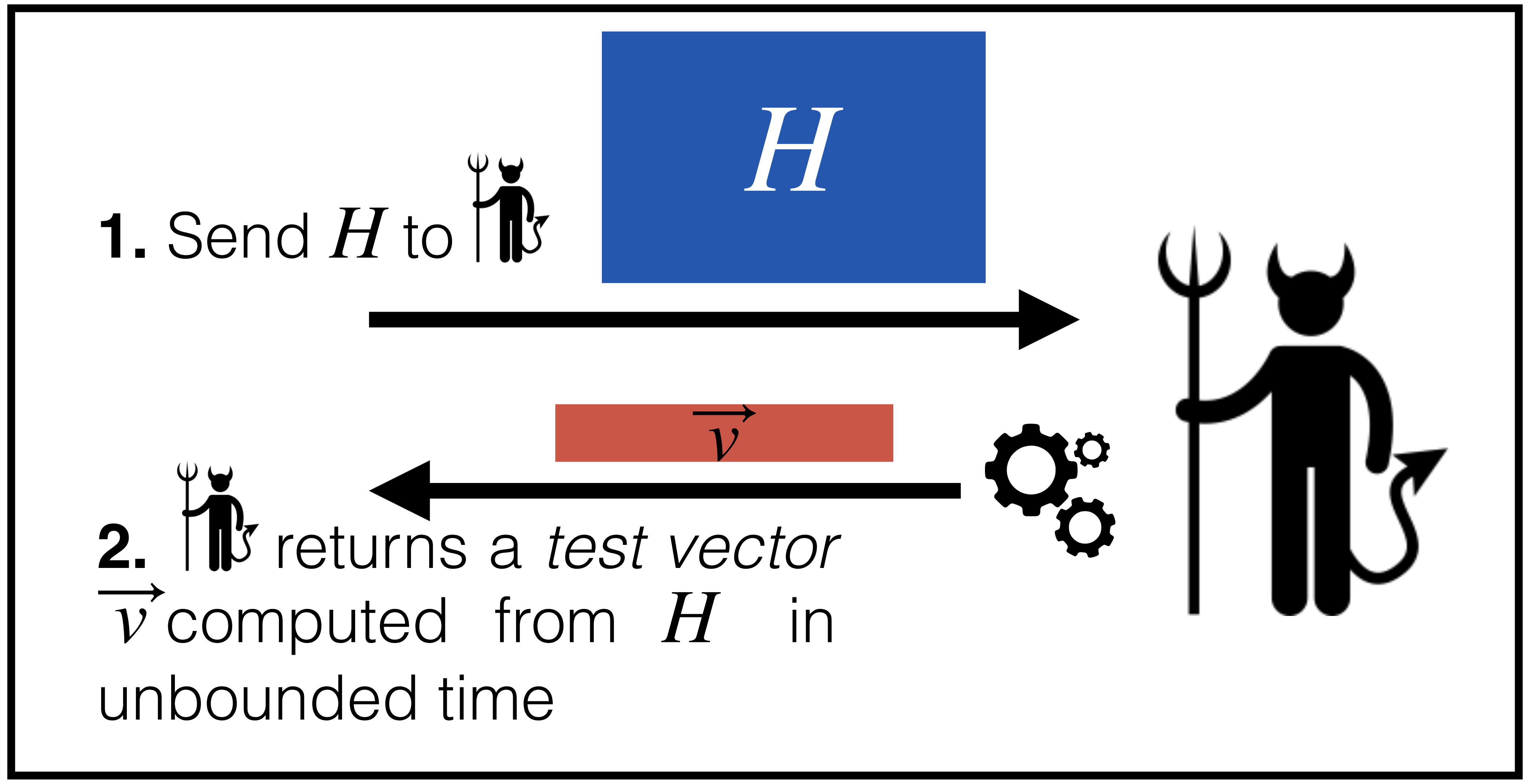


Linear Test Framework

- **Gaussian Elimination attacks**
  - Standard gaussian elimination
  - Blum-Kalai-Wasserman [J.ACM:BKW03]
  - Sample-efficient BKW [A-R:Lyu05]
  - Pooled Gauss [CRYPTO:EKM17]
  - Well-pooled Gauss [CRYPTO:EKM17]
  - Leviel-Fouque [SCN:LF06]
  - Covering codes [JC:GJL19]
  - Covering codes+ [BTV15]
  - Covering codes++ [BV:AC16]
  - Covering codes+++ [EC:ZJW16]
- **Statistical Decoding Attacks**
  - Jabri's attack [ICCC:Jab01]
  - Overbeck's variant [ACISP:Ove06]
  - FKI's variant [Trans.IT:FKI06]
  - Debris-Tillich variant [ISIT:DT17]
- **Information Set Decoding Attacks**
  - Prange's algorithm [Prange62]
  - Stern's variant [ICIT:Stern88]
  - Finiasz and Sendrier's variant [AC:FS09]
  - BJMM variant [EC:BJMM12]
  - May-Ozerov variant [EC:MO15]
  - Both-May variant [PQC:BM18]
  - MMT variant [AC:MMT11]
  - Well-pooled MMT [CRYPTO:EKM17]
  - BLP variant [CRYPTO:BLP11]
- **Other Attacks**
  - Generalized birthday [CRYPTO:Wag02]
  - Improved GBA [Kirchner11]
  - Linearization [EC:BM97]
  - Linearization 2 [INDO:Saa07]
  - Low-weight parity-check [Zichron17]
  - Low-deg approx [ITCS:ABGKR17]

Crucial observation: most attacks fit in the same framework, the *linear test framework. (*)*

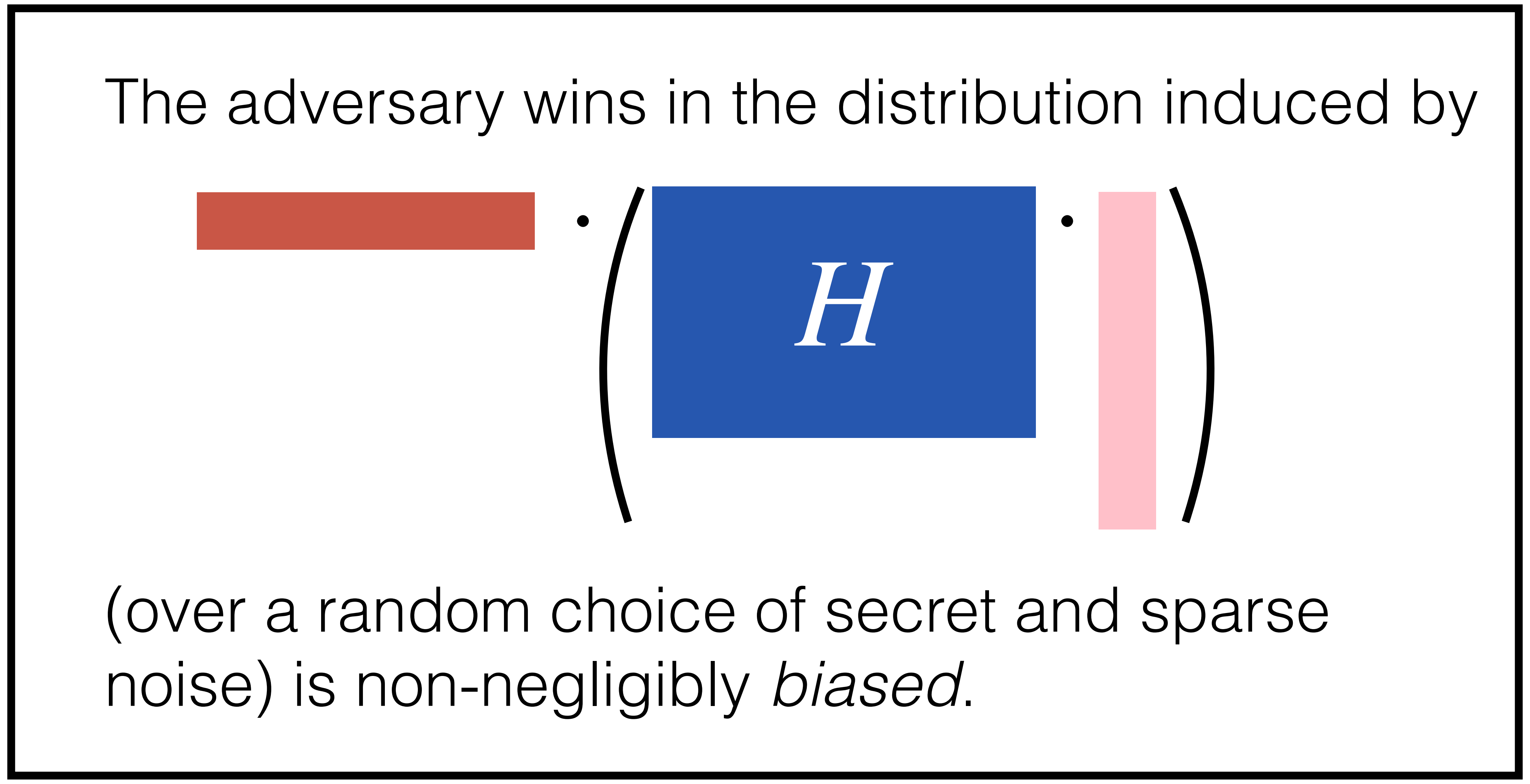
Game



1. Send H to 😈

2. 😈 returns a *test vector* $\vec{v}$ computed from H in unbounded time

Check

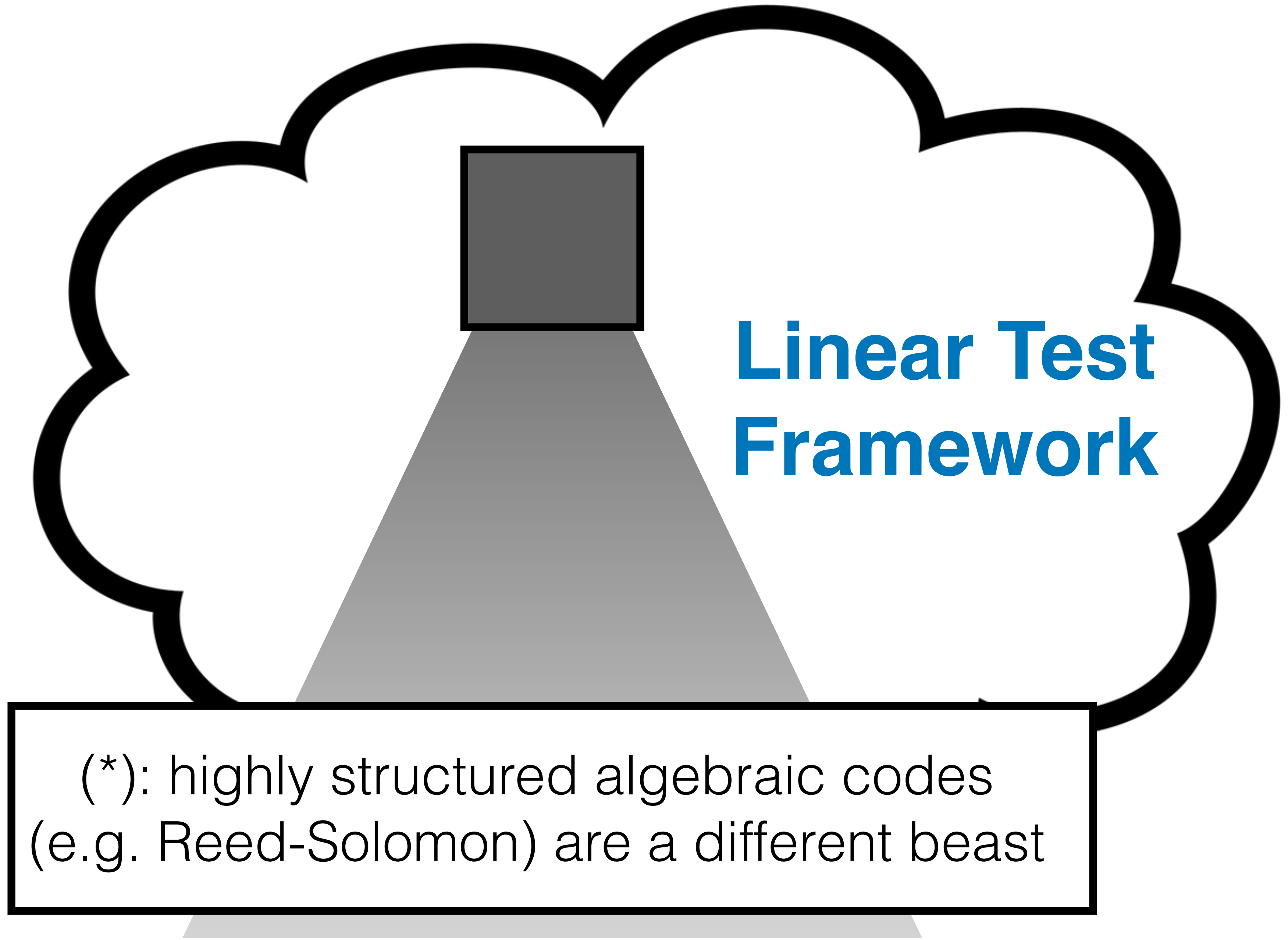


The adversary wins in the distribution induced by

(over a random choice of secret and sparse noise) is non-negligibly *biased*.

Security of (variants of) LPN - Linear Tests

A tremendous number of attacks on LPN have been published…



(*): highly structured algebraic codes (e.g. Reed-Solomon) are a different beast

- **Gaussian Elimination attacks**
  - Standard gaussian elimination
  - Blum-Kalai-Wasserman [J.ACM:BKW03]
  - Sample-efficient BKW [A-R:Lyu05]
  - Pooled Gauss [CRYPTO:EKM17]
  - Well-pooled Gauss [CRYPTO:EKM17]
  - Leviel-Fouque [SCN:LF06]
  - Covering codes [JC:GJL19]
  - Covering codes+ [BTV15]
  - Covering codes++ [BV:AC16]
  - Covering codes+++ [EC:ZJW16]
- **Statistical Decoding Attacks**
  - Jabri's attack [ICCC:Jab01]
  - Overbeck's variant [ACISP:Ove06]
  - FKI's variant [Trans.IT:FKI06]
  - Debris-Tillich variant [ISIT:DT17]
- **Information Set Decoding Attacks**
  - Prange's algorithm [Prange62]
  - Stern's variant [ICIT:Stern88]
  - Finiasz and Sendrier's variant [AC:FS09]
  - BJMM variant [EC:BJMM12]
  - May-Ozerov variant [EC:MO15]
  - Both-May variant [PQC:BM18]
  - MMT variant [AC:MMT11]
  - Well-pooled MMT [CRYPTO:EKM17]
  - BLP variant [CRYPTO:BLP11]
- **Other Attacks**
  - Generalized birthday [CRYPTO:Wag02]
  - Improved GBA [Kirchner11]
  - Linearization [EC:BM97]
  - Linearization 2 [INDO:Saa07]
  - Low-weight parity-check [Zichron17]
  - Low-deg approx [ITCS:ABGKR17]

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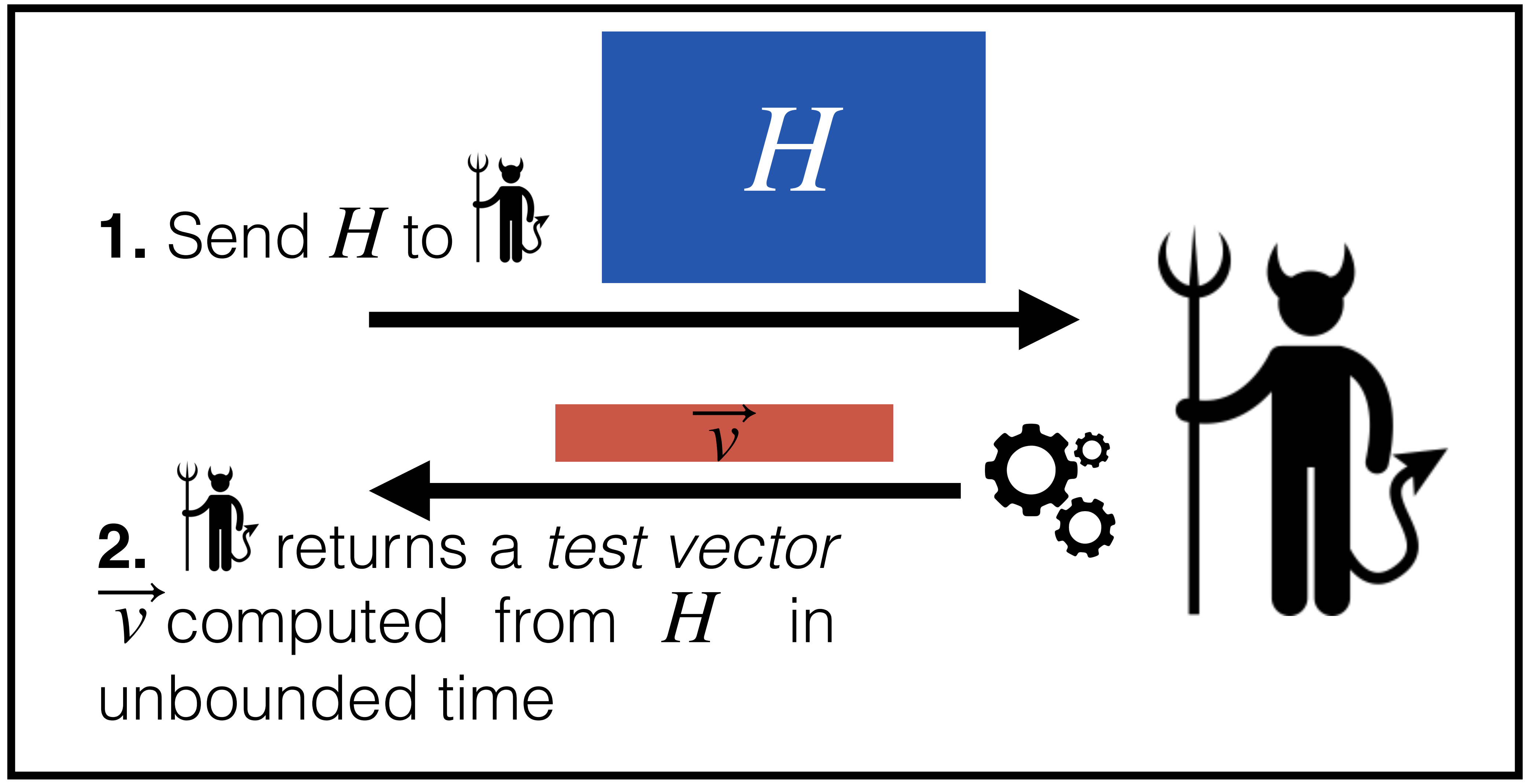

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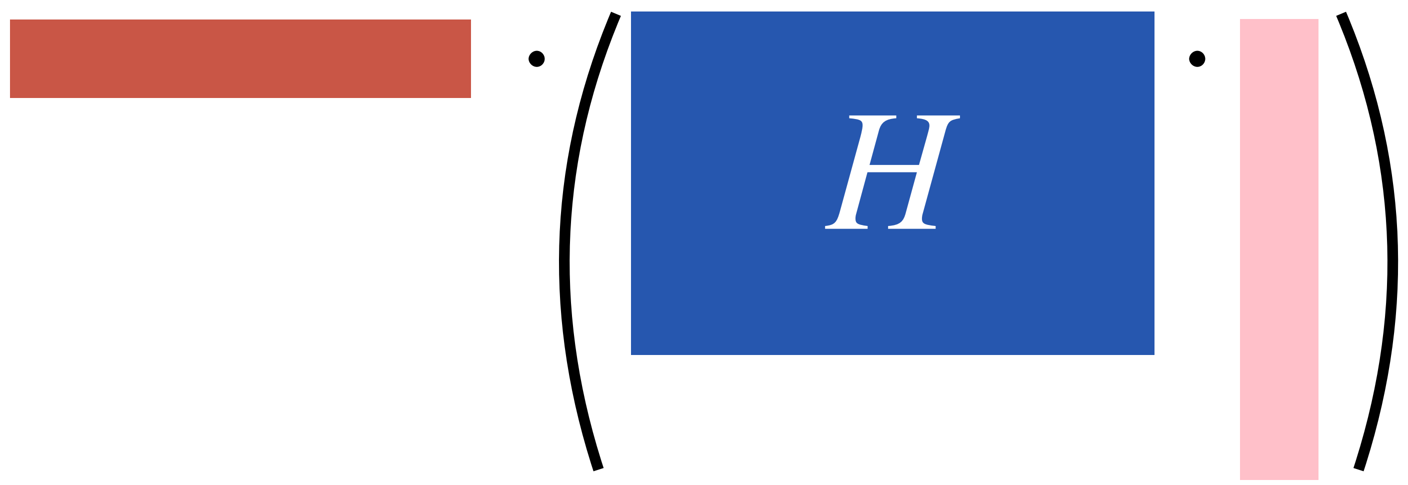

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Withstanding Linear Tests

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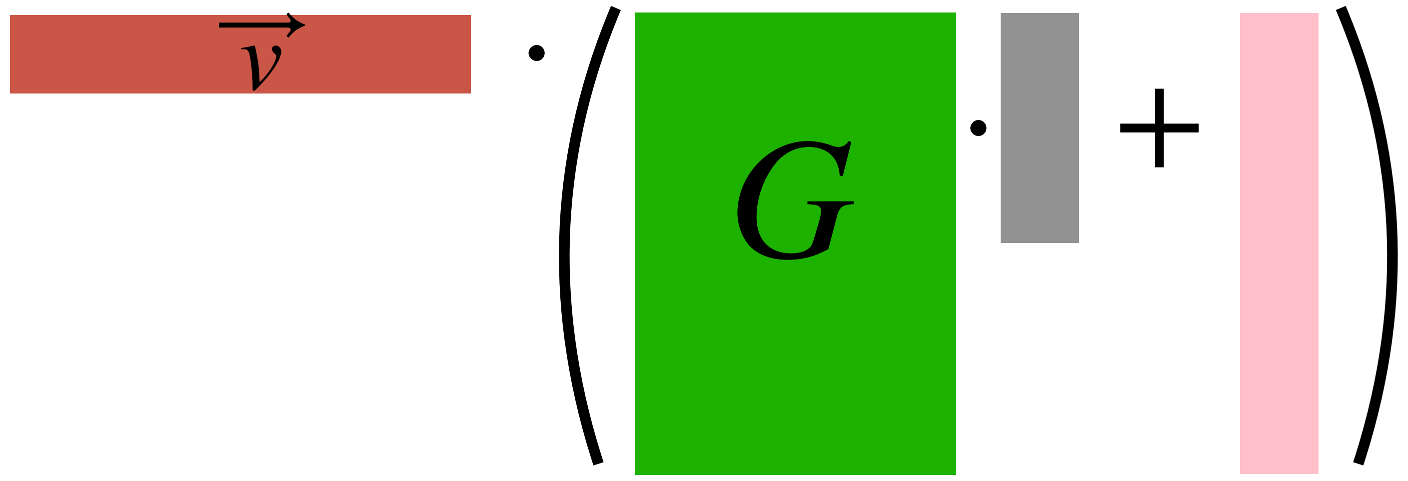

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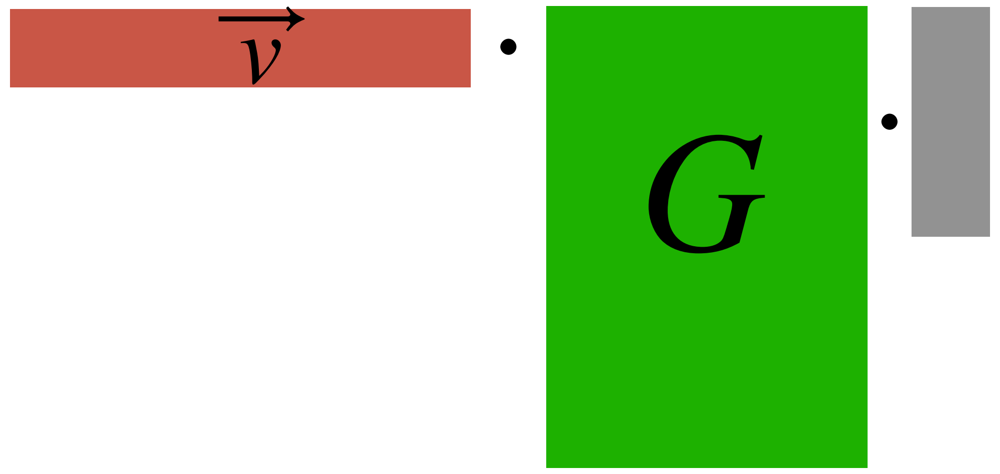
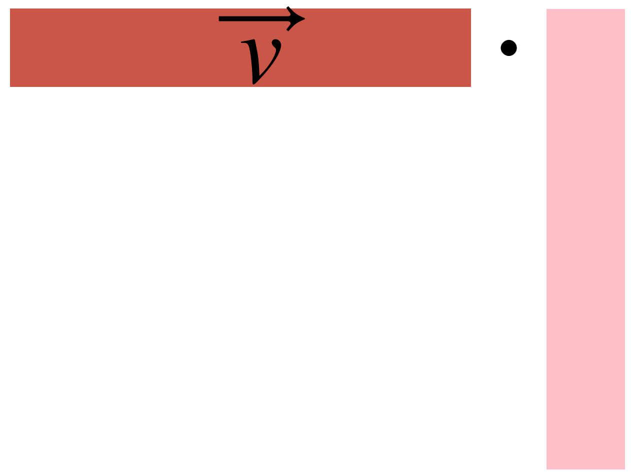
We have a sum of two distributions:

Induced by the *codeword*



Protects against *light* linear tests

Induced by the *noise vector*



Protects against *heavy* linear tests

Claim: Assume t (number of noisy coordinates) is set to a security parameter. If there is a constant c such that every subset of $c \cdot n$ rows of G is linearly independent, no linear test can distinguish $G \cdot \vec{s} + \vec{e}$ from random.

Withstanding Linear Tests

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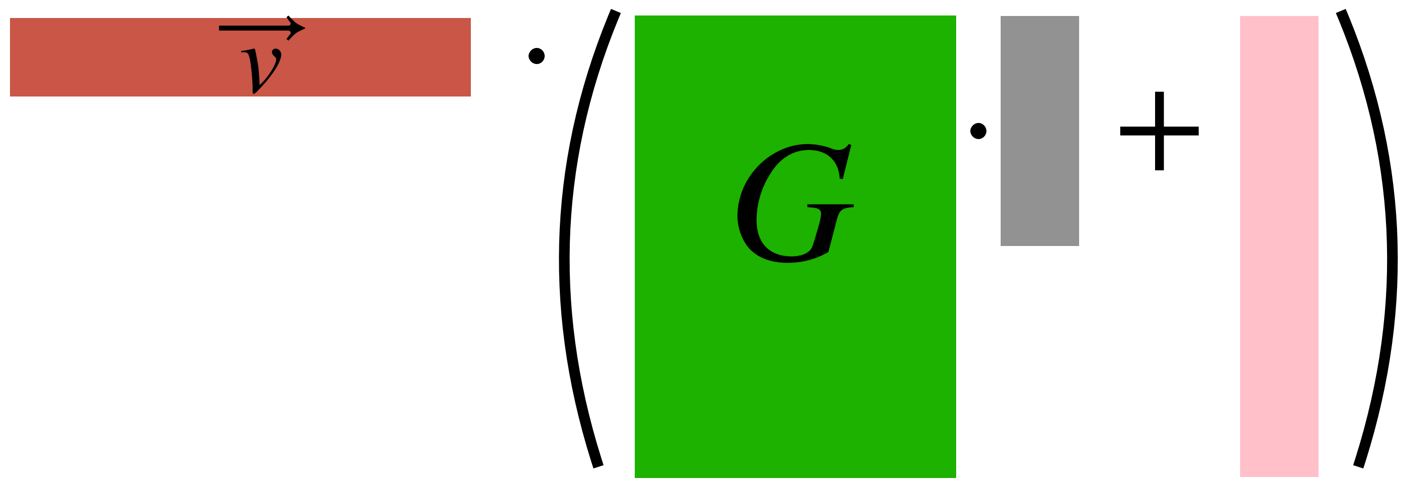

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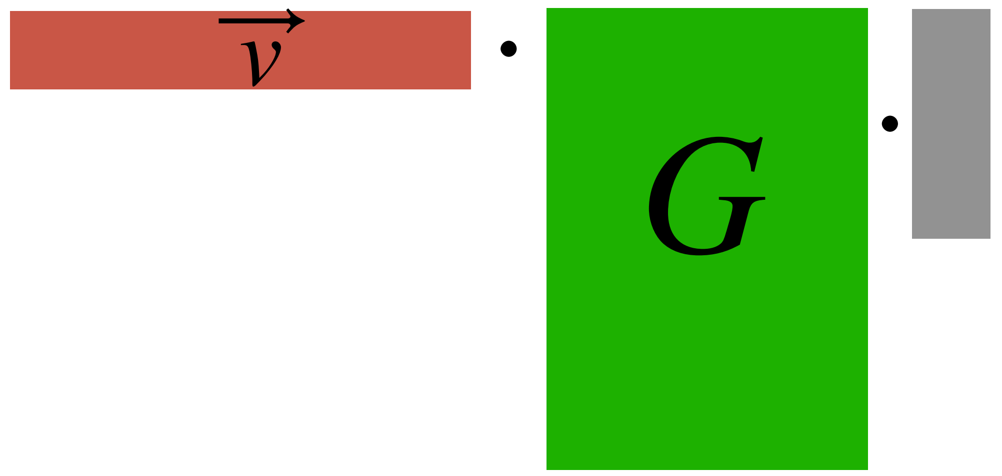

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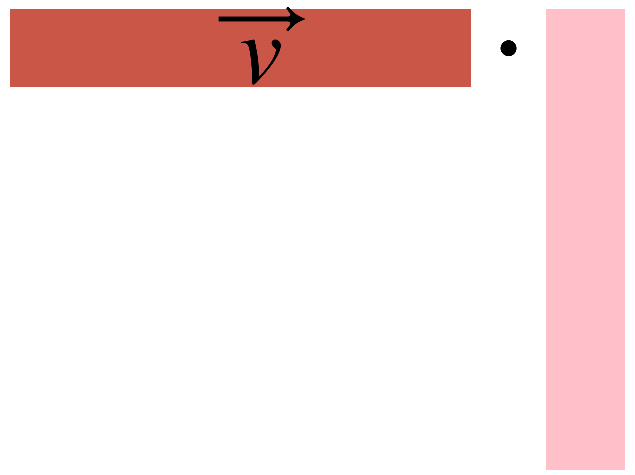

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Rephrasing the sufficient condition:

Every subset of $O(n)$ rows of G is linearly independent
$\Longleftrightarrow$ the left-kernel of G does not contain nonzero vector of weight less than $O(n)$
$\Longleftrightarrow$ the *dual code* of G, i.e., the code generated by *the transpose of its parity check matrix* H, has linear minimum distance

Pseudorandom Correlation Generators - Efficiency

Goal: computing  fast, such that the code  is LPN-friendy

We want to find a matrix $M = H^\intercal$ such that (1) the code generated by M is a good code, and (2) computing ~~$M^\intercal \cdot \vec{v}$~~ takes time $O(n)$ for any $\vec{v}$

$M \cdot \vec{v}$ (this is the *transposition principle*)

$\Longrightarrow$ We need to find a *good* and *linear-time encodable* code. And we want it concretely efficient!

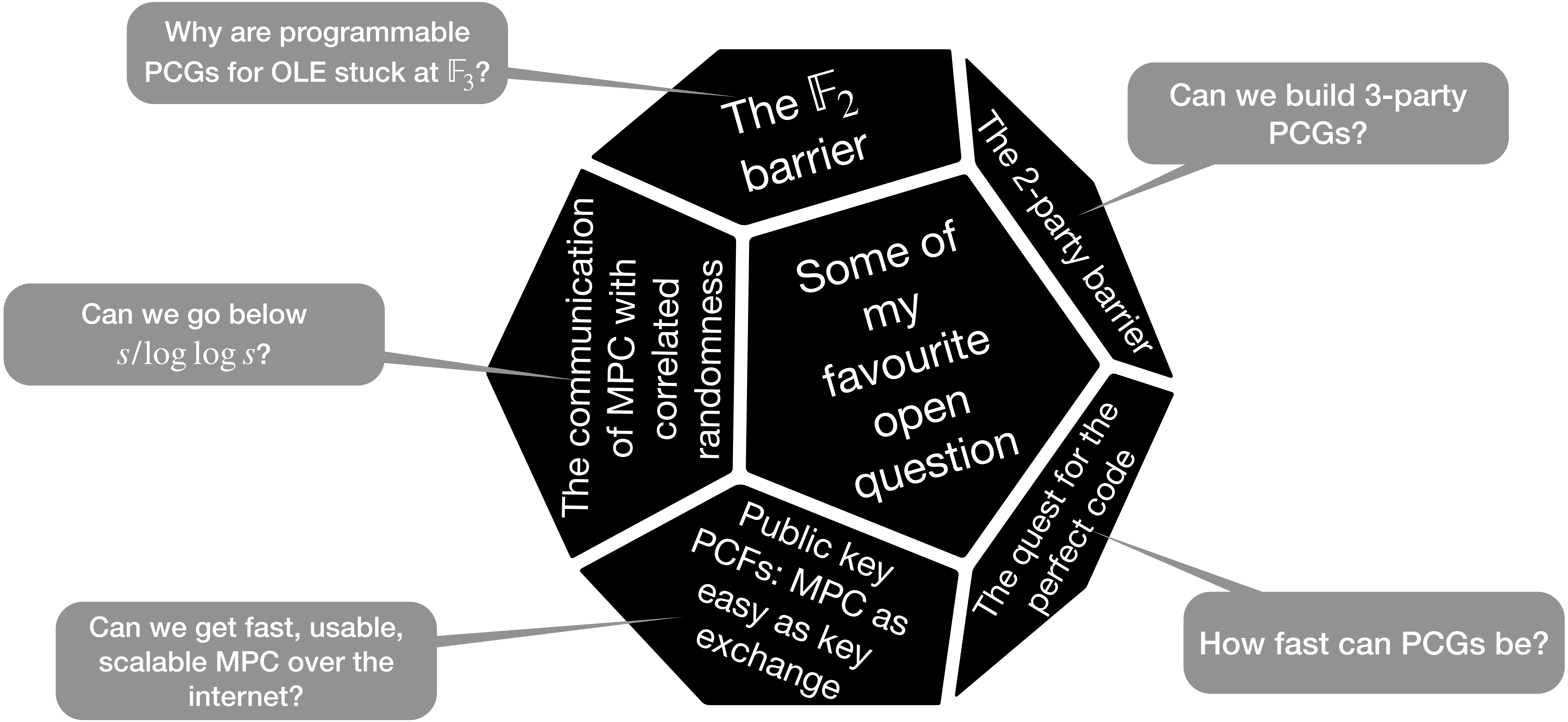
Pseudorandom Correlation Generators - Efficiency

There is an ongoing and exciting quest for pinpointing the *right* code for PCG applications:

- CCS:Boyle-C-Gilboa-Ishai'18 suggested using LDPC code
- CCS:Boyle-C-Gilboa-Ishai-Kohl-Rindal-Scholl'19 moved to quasi-cyclic codes

  due to concern regarding linear-time encoding of LDPC codes
- Crypto:C-Raghuraman-Rindal'21: tailored LDPC with heuristic & experimental support
- Crypto:Boyle-C-Gilboa-Ishai-Kohl-Resch—Scholl'22: Expand-Accumulate codes
- Latest news: there's apparently a new proposal that suggests Expand-Convolute codes instead (and which breaks Silver along the way!)
- There are a few more codes I'd like to investigate, the quest continues!

Some of my Favourite Open Questions



Some of my favourite open question

The $\mathbb{F}_2$ barrier

Why are programmable PCGs for OLE stuck at $\mathbb{F}_3$?

The 2-party barrier

Can we build 3-party PCGs?

The quest for the perfect code

How fast can PCGs be?

Public key PCFs: MPC as easy as key exchange

Can we get fast, usable, scalable MPC over the internet?

The communication of MPC with correlated randomness

Can we go below $s/\log\log s$?

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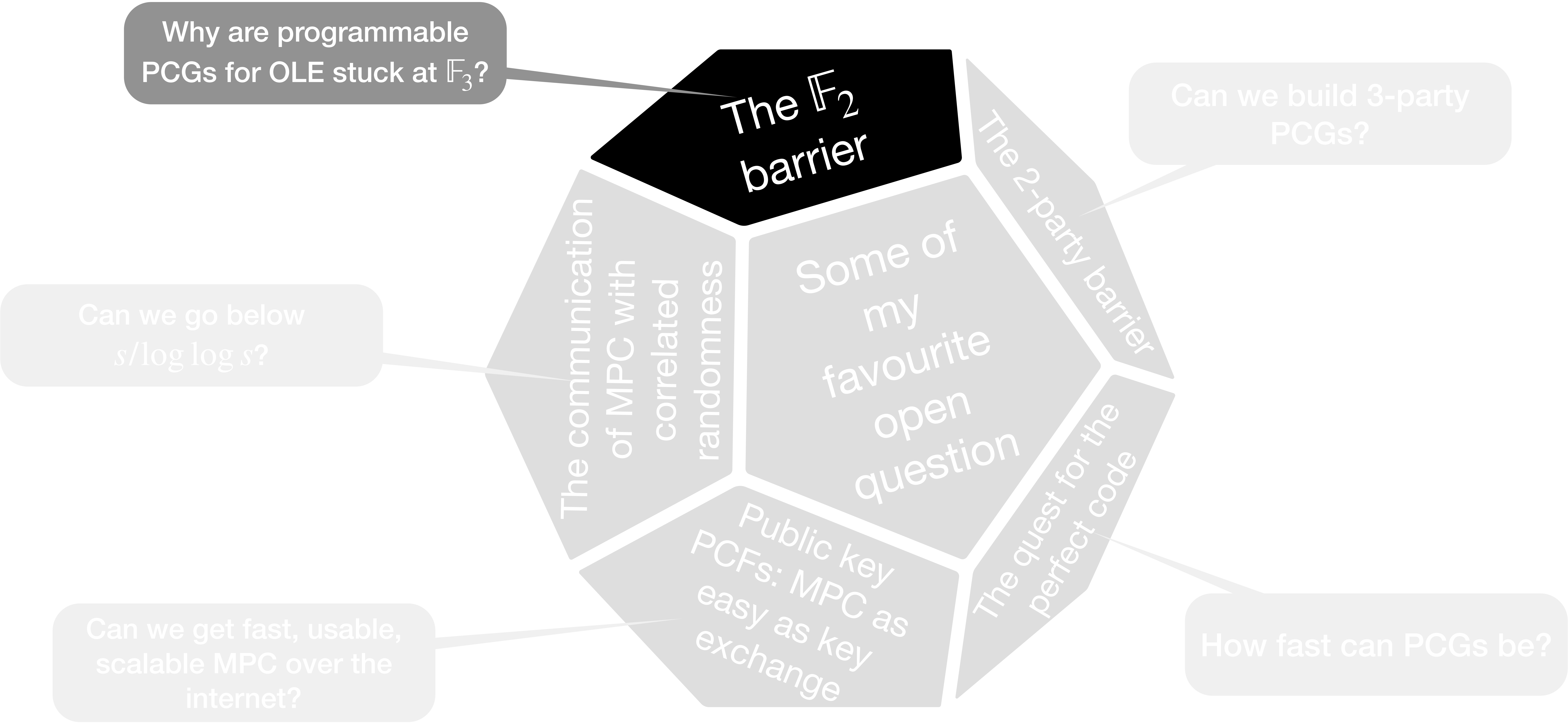

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OLE Correlations

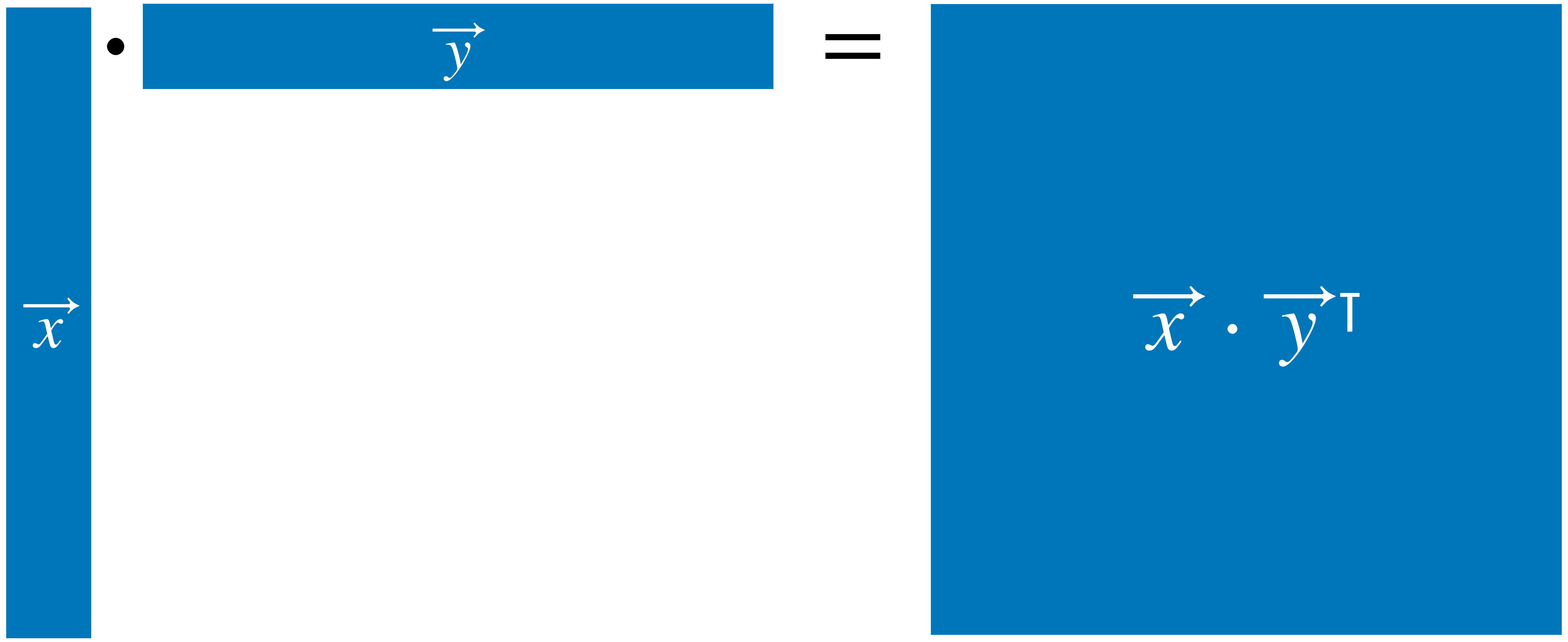
OLE over $\mathbb{F}$ is the type of correlation we want to do (semi-honest) secure computation of arithmetic circuits over $\mathbb{F}$.

In an OLE, Alice gets $a \leftarrow \mathbb{F}$, Bob gets $b \leftarrow \mathbb{F}$, and Alice and Bob get random shares of $a \cdot b$.

OLE Correlations, the LPN Way

Goal:

- Alice gets a pseudorandom vector $\overrightarrow{x}$
- Bob gets a pseudorandom vector $\overrightarrow{y}$
- Alice and Bob get shares of $\overrightarrow{x} \odot \overrightarrow{y}$



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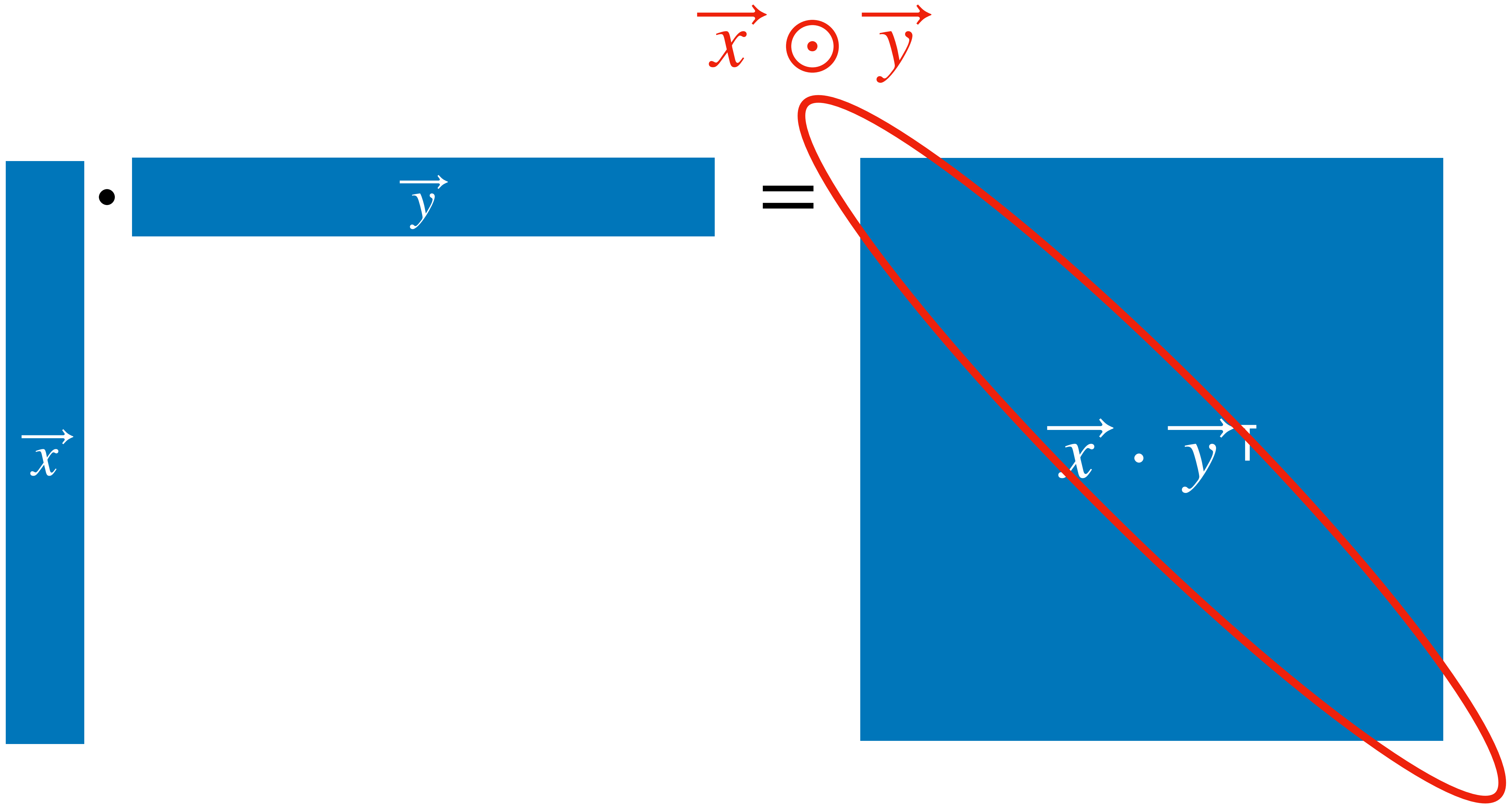

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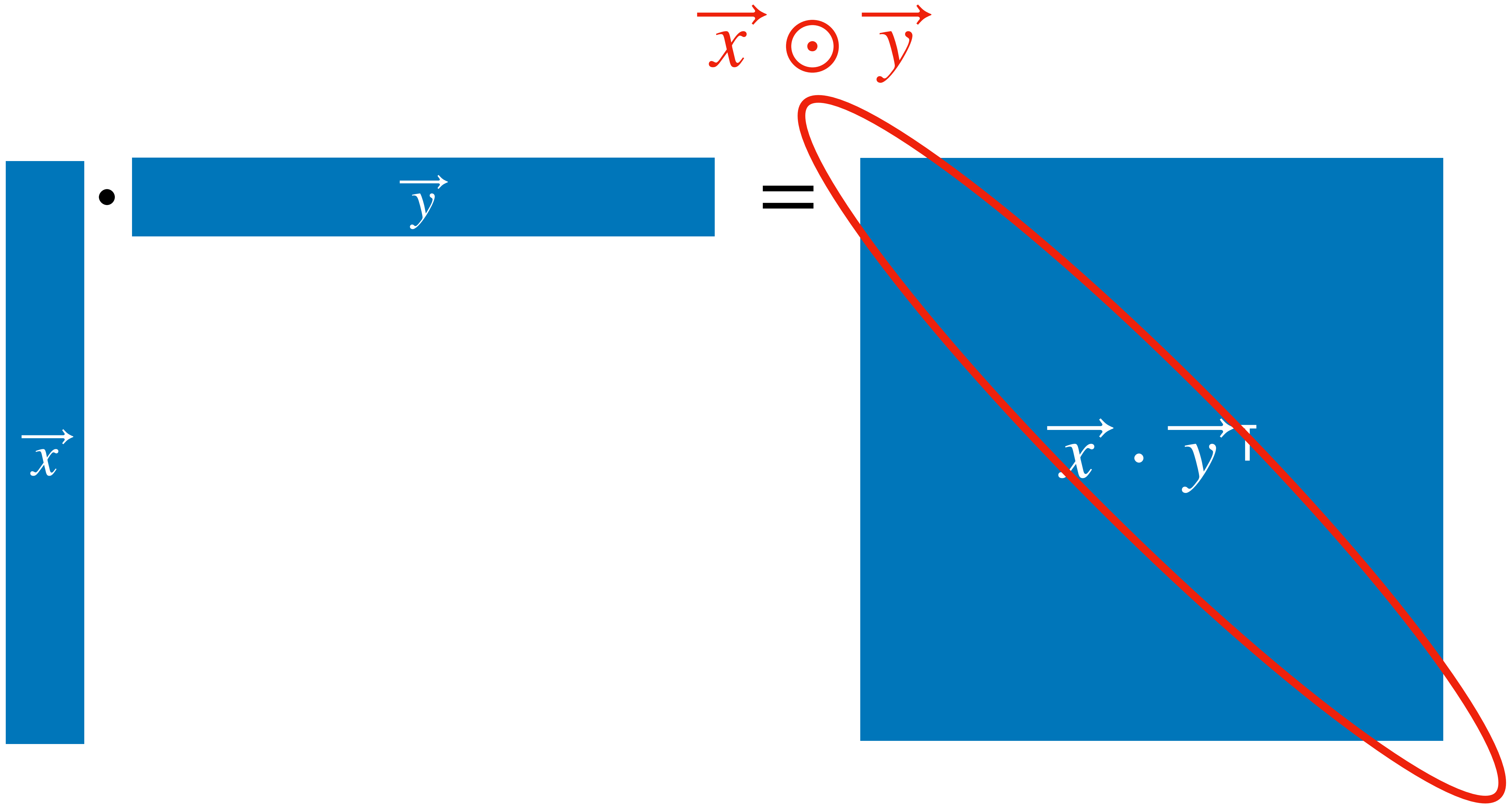

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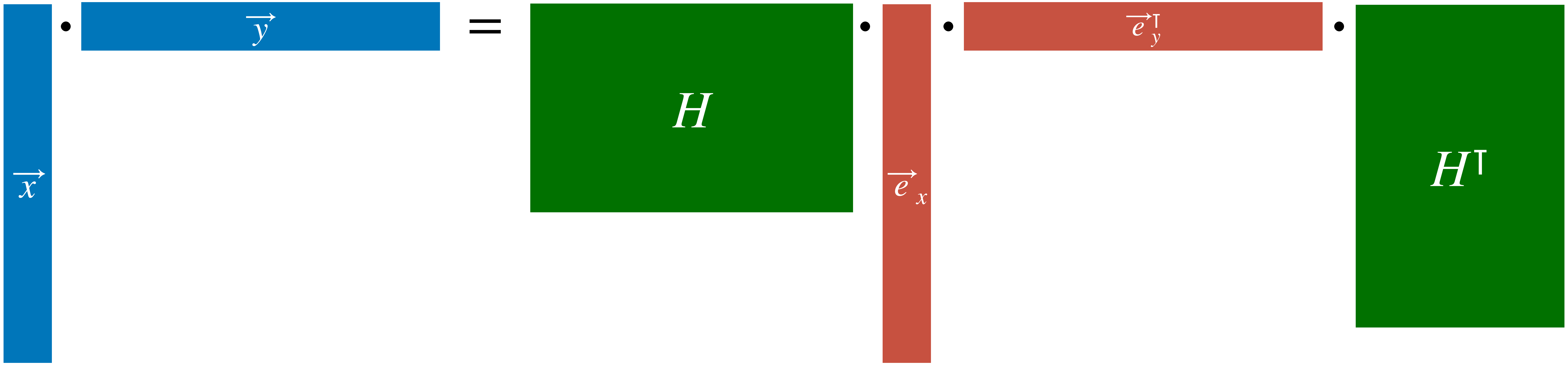

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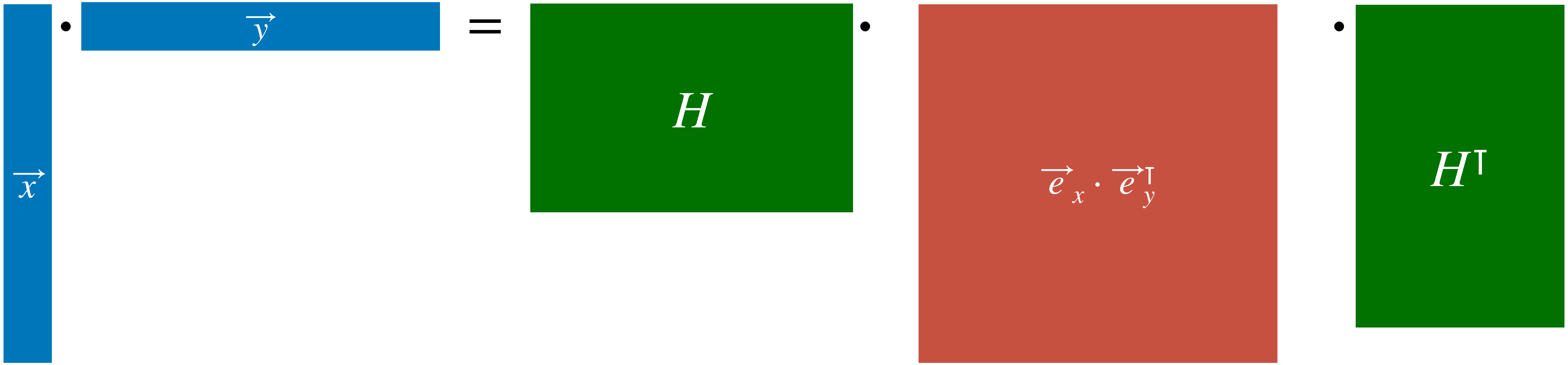

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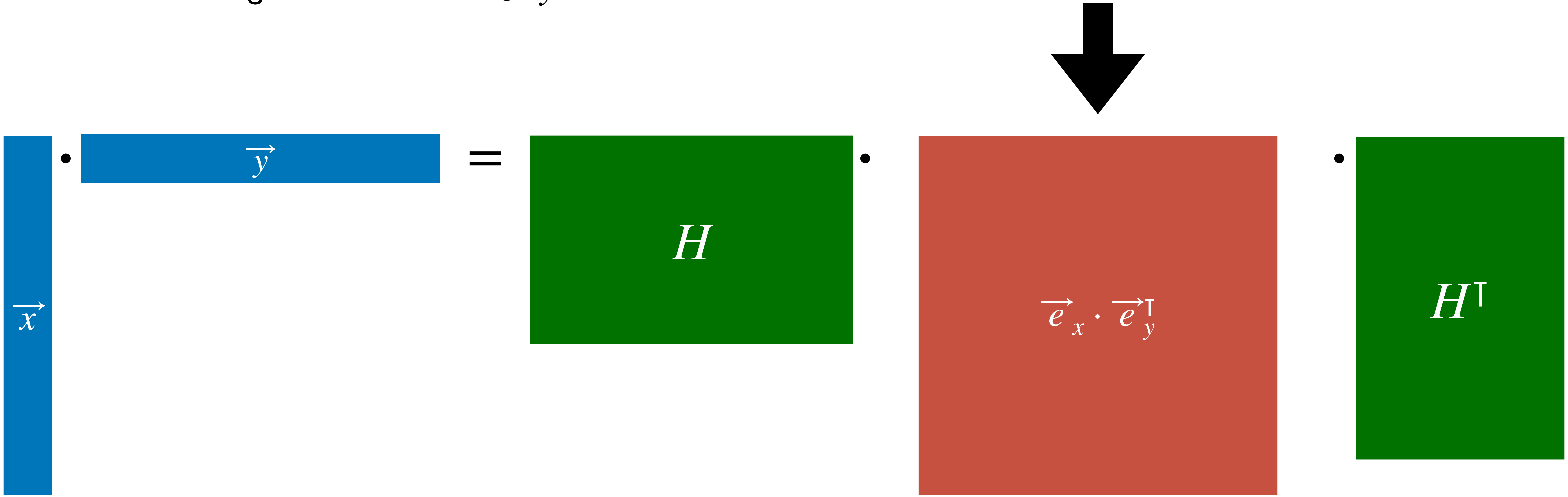
This is a t^2-sparse matrix, *i.e.* a sum of t^2 point functions!

$\Longrightarrow$ can be generated with comm. $O(\lambda t^2 \log n)$



$$\vec{x} \cdot \vec{y} = H \cdot \vec{e}_x \cdot \vec{e}_y^\intercal \cdot H^\intercal$$

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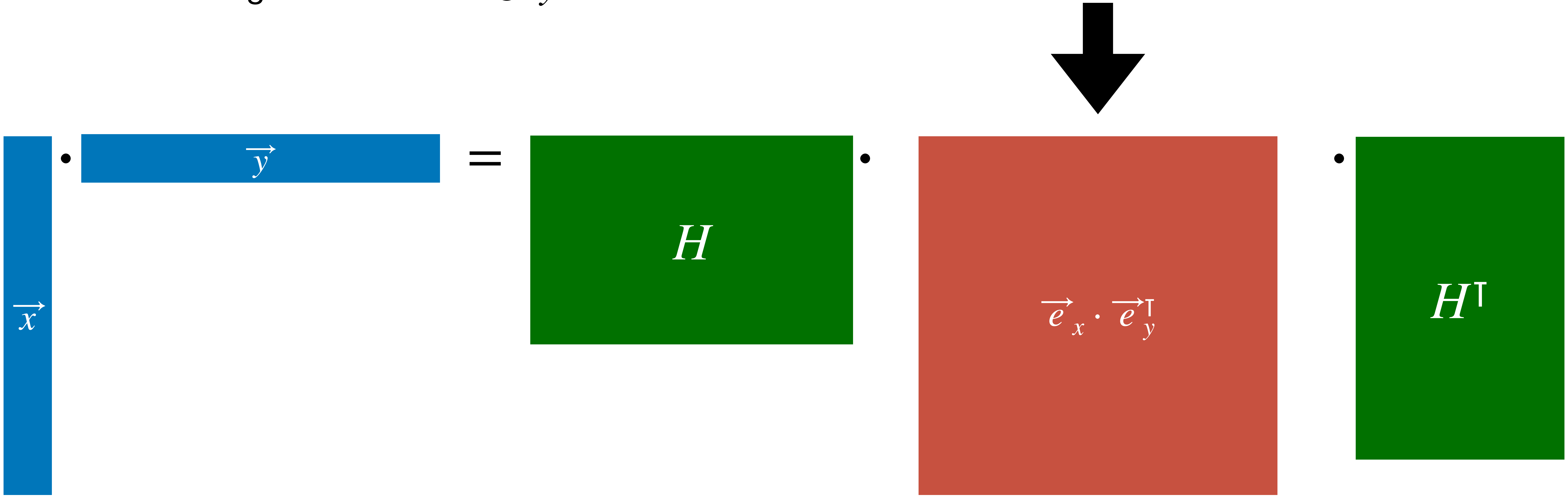

Uses only LPN

Costs $\omega(n^2)$! (Think: $n \sim 2^{30}\ldots$)

OLE Correlations, the *Ring*-LPN Way

Crypto: Boyle-C-Gilboa-Ishai-Kohl-Scholl'20

Let $\mathscr{R}$ be the ring $\mathbb{Z}_p/F(X)$ where $F(X)$ is a degree-n polynomial that splits entirely, and $p > n$.

Ring-LPN assumption: $(a, b) \sim (a, a \cdot e + f)$ where $(a, b) \leftarrow \mathscr{R}$ and (e, f) are random t-*sparse* polynomials.

Observation: we can get n OLE correlations from a single 'ring-OLE' correlation $(x, y, \langle x \cdot y \rangle)$ over $\mathscr{R}$: the OLE correlations are obtained by reducing x, y, and $x \cdot y$ modulo each of the linear factors F_i of F.

Construction:

- Alice gets a pseudorandom polynomial $x = a \cdot e_x + f_x$ where (e_x, f_x) are t-sparse polynomials over $\mathscr{R}$
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The polynomials a^2, a are public, and $e_x e_y, e_x f_y, f_x e_y, f_x f_y$ are all t^2-sparse polynomials

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☺ Costs only $O(n \cdot \log n)$

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- 'Splittable ring-LPN' deserves further study

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Costs only $O(n \cdot \log n)$

- 'Splittable ring-LPN' deserves further study
- $\mathbb{F}$ must be large!

OLE Correlations, from Quasi-Abelian Syndrome Decoding

How do we break this ‘field-size barrier’? An answer in our recent Crypto paper (Bombar-C-Couvreur-Ducros’23): we move to *quasi-abelian* codes, which are defined over group algebras.

High level intuition

The group algebra structure gives a suitable framework to find the *right* polynomial P to instantiate an LPN variant over a ring $\mathscr{R} = \mathbb{F}[X_1, \cdots, X_d]/P(X_1, \cdots, X_d)$ such that

- $\mathscr{R} \sim \mathbb{F} \times \cdots \times \mathbb{F}$ (*i.e.* we get many copies of an OLE over $\mathbb{F}$)
- The underlying assumption is plausibly secure (*i.e.* resists linear attacks)

Using multivariate rings gives us many more roots of P even for a small $\mathbb{F}$! In fact, we can get up to $(|\mathbb{F}| - 1)^d$ copies of an OLE over $\mathbb{F}$.

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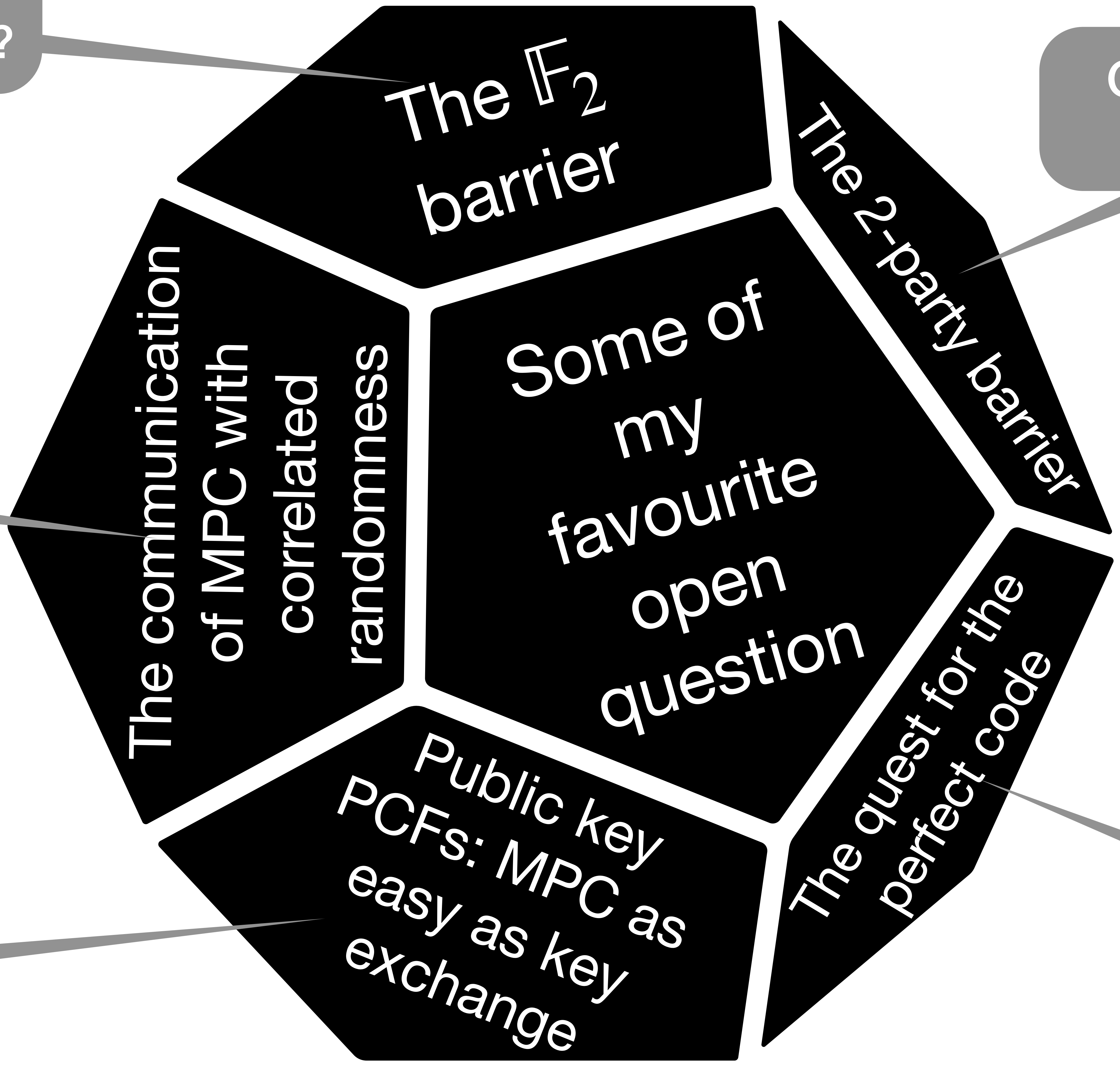
This only gives something meaningful up to $\mathbb{F}_3$!

Some of my Favourite Open Questions



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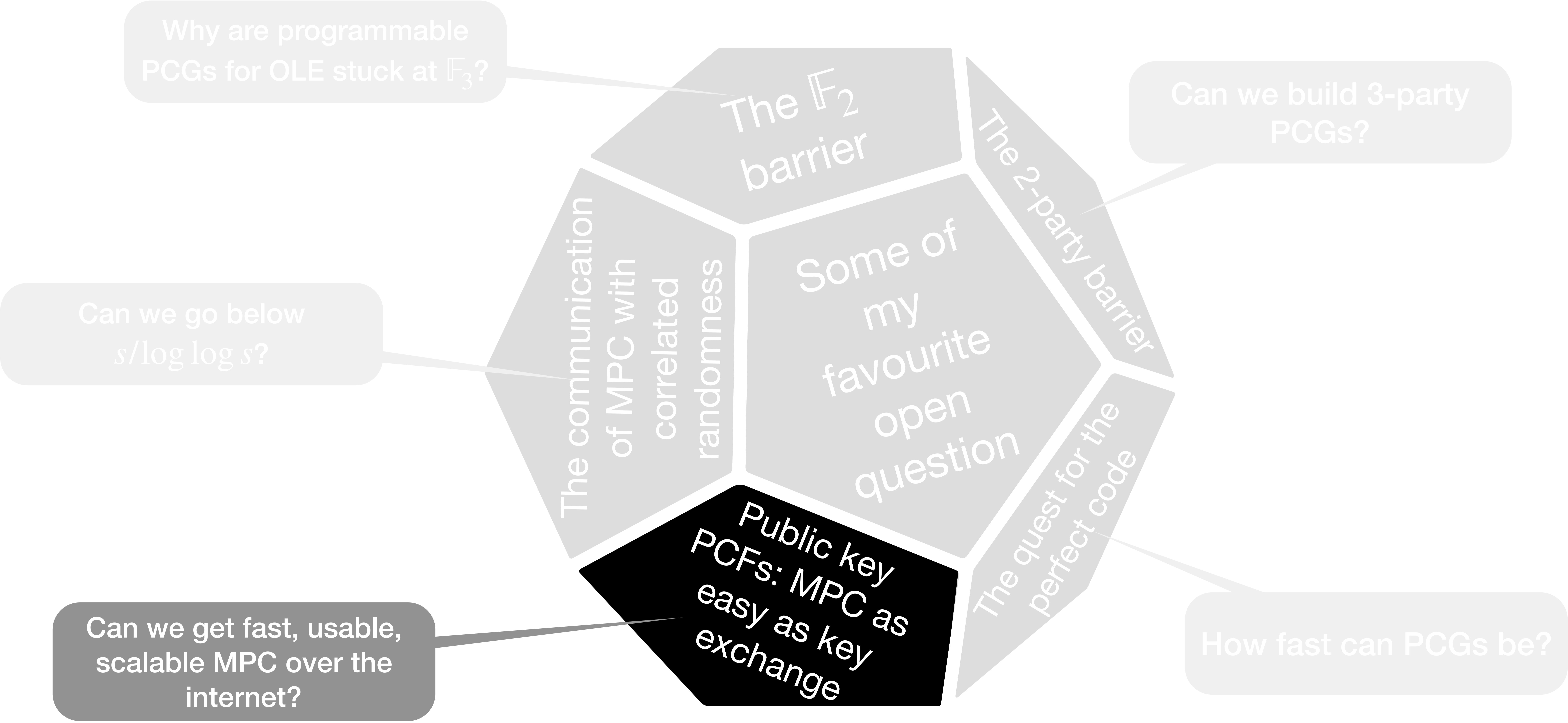

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A Closer Look at Secure Communication

Our ultimate goal is *practical* MPC that can be deployed and used over the web

Secure communication is already widely deployed and in use

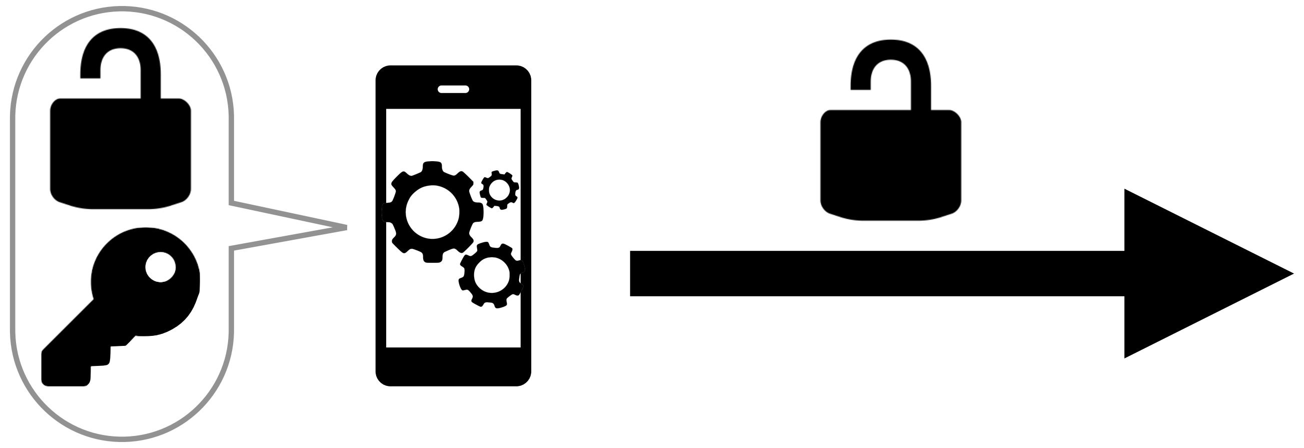
> 85% of the total internet traffic is encrypted

⟹ Let us look at secure communication’s recipe for success!

A Closer Look at Secure Communication

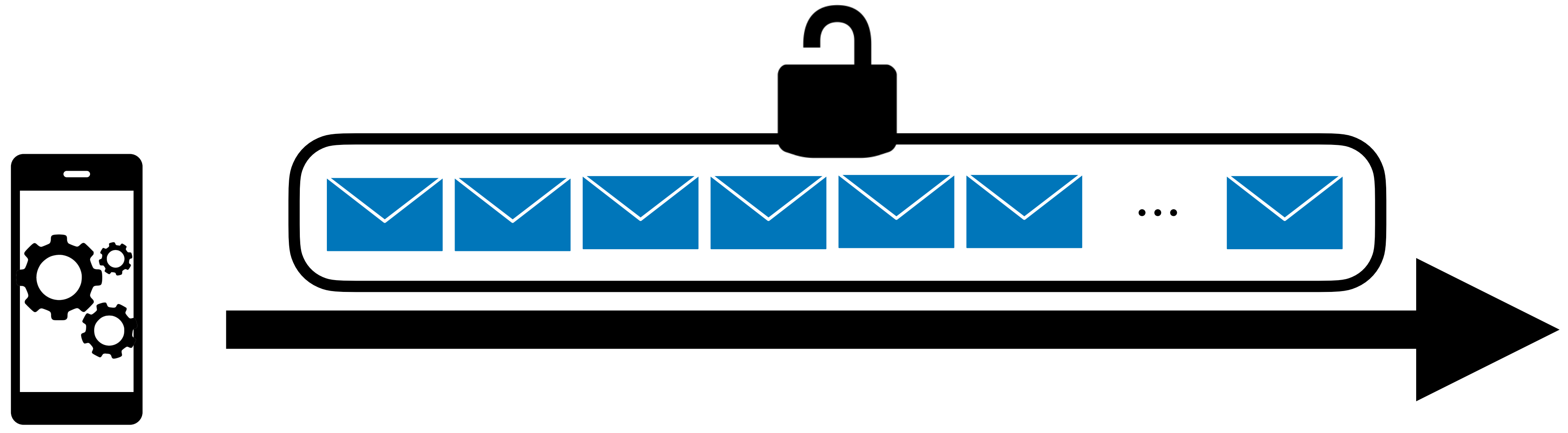
Two Phases:

Key exchange phase



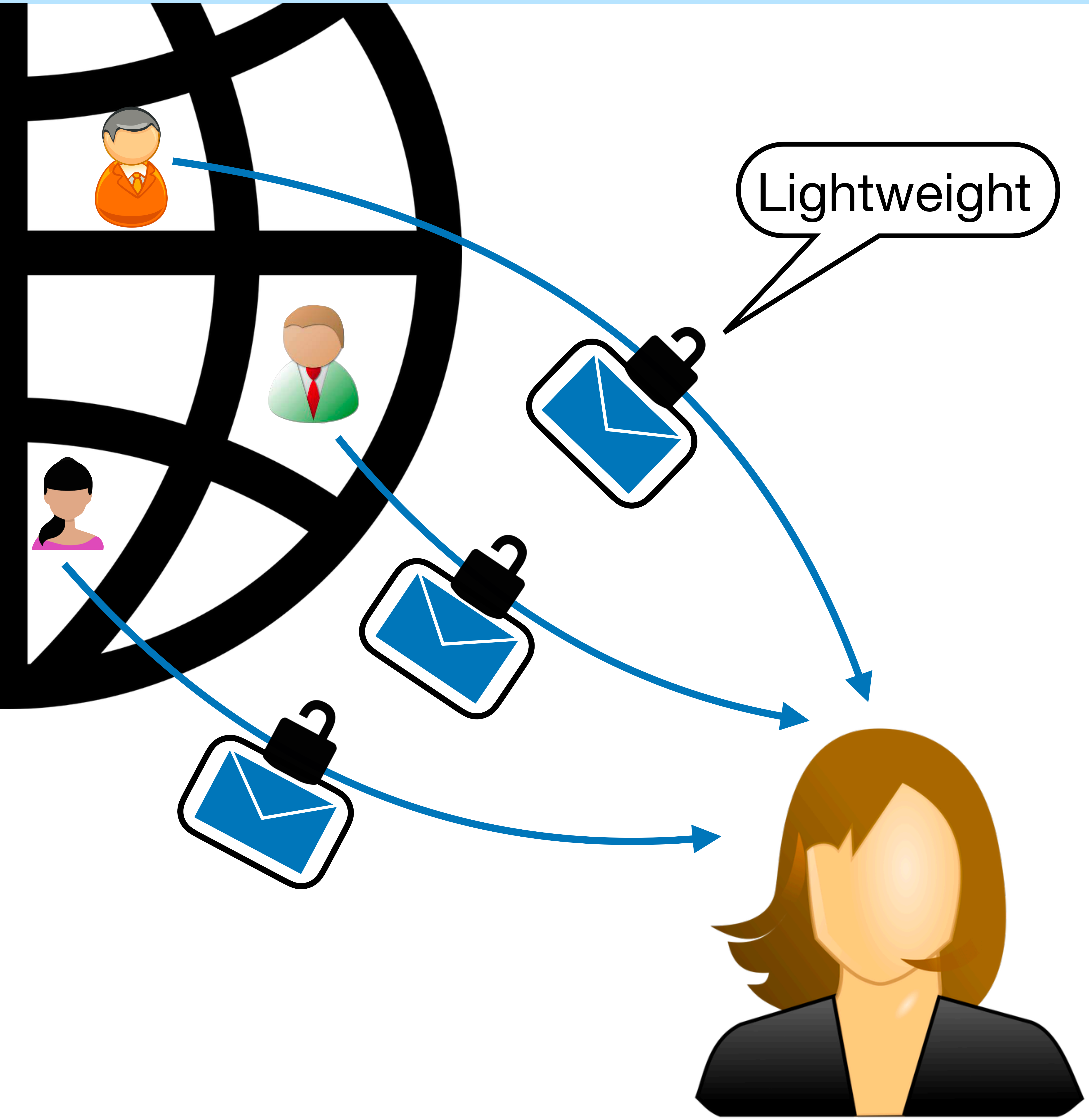
- One-time, *simultaneous* interaction
- Heavy (public key) computations
- Low communication $n \cdot |$🔒$|$ (not n^2)

Encryption phase



- Lightweight (symmetric) computations
- Optimal message-to-cipher ratio

A Closer Look at Secure Communication



Two Phases:

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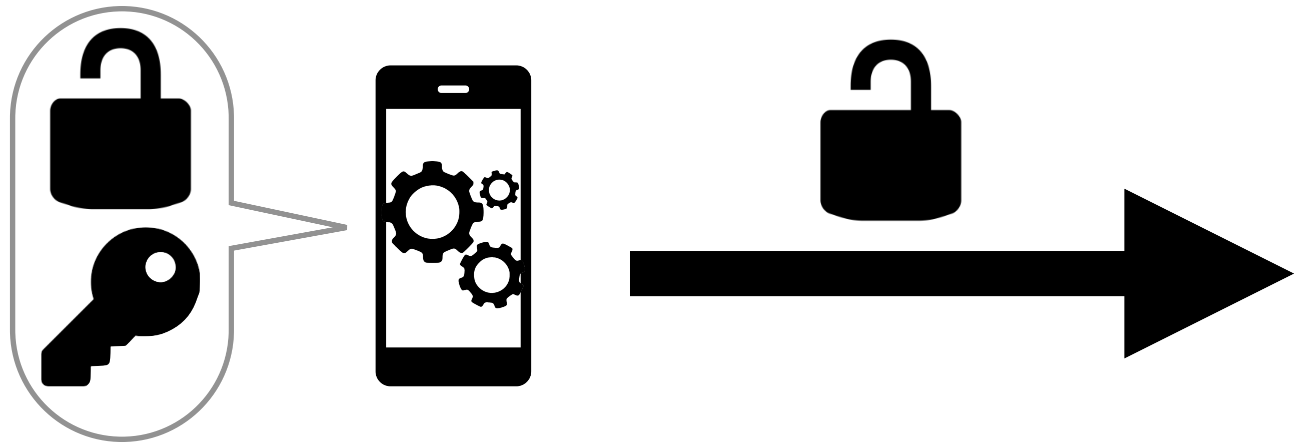

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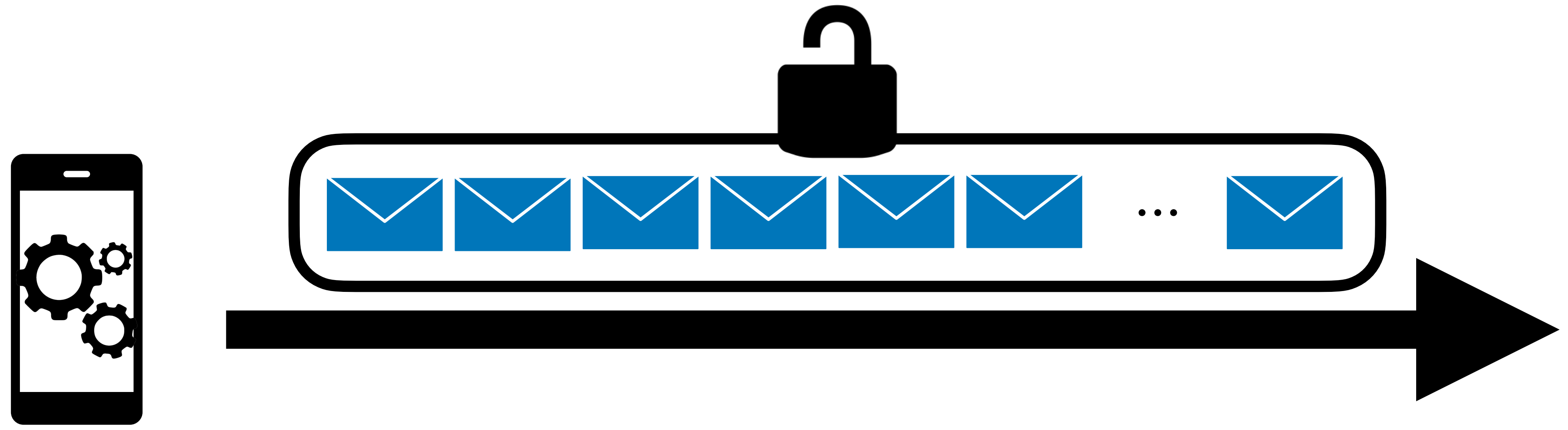

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Back to the PCG Template Again

$$\underbrace{\mathrm{FSS}}_{\text{Function secret sharing}}(\overbrace{C \circ \mathrm{PRG}}^{\text{Public function}}(\underbrace{\mathrm{seed}}_{\text{Short seed}}))$$

Using a PRG enables a *one-time* generation
of a *fixed* amount of correlations

Back to the PCG Template Again

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A pseudorandom correlation *function* is to a PCG what a PRF is to a PRG

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Are there any FSS-friendly PRFs?

Back to the PCG Template Again

$$\underbrace{\mathsf{FSS}}_{\text{Function secret sharing}}(\overbrace{C \circ \mathsf{PRF}}^{\text{Public function}}(\underbrace{\mathsf{seed}}_{\text{Short seed}}))$$

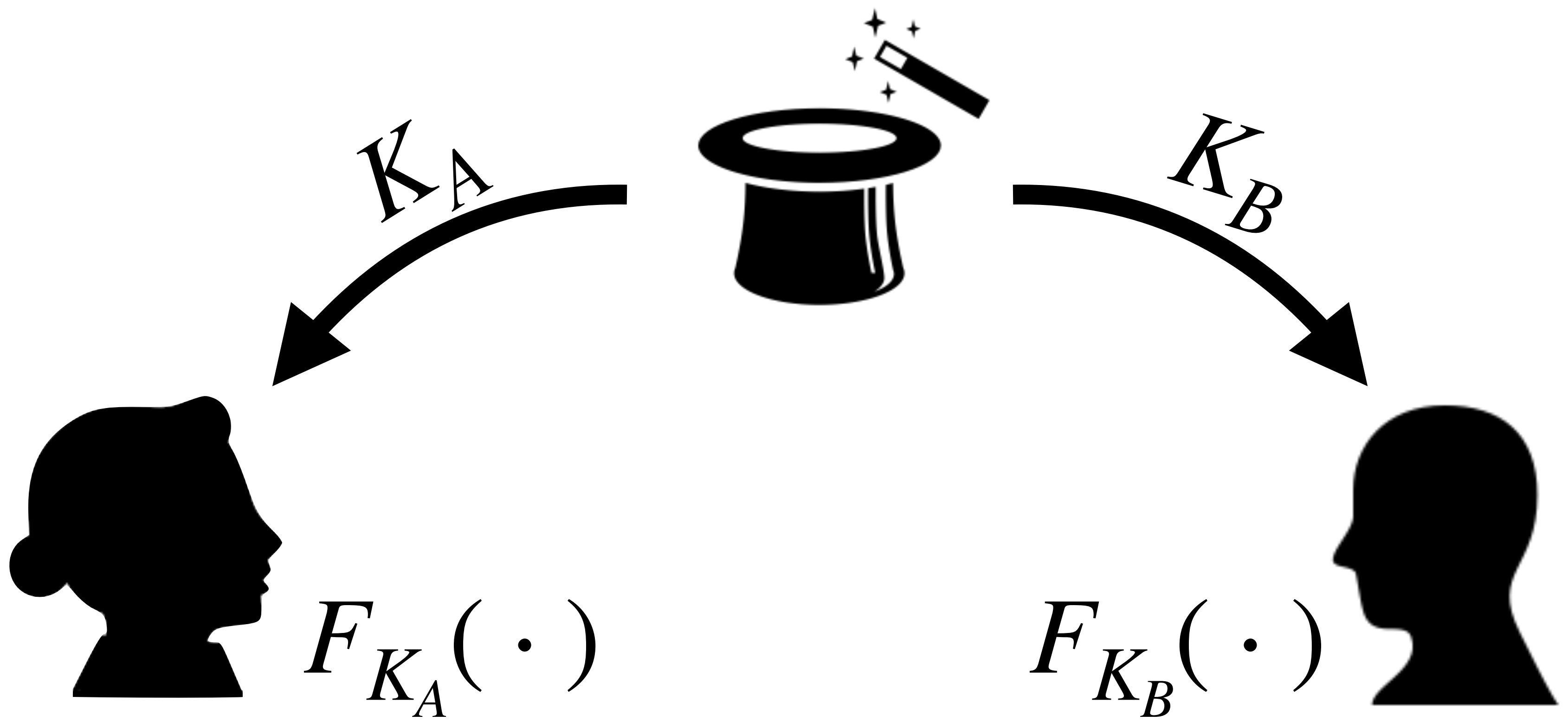
A pseudorandom correlation *function* is to a PCG what a PRF is to a PRG

The existence of PRFs in low complexity classes yields strong limitations for learning theory

Are there any FSS-friendly PRFs?

FOCS:BCGIKS20 and Crypto:BCGIKRS22 give plausible candidates

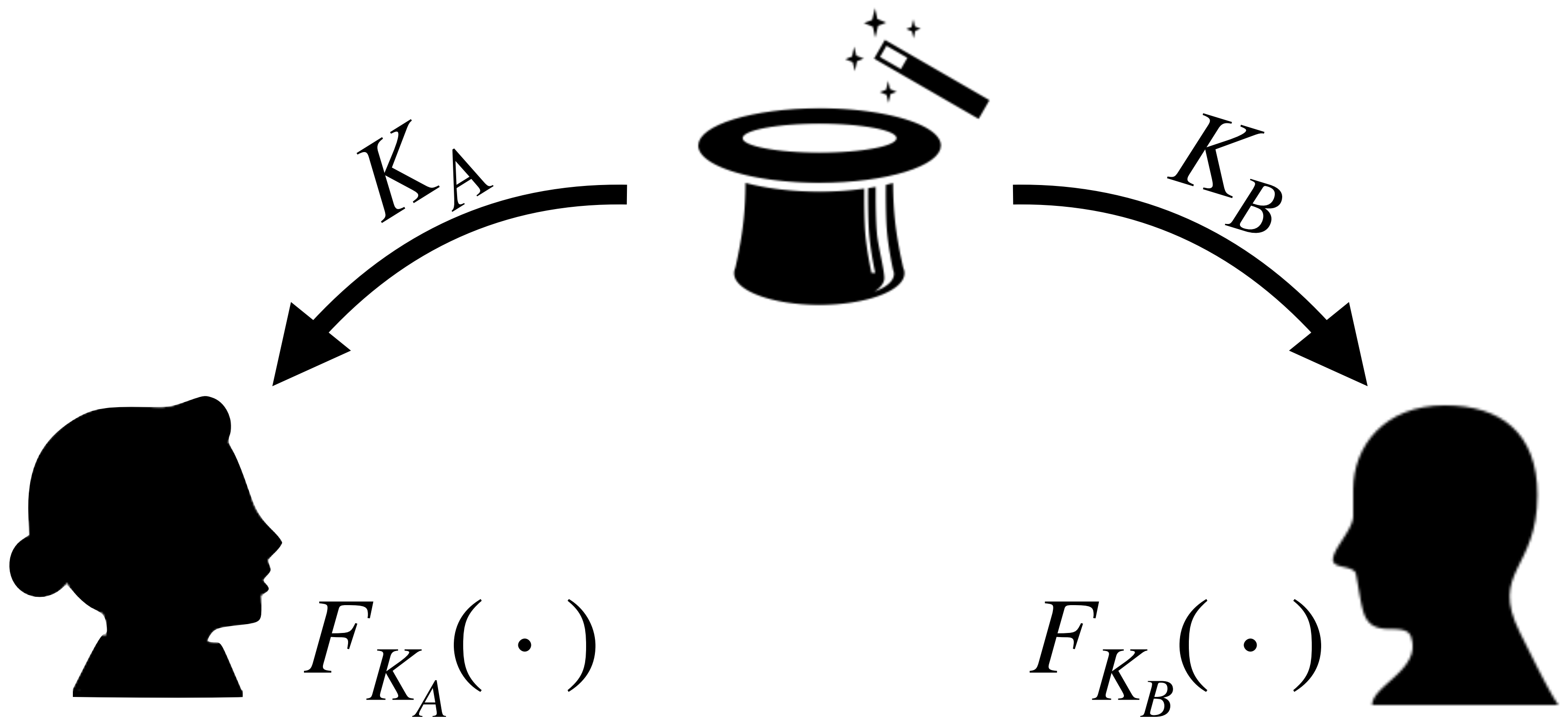
Pseudorandom Correlation Functions



Correctness & security:

- Black-box access to samples of the form $(F_{K_A}(x), F_{K_B}(x))$ are indistinguishable from black-box access to random samples from a target correlation.
- From the viewpoint of Alice, each $F_{K_B}(x)$ is indistinguishable from a random value sampled *conditioned on satisfying the correlation with* $F_{K_A}(x)$.
- Same condition in the other direction.

Public-Key Pseudorandom Correlation Functions



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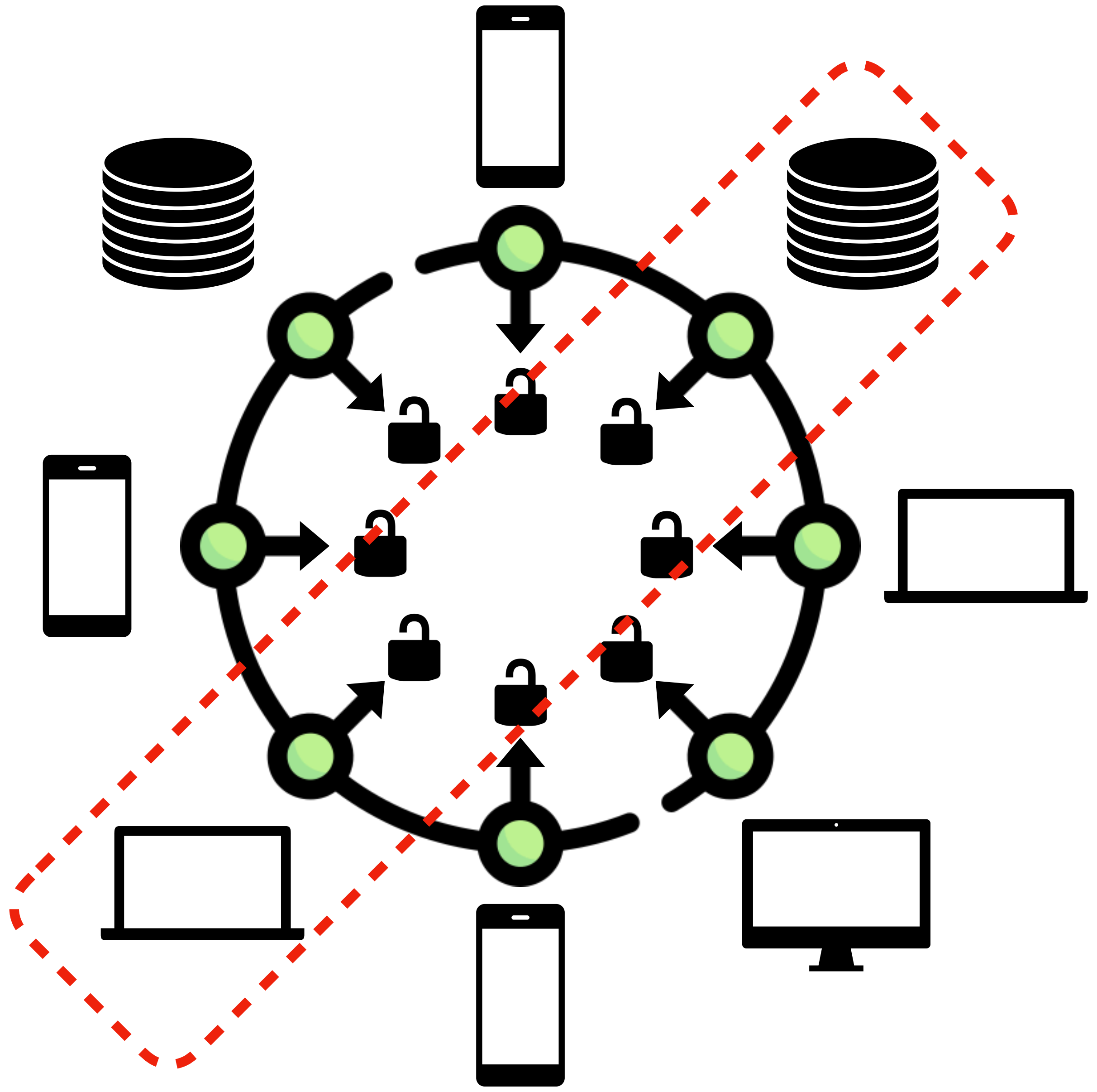
Achieving *non-interactive* silent key generation



Formally:

- KeyGen → (pk, sk) generates public and private PCF keys
- KeyDer(pk$_A$, sk$_B$) → K_B^{AB} yields Bob's PCF key w.r.t. Alice's key
- Eval$(K, x) \rightarrow y$ yields a pseudorandom sample

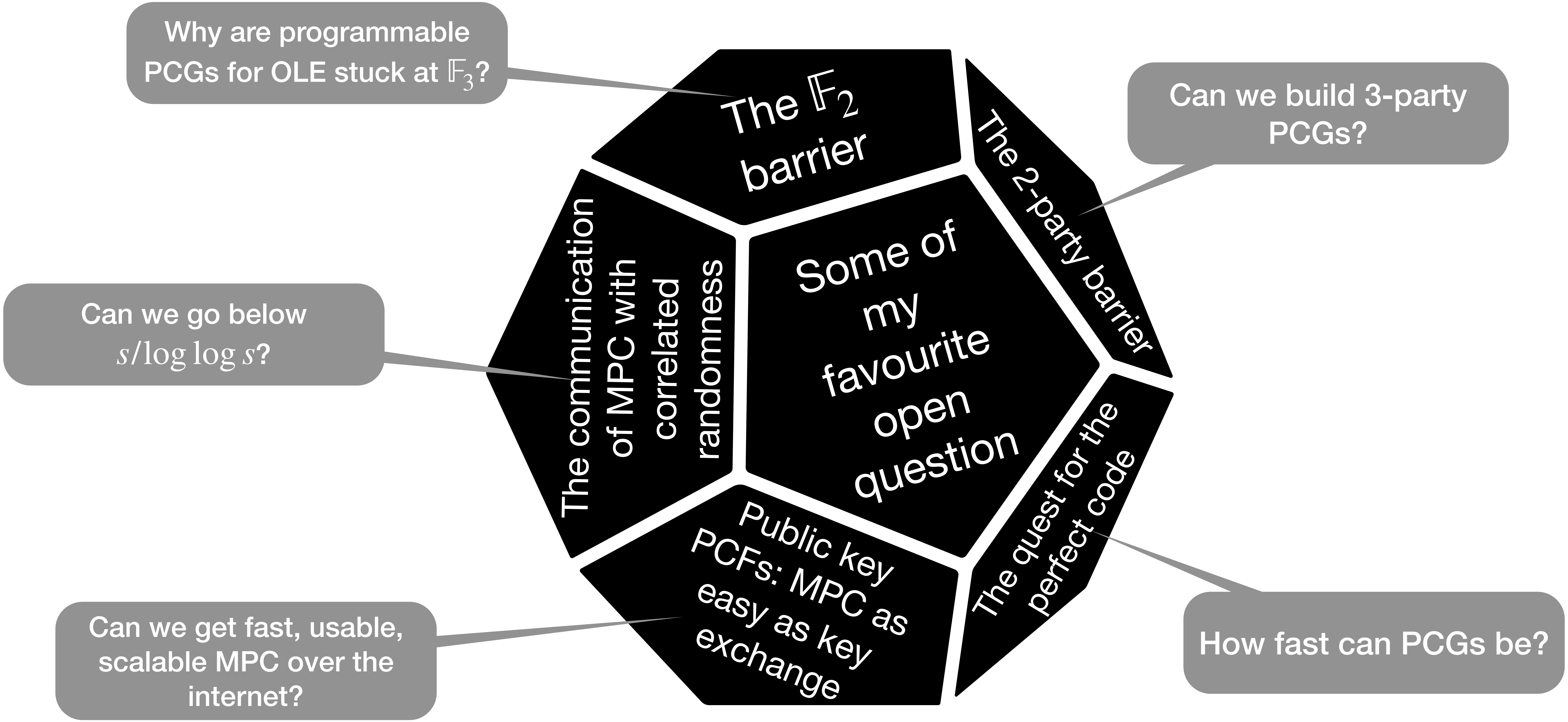
Public-Key Pseudorandom Correlation Functions

Public-key PCFs are exactly the *right tool* to enable scalable, on-demand 2-party secure computation over the Internet, with a communication and computation pattern close to that of secure *communication* over the web.

Building efficient public-key PCF is essentially a wide-open question: the recent work of EC:Orlandi-Scholl-Yakoubov'21 gets it for OT from QR, but efficiency is quite bad.

(Teaser) Coming soon: we have some exciting progress in this line of work, which does not fully solve the problem, but is a big step forward!

Some of my Favourite Open Questions



Some of my favourite open question

The $\mathbb{F}_2$ barrier

Why are programmable PCGs for OLE stuck at $\mathbb{F}_3$?

The 2-party barrier

Can we build 3-party PCGs?

The communication of MPC with correlated randomness

Can we go below $s/\log\log s$?

Public key PCFs: MPC as easy as key exchange

Can we get fast, usable, scalable MPC over the internet?

The quest for the perfect code

How fast can PCGs be?

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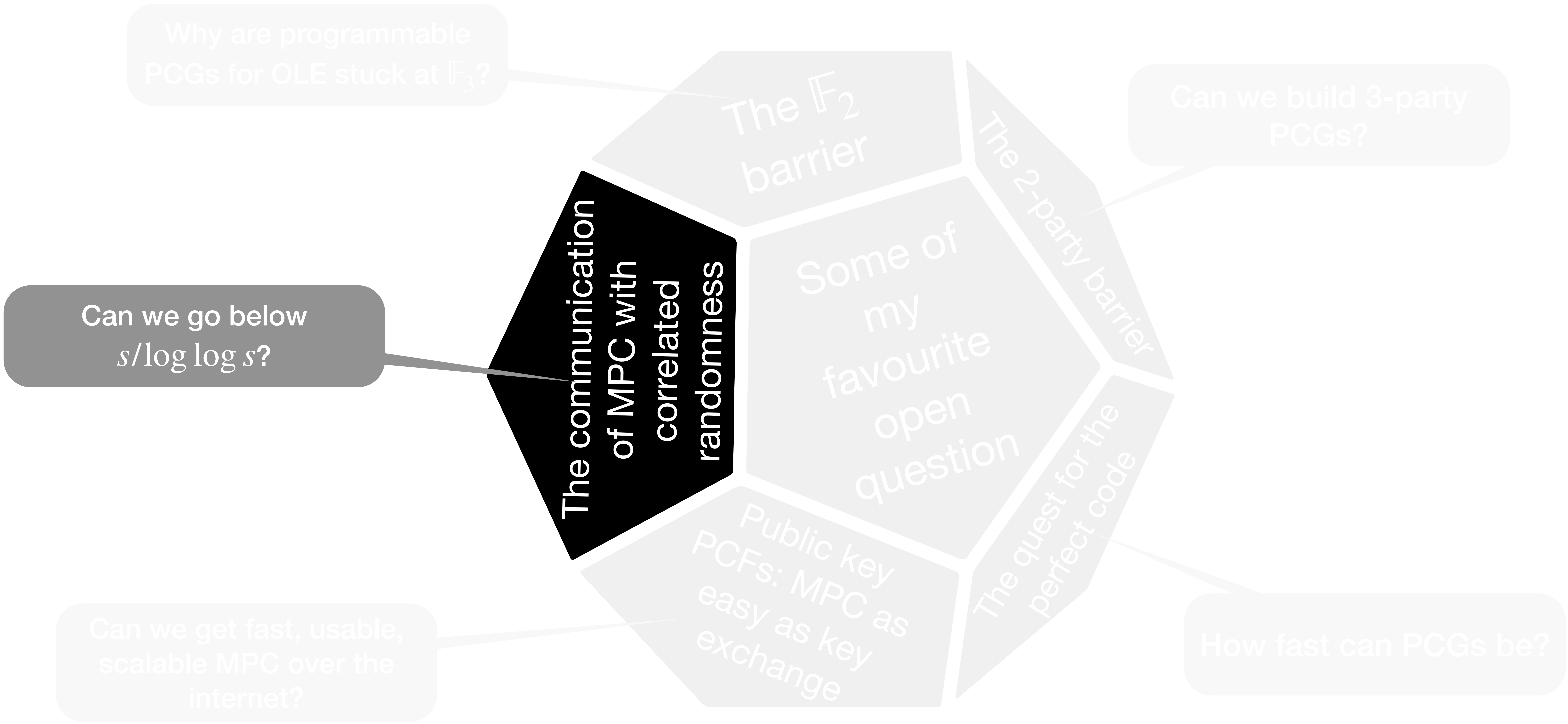

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No time left for that, but I'd be happy to discuss it over dinner tonight!

Other cool things to check out that I don't have time to discuss:

- People have been doing great things in zero-knowledge using these PCG techniques (incl. right here in Aarhus!)
- Everything we have so far works only for two parties!
- … And many more

Questions?