# A Note on the Communication Complexity of Multiparty Computation in the Correlated Randomness Model

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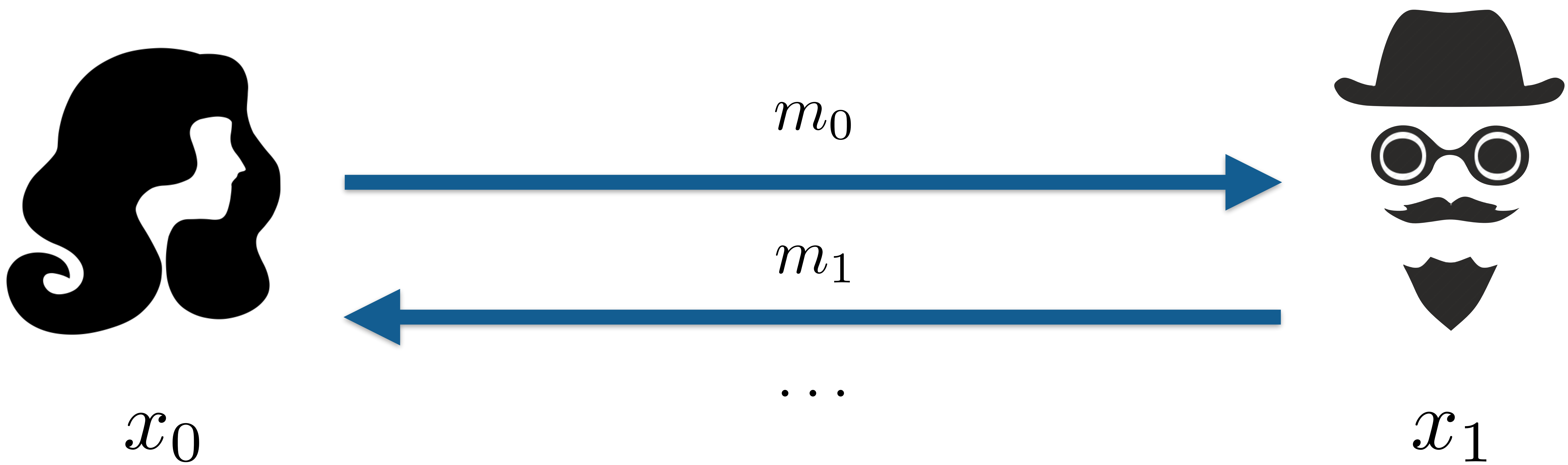
# The Quest for MPC with Low Communication




$$y = f(x_0, x_1)$$

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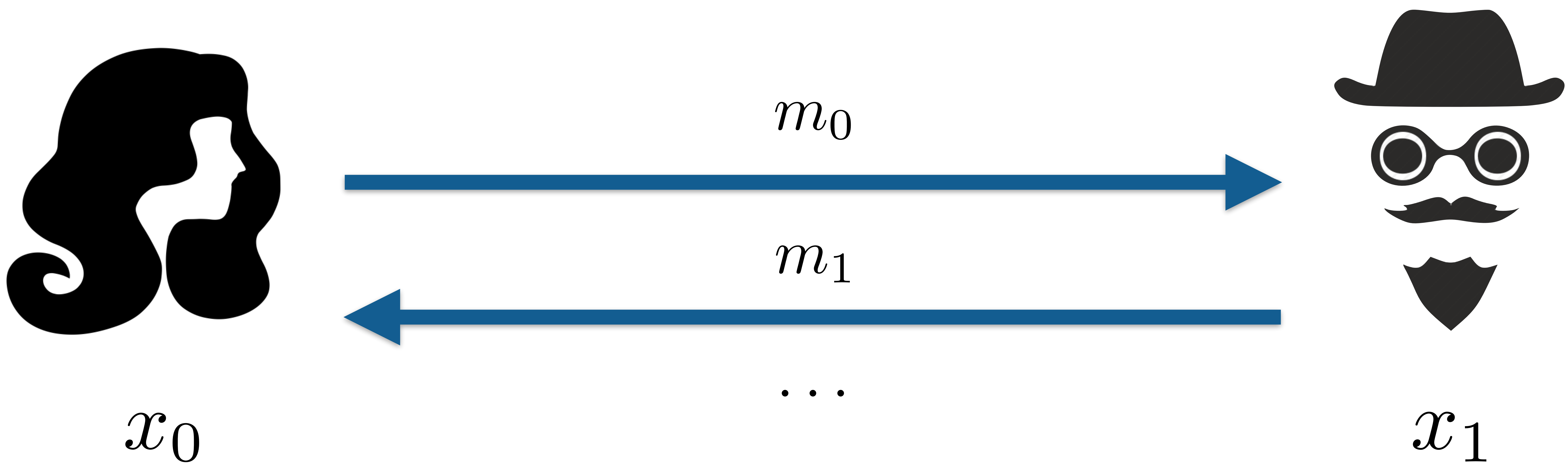


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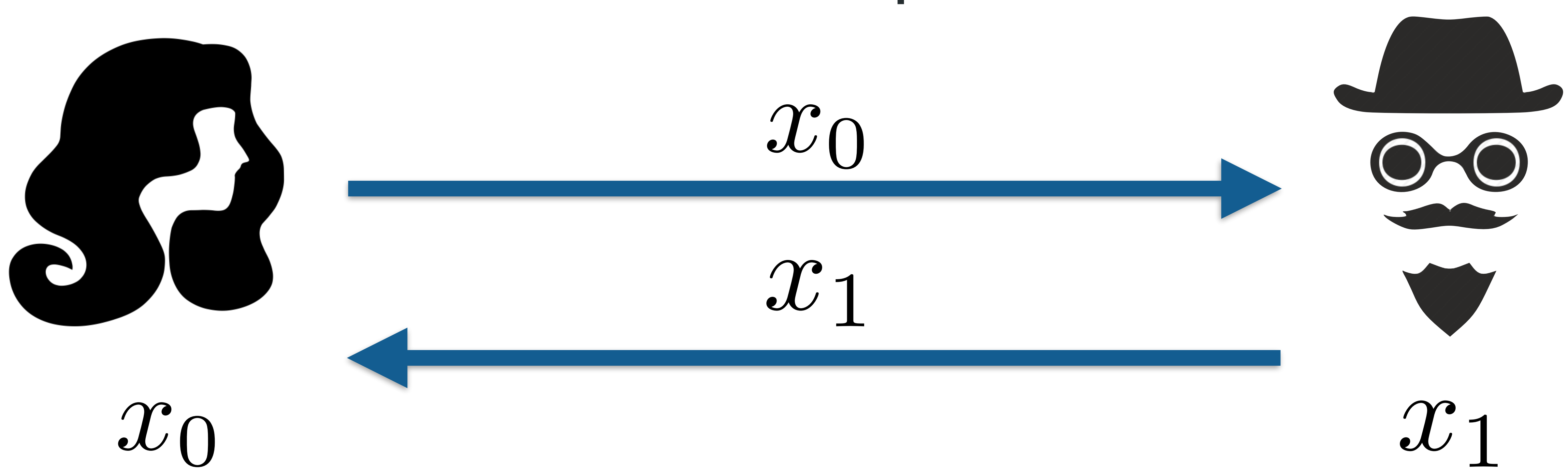
- Correctness: the parties learn the correct output
- Privacy: the parties learn nothing more than the output

# The Quest for MPC with Low Communication

Insecure computation:

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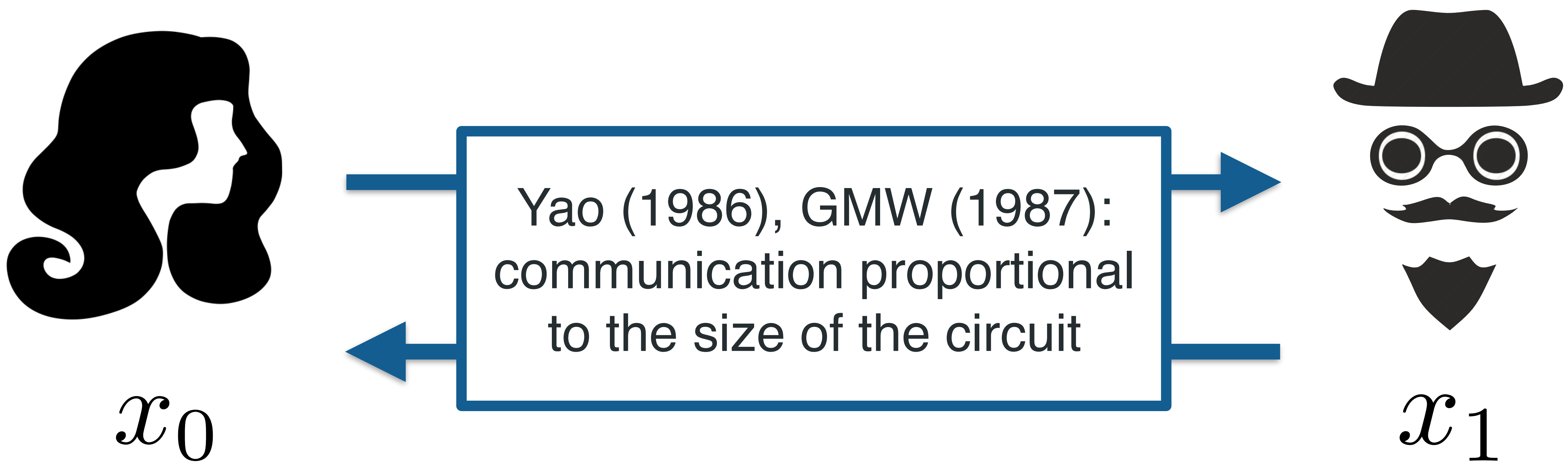


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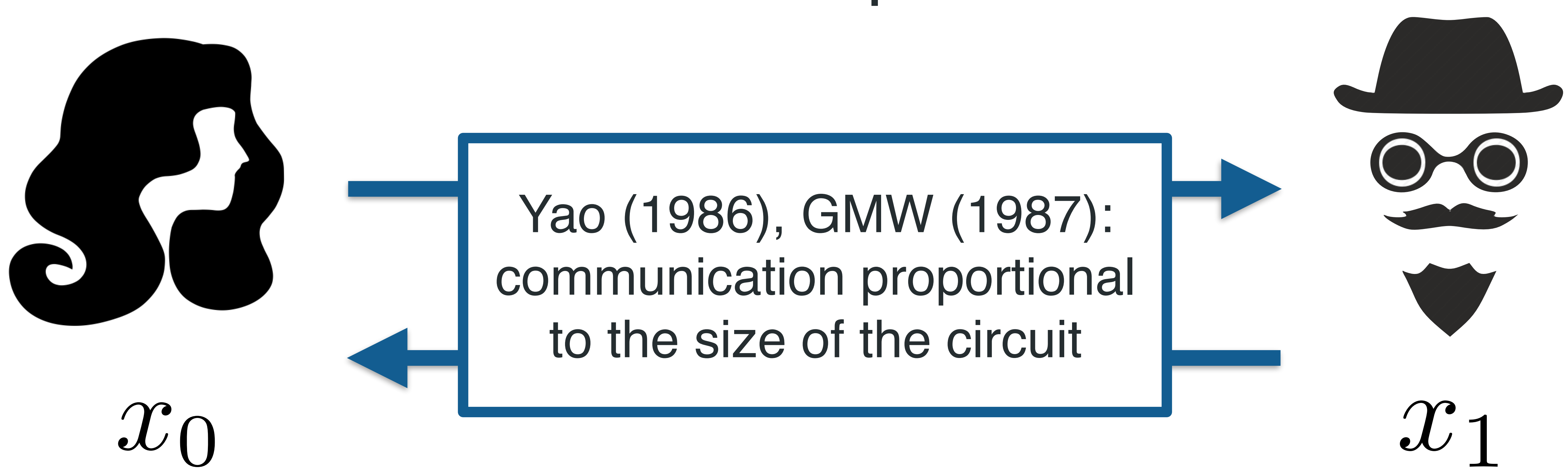


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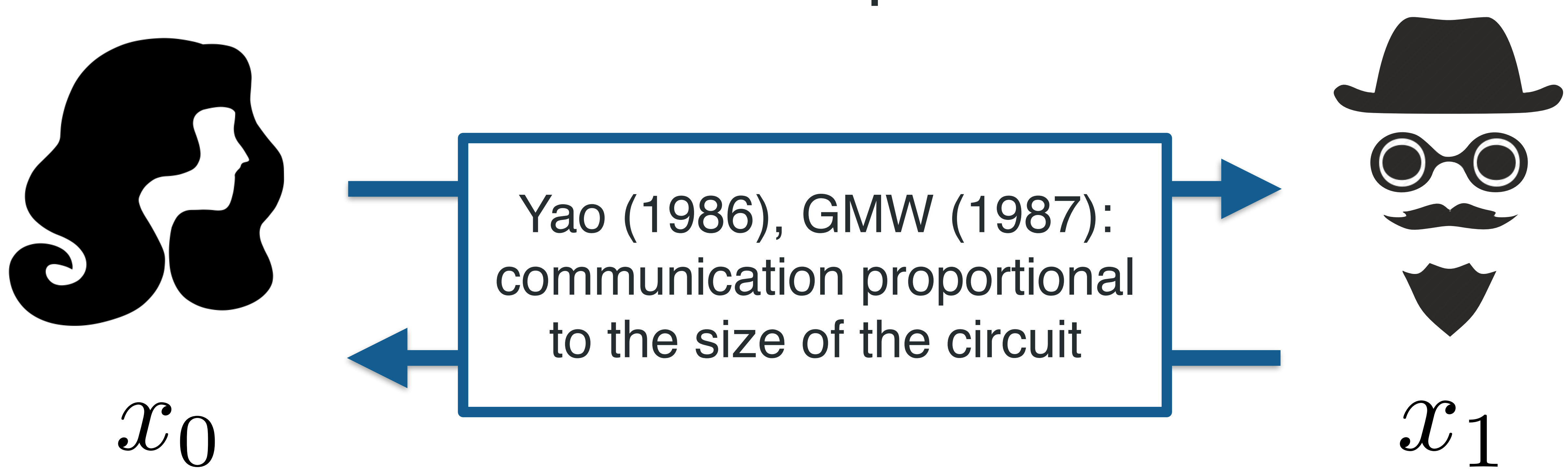


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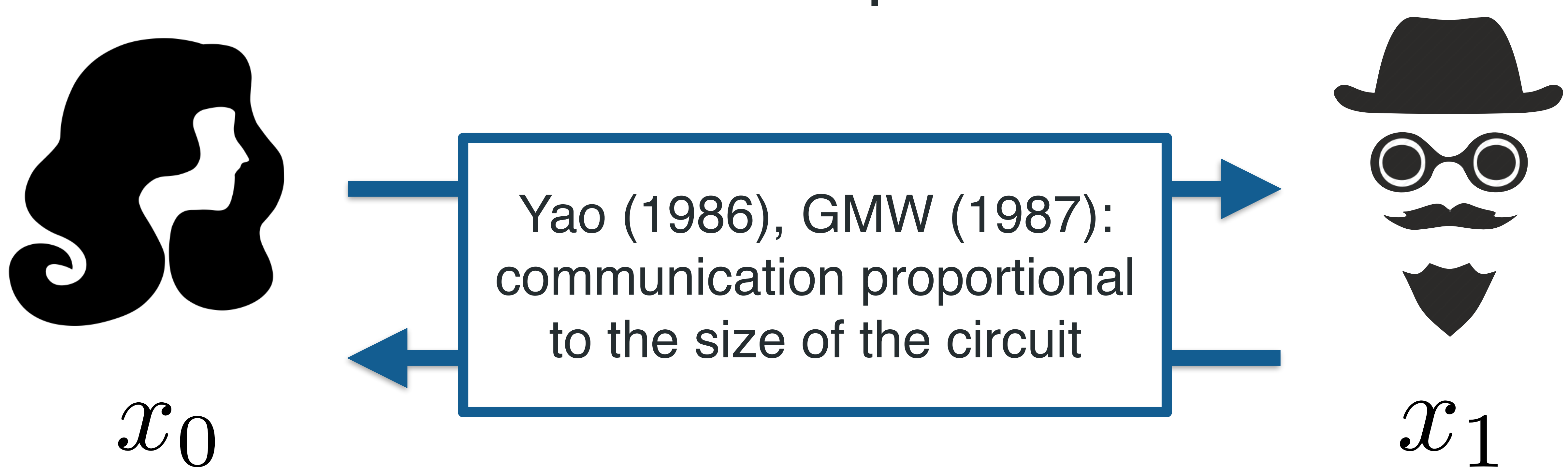


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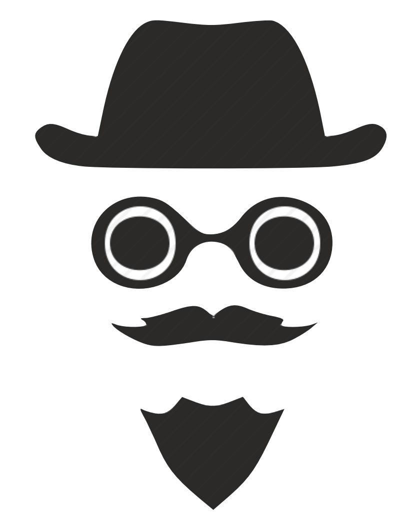
This work: revisiting this question for MPC with correlated randomness

# MPC with Correlated Randomness

Generates and distributes correlated random coins,
independent of the inputs of the parties

$x_0$



$x_1$

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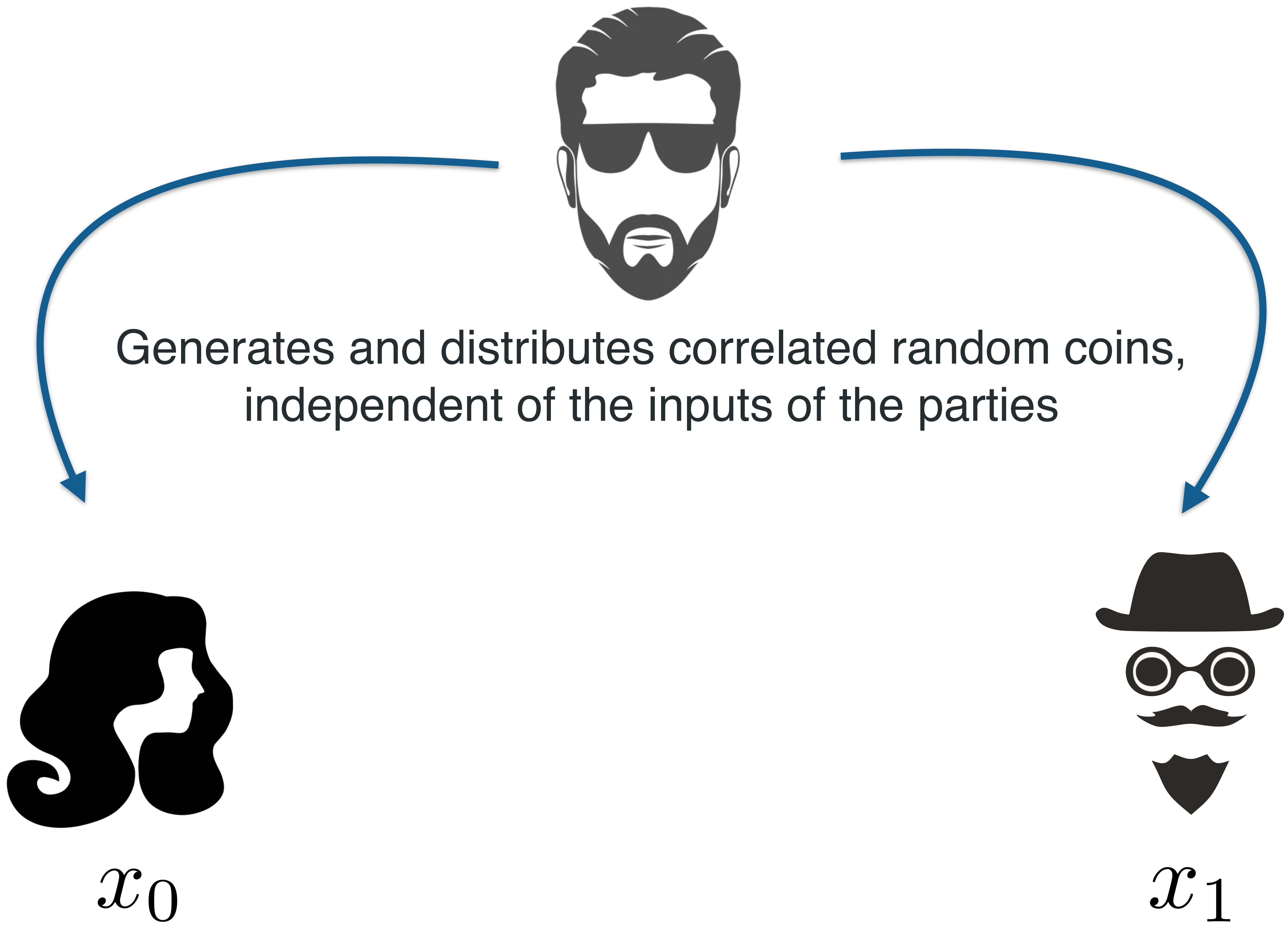

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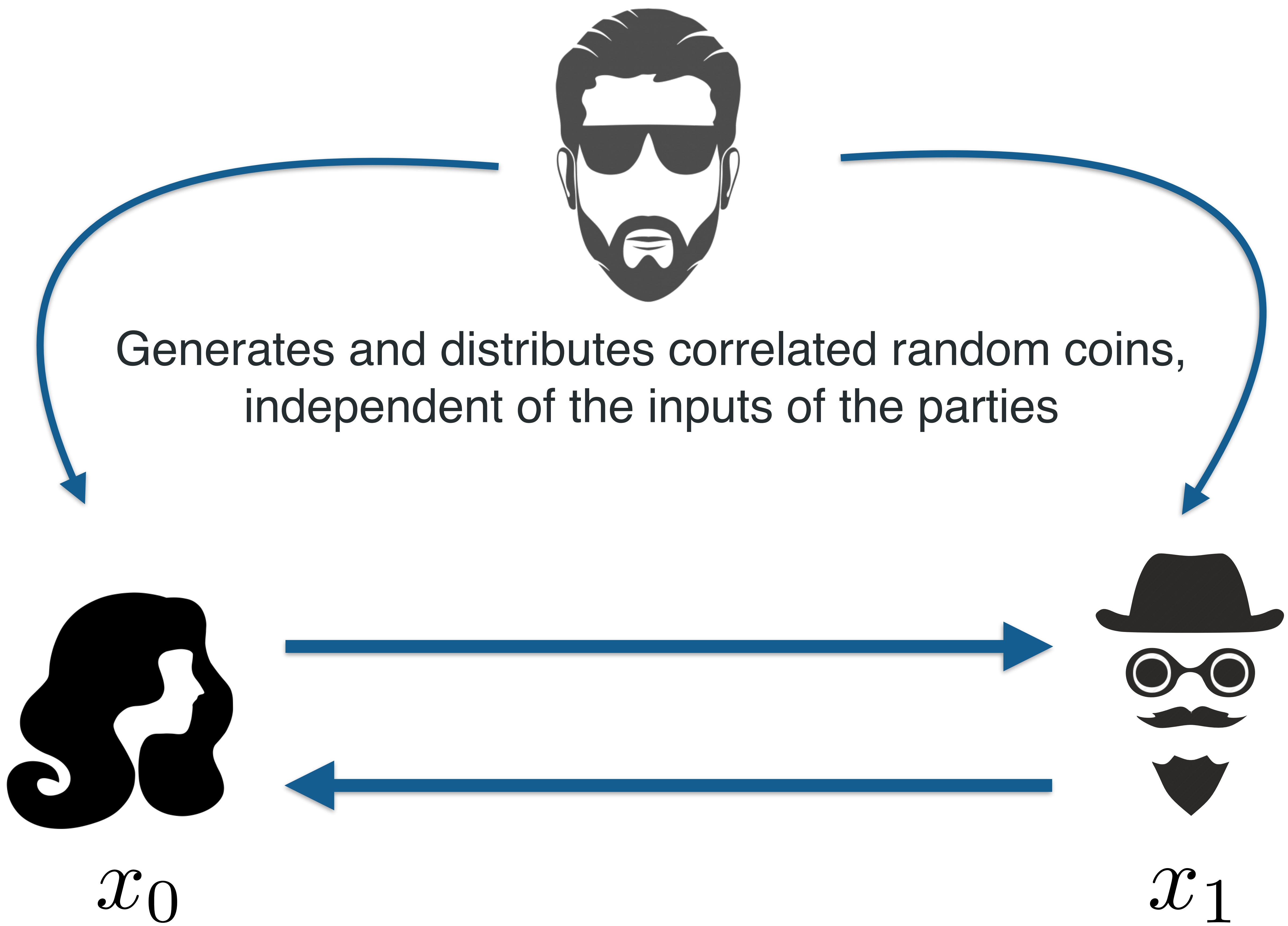

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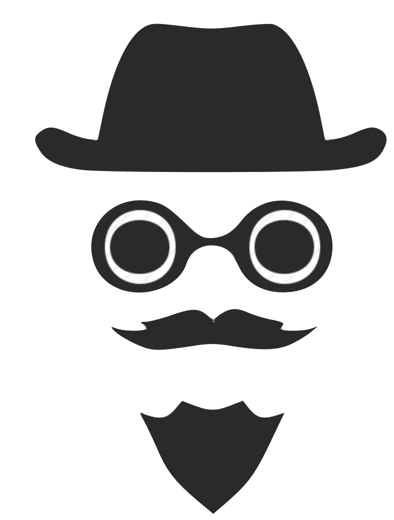
Beaver (1991): this allows for information-theoretically secure MPC in the online phase



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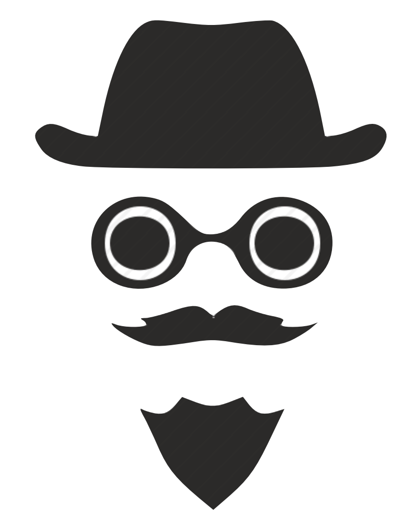

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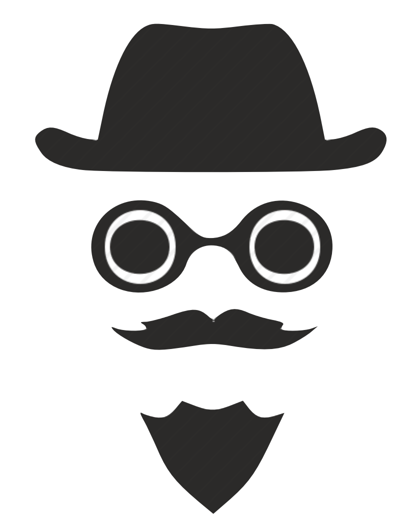

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All known protocols in the correlated randomness model have communication proportional to the circuit size

DNPR16: this is inherent for gate-by-gate protocols

# Our Result

For any layered boolean circuit $C$ of size $s$ with $n$ inputs and $m$ outputs, there exists an $N$-party protocol which securely evaluates $C$ in the (function-dependent) correlated randomness model against malicious parties, with adaptive security, and without honest majority, using a polynomial number of correlated random coins and with communication

$$O\left(n + N \cdot \left(m + \frac{s}{\log \log s}\right)\right).$$

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+ Concrete efficiency improvements for TinyTable

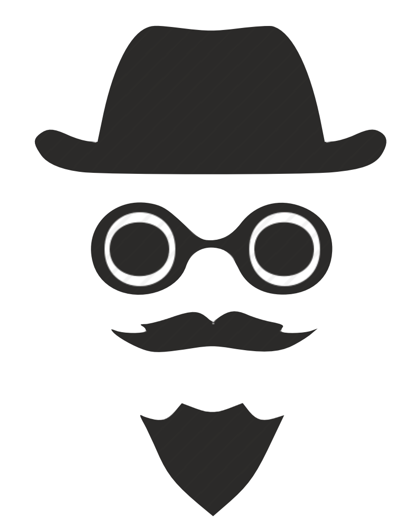
We'll focus on 2 parties & semi-honest security here

# Sharing Truth-Table Correlations

$$f(x) = f(x_0 + x_1)$$

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $M =$ | f(0) | f(1) | f(2) | f(3) | f(4) | f(5) | … | … | … | … | f(N-5) | f(N-4) | f(N-3) | f(N-2) | f(N-1) | f(N) |

$x_0$



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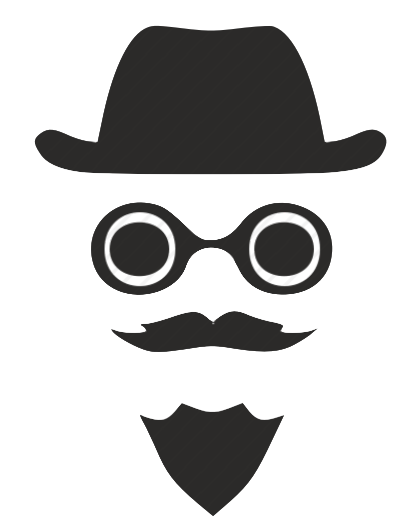
$r$

picks a random offset
$r = r_0 + r_1$

$x_0$



$x_1$

# Sharing Truth-Table Correlations

$$f(x + r) = f((x_0 + r_0) + (x_1 + r_1))$$

$M' =$

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
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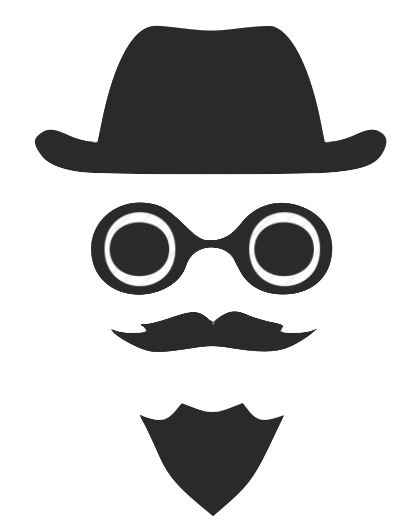

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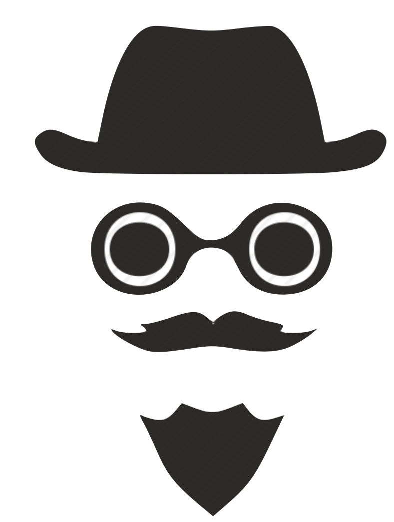
picks a random offset
$r = r_0 + r_1$

shares M’ into
$M' = M'_0 + M'_1$

$x_0$



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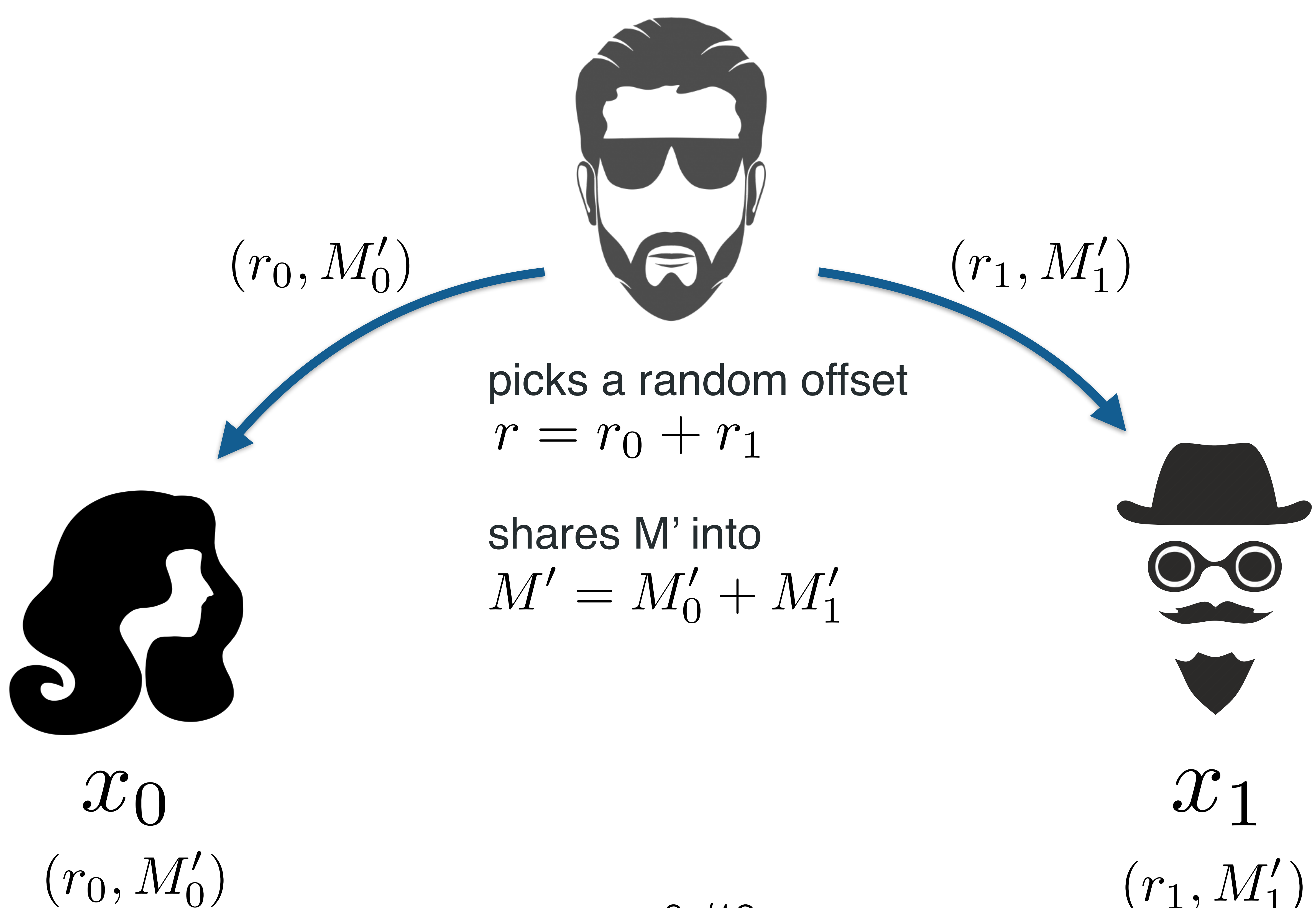

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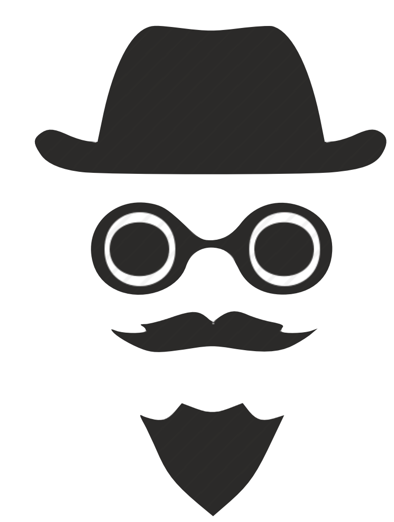
$x_0$

$(r_0, M'_0)$



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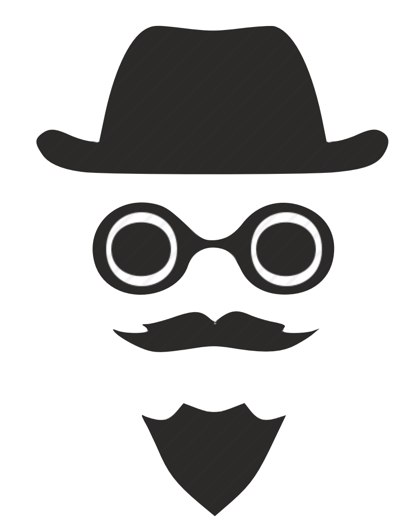

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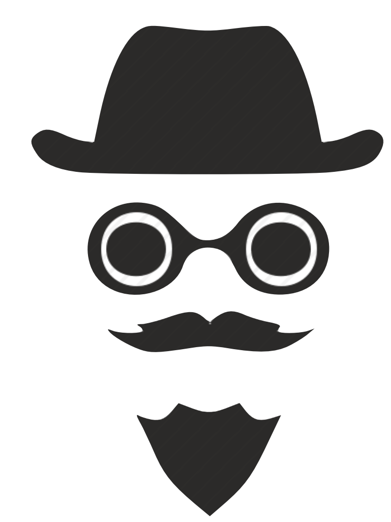

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$y_0 + y_1 = M'|_{x+r} = f(x)$

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communication: $2n$

storage: $m \cdot 2^n + n$

$y_0 \leftarrow M'_0|_{u_0+u_1}$

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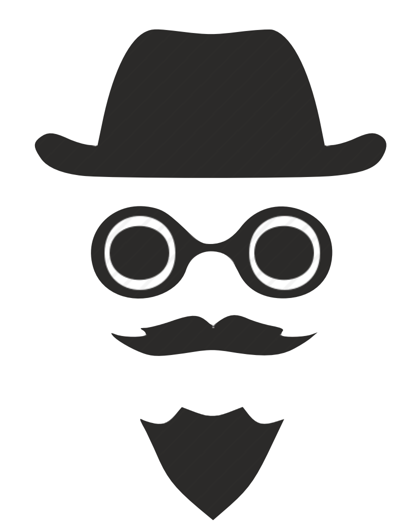

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that's great

communication: $2n$

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that's bad

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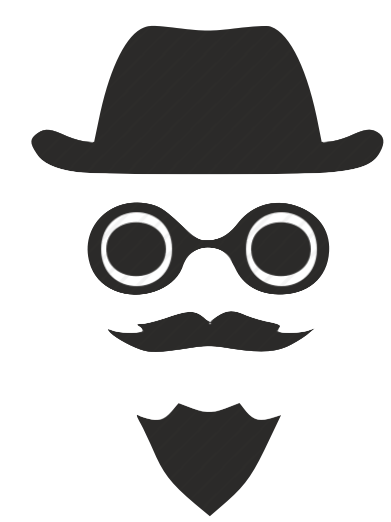

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communication: $2n$

storage: $m \cdot 2^n + n$

that's bad

IKMOP (2013): a polynomial storage for all functions would imply a breakthrough in information-theoretic PIR

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$u_0 = x_0 + r_0$

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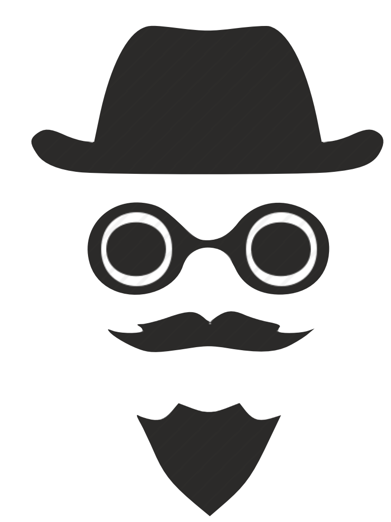

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# The Core Lemma

Let $f$ be a $c$-local function, with input of size $n$ and output of size $m$. Then there exists a protocol $\Pi$ which securely computes shares of $f$ in the correlated randomness model, with optimal communication $O(n)$ and storage $m \cdot 2^c + n$.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
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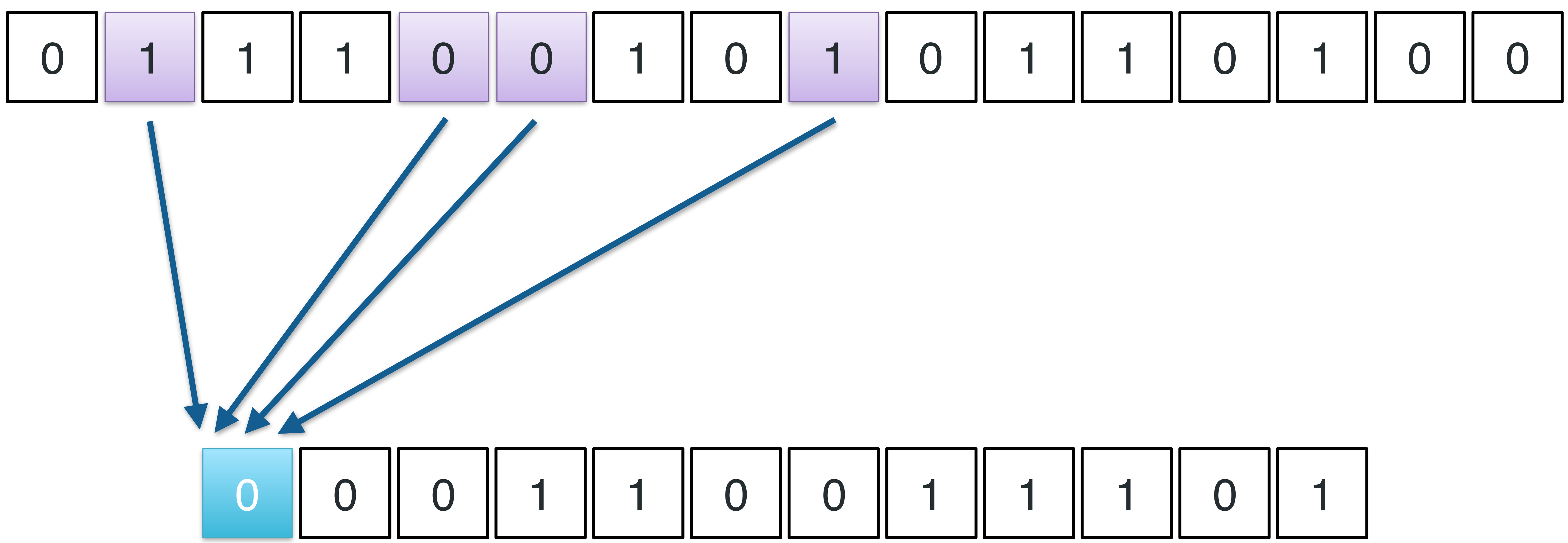


$$f(x) = (f_1(x[S_1]), f_2(x[S_2]), \cdots, f_m(x[S_m]))$$
$$\forall i, \ |S_i| = c$$

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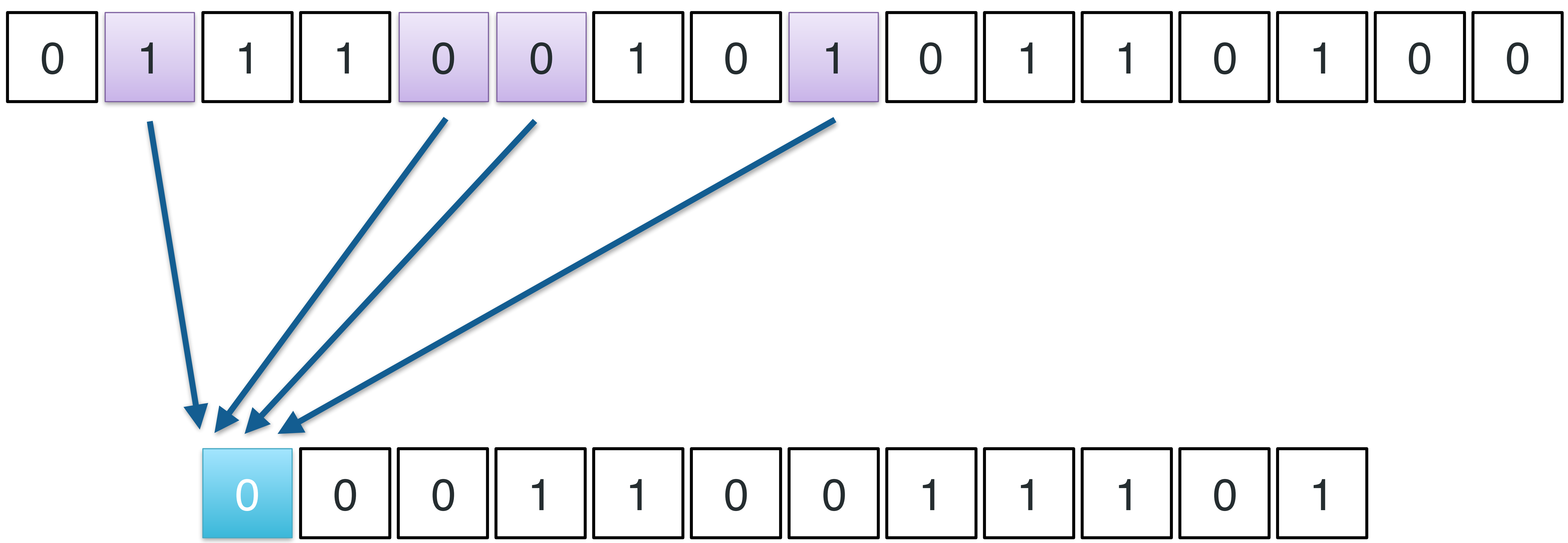
instead of $m \cdot 2^n + n$

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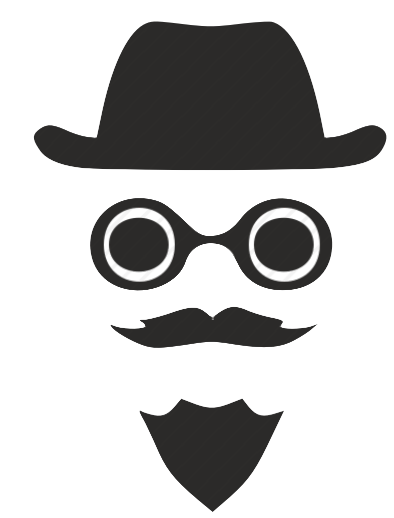
$x_0$



$x_1$

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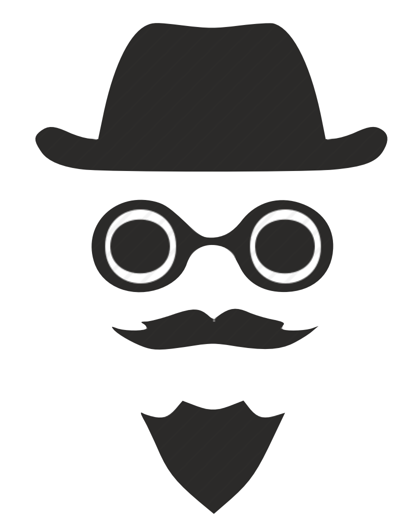
$(M_0, M_1, \cdots, M_m)$



$x_0$ $x_1$

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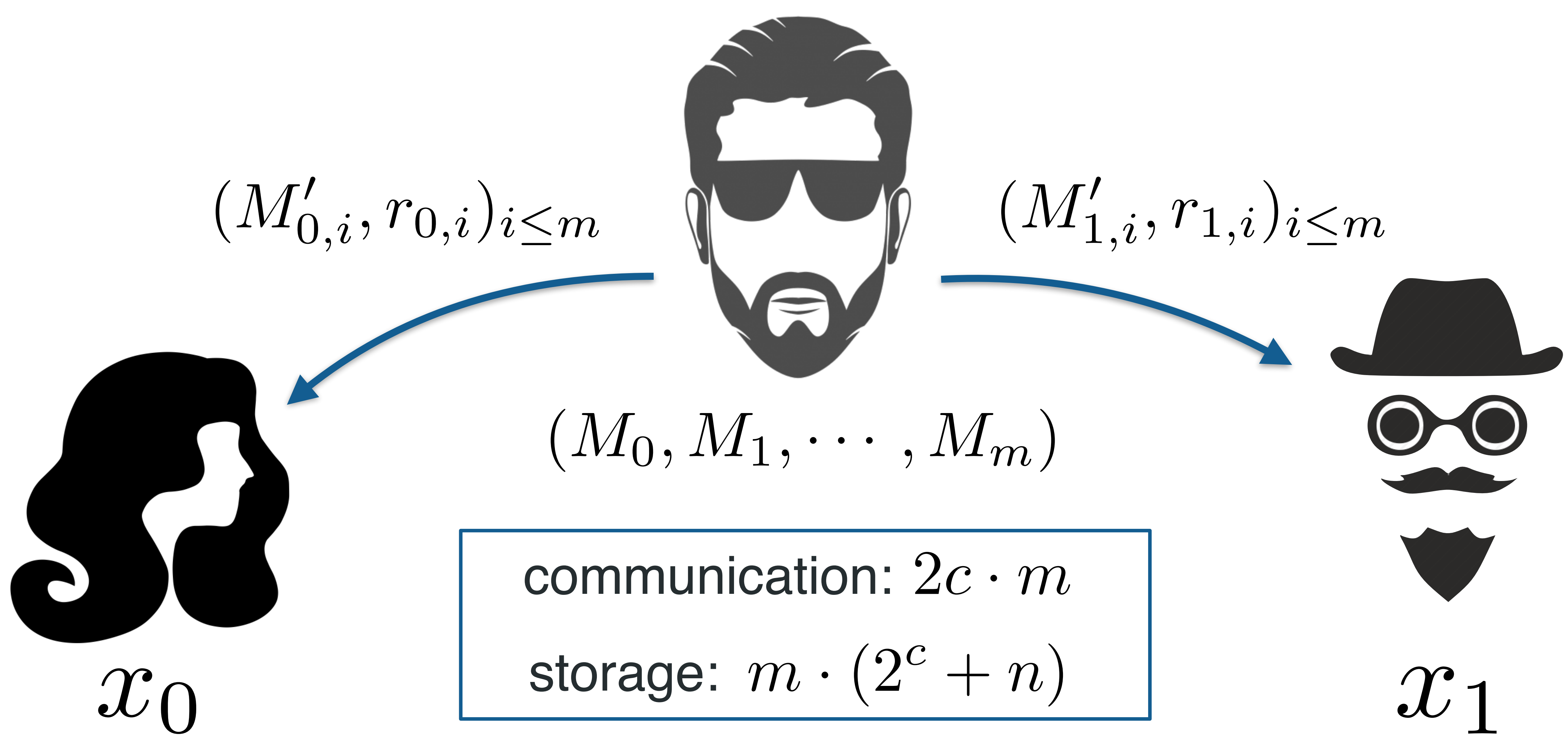


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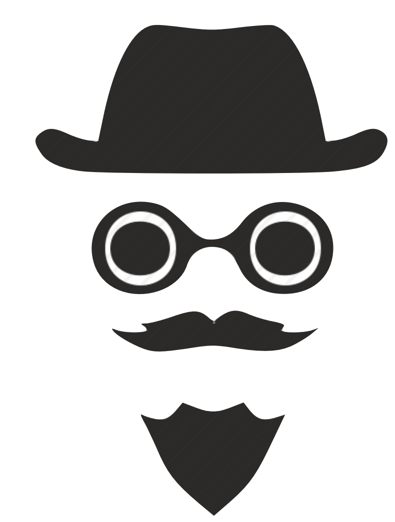
$M_1$ , $M_2$ ... $M_m$

| $f_1(1)$ | $f_1(2)$ | ... | $f_1(2^c)$ | , | $f_2(1)$ | $f_2(2)$ | ... | $f_2(2^c)$ | ... | $f_m(1)$ | $f_m(2)$ | ... | $f_m(2^c)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

$\xrightarrow{\quad r_1 \quad}$ $\xrightarrow{\ r_2\ }$ $\xrightarrow{\quad r_m \quad}$

$$\forall i, \ |r_i| = c$$

$(x_0[s_i] + r_{0,i})_i$



$(x_1[s_i] + r_{1,i})_i$

# The Core Lemma

Let $f$ be a $c$-local function, with input of size $n$ and output of size $m$. Then there exists a protocol $\Pi$ which securely computes shares of $f$ in the correlated randomness model, with optimal communication $O(n)$ and storage $m \cdot 2^c + n$.

$$f(x) = (f_1(x[S_1]), f_2(x[S_2]), \cdots, f_m(x[S_m]))$$

$M_1$ , $M_2$ ... $M_m$

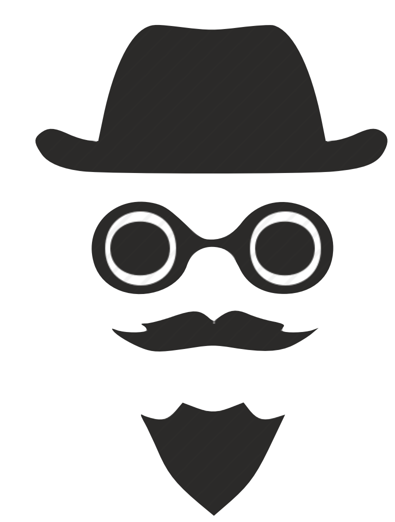
| $f_1(1)$ | $f_1(2)$ | ... | $f_1(2^c)$ | , | $f_2(1)$ | $f_2(2)$ | ... | $f_2(2^c)$ | ... | $f_m(1)$ | $f_m(2)$ | ... | $f_m(2^c)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

$\xrightarrow{\hspace{4em}}$ $r_1$ $\quad$ $\xrightarrow{\hspace{1em}}$ $r_2$ $\quad$ $\xrightarrow{\hspace{3em}}$ $r_m$

$$\forall i, \ |r_i| = c$$

$x_0 + r_0$

Idea: pick a single global offset $r$, and set $r_i \leftarrow r[S_i]$



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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

$r_1$ $r_2$ $r_m$

$$\forall i, \ |r_i| = c$$

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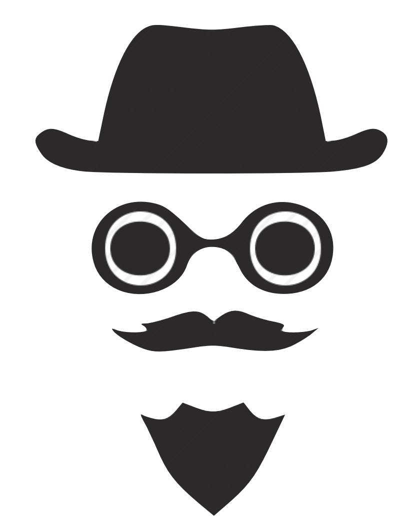
Idea: pick a single global offset $r$, and set $r_i \leftarrow r[S_i]$

communication: $2n$

storage: $m \cdot 2^c + n$



$x_1 + r_1$

# The Core Lemma

Let $f$ be a $c$-local function, with input of size $n$ and output of size $m$. Then there exists a protocol $\Pi$ which securely computes shares of $f$ in the correlated randomness model, with optimal communication $O(n)$ and storage $m \cdot 2^c + n$.

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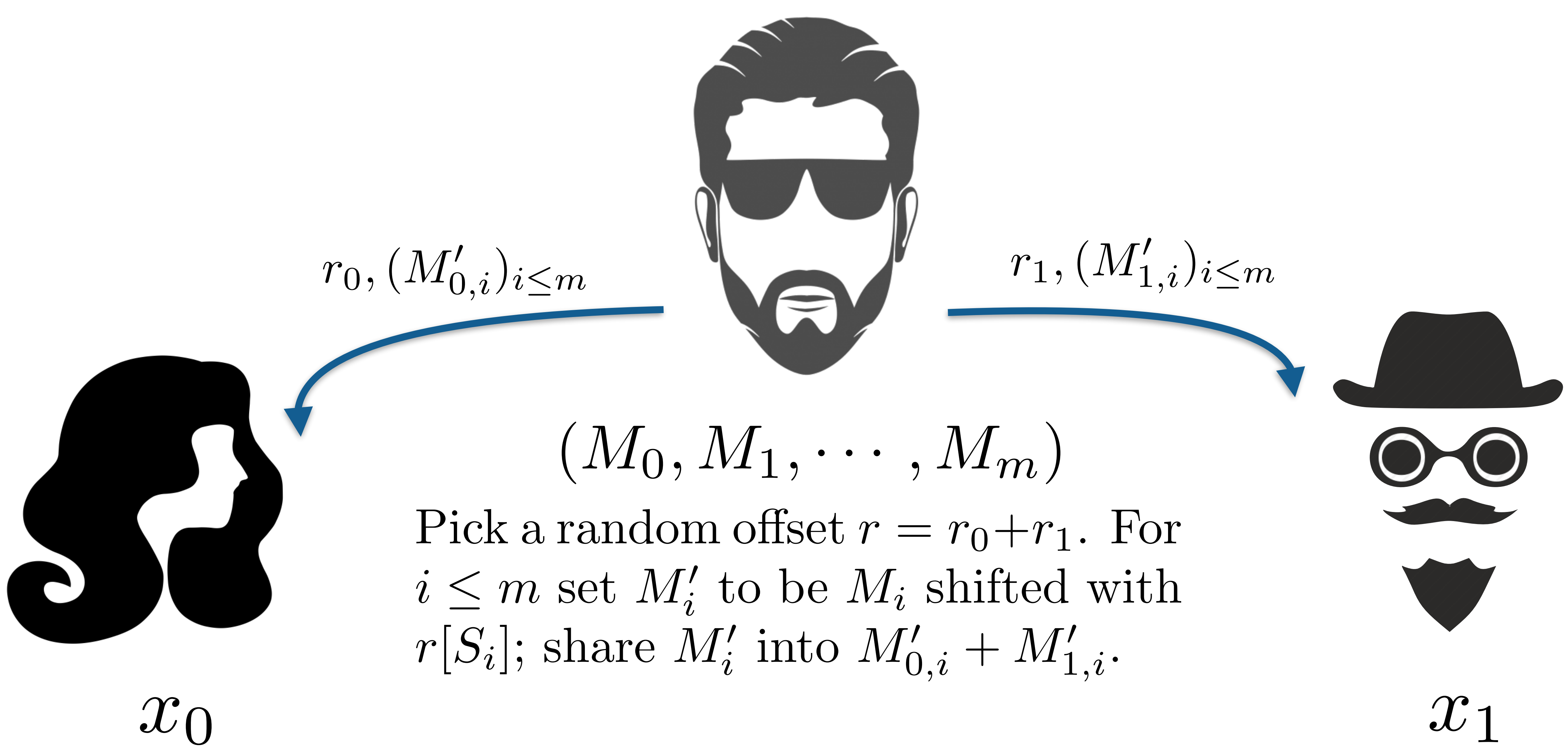

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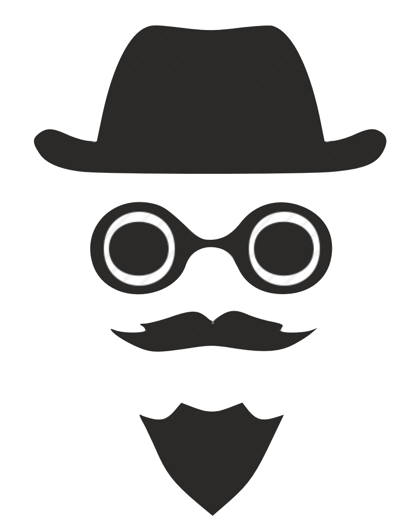
$$f(x) = (f_1(x[S_1]), f_2(x[S_2]), \cdots, f_m(x[S_m]))$$

$y_{0,i} \leftarrow M'_{0,i}|_{u[S_i]}$

$y_{1,i} \leftarrow M'_{1,i}|_{u[S_i]}$

$u_0 = x_0 + r_0$

$u_1 = x_1 + r_1$



$x_0$

$r_0, (M'_{0,i})_{i \leq m}$

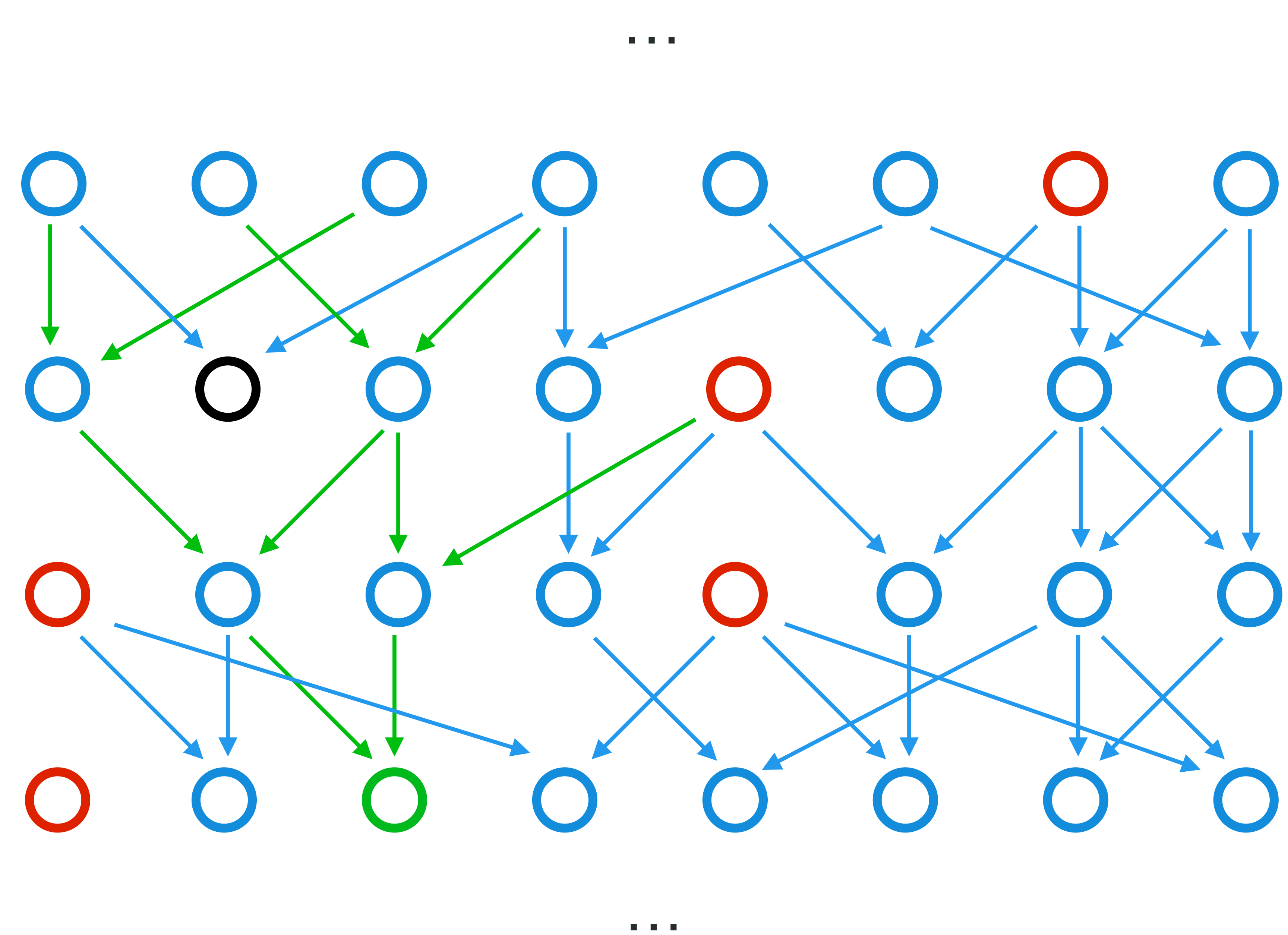
$u \leftarrow u_0 + u_1$

$x_1$

$r_1, (M'_{1,i})_{i \leq m}$

# Construction

Layered boolean circuit, size $s$, depth $d$, width $w$, $n$ inputs and $m$ outputs



: node

: input node

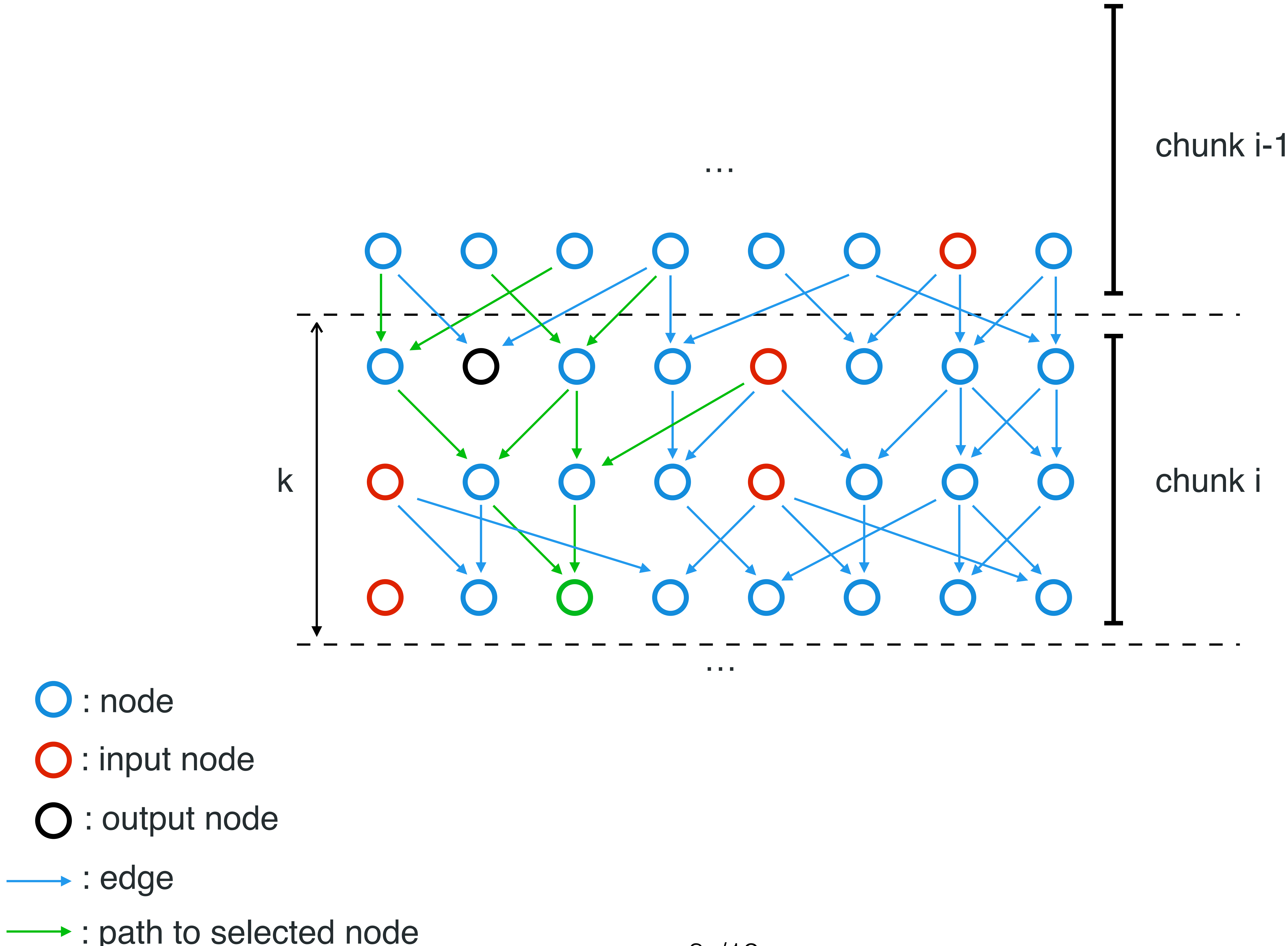
: output node

: edge

: path to selected node

# Construction

Layered boolean circuit, size $s$, depth $d$, width $w$, $n$ inputs and $m$ outputs



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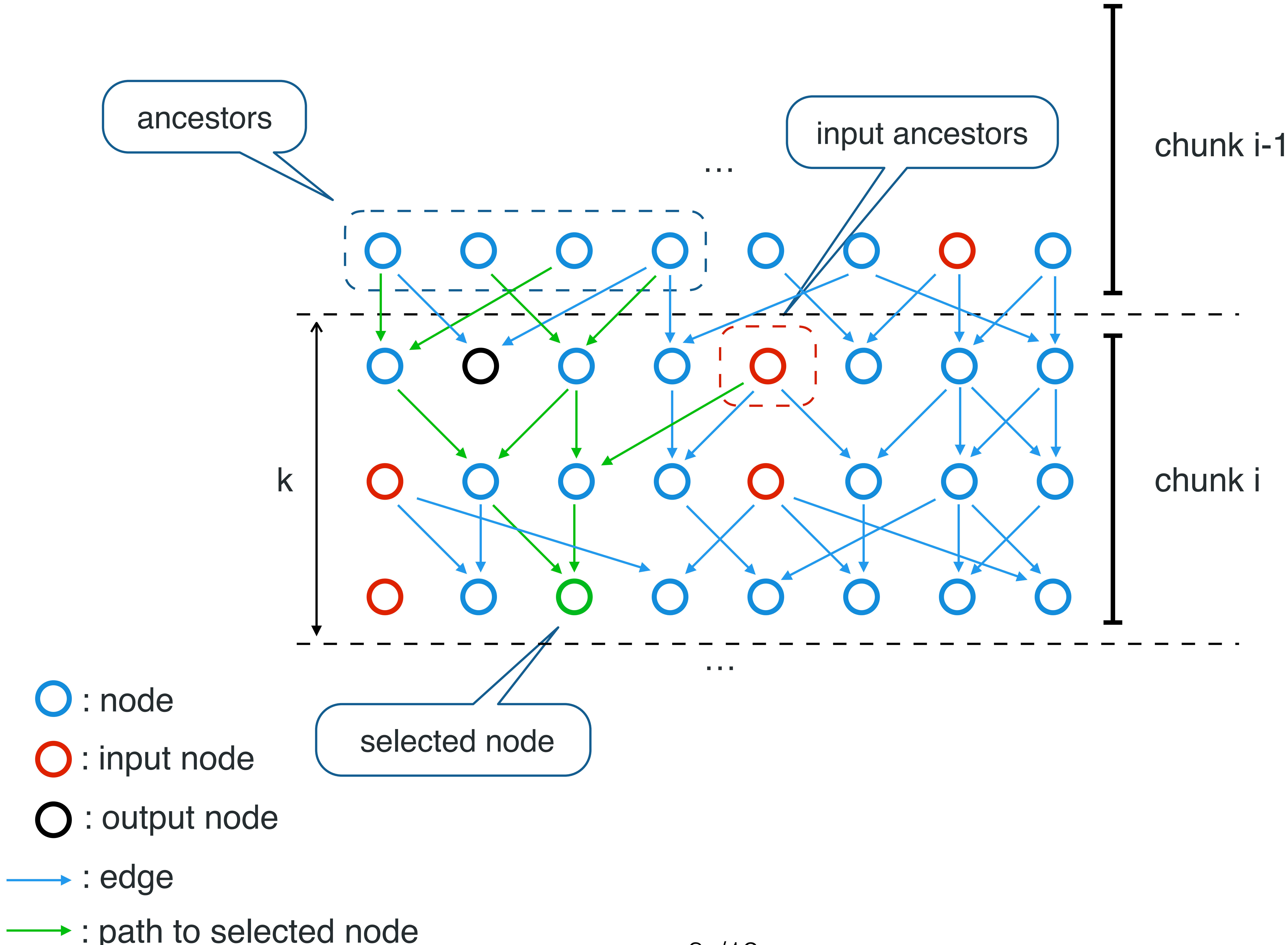

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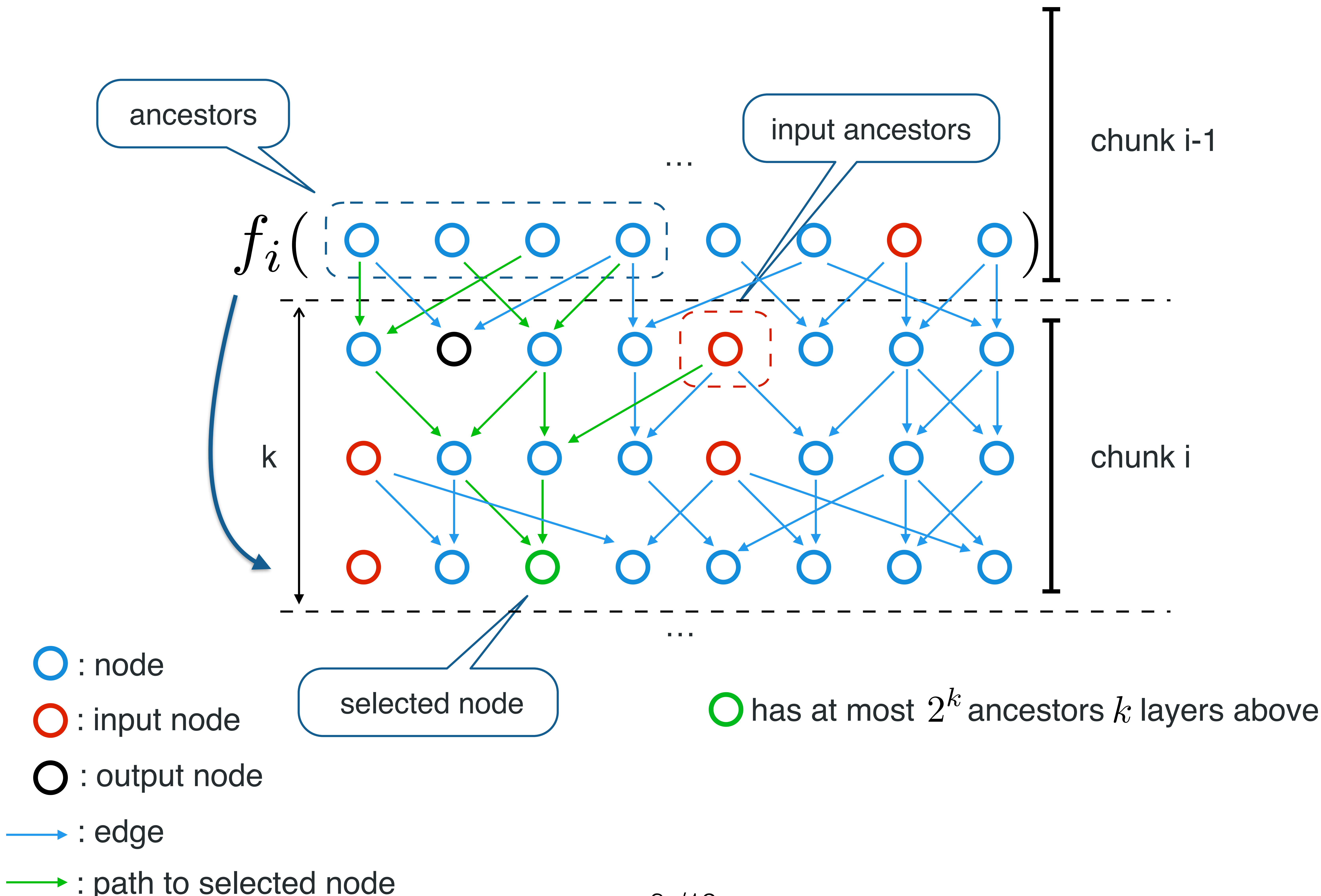

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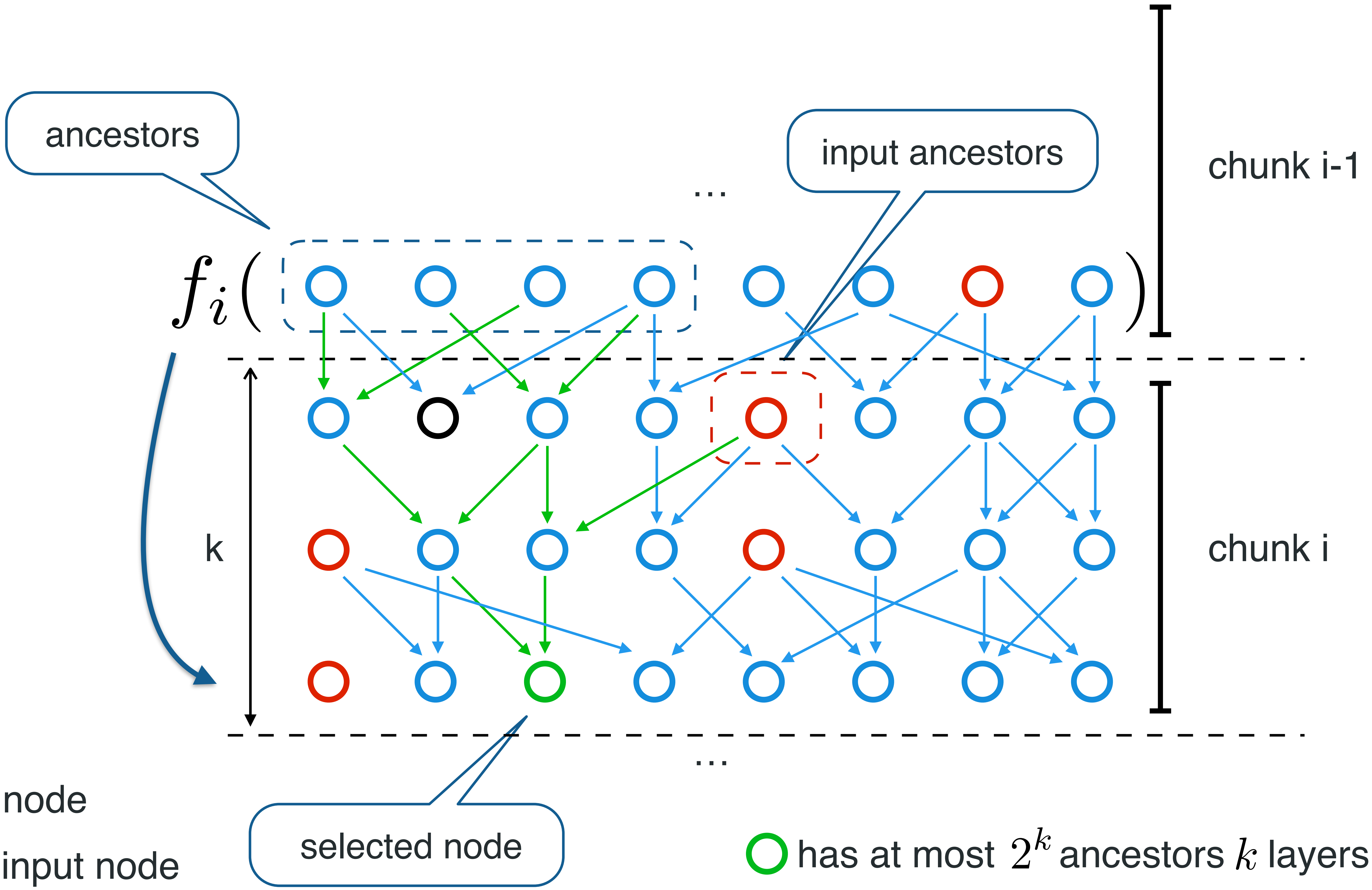


: node

: input node

: output node

: edge

: path to selected node

has at most $2^k$ ancestors $k$ layers above

$f_i$ is a $2^k$-local function with $w$ inputs and outputs

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$f_i$ is a $2^k$-local function with $w$ inputs and outputs

We can securely compute shares of $f_i$ with communication $O(w)$ and storage $O(w \cdot 2^{2^k})$

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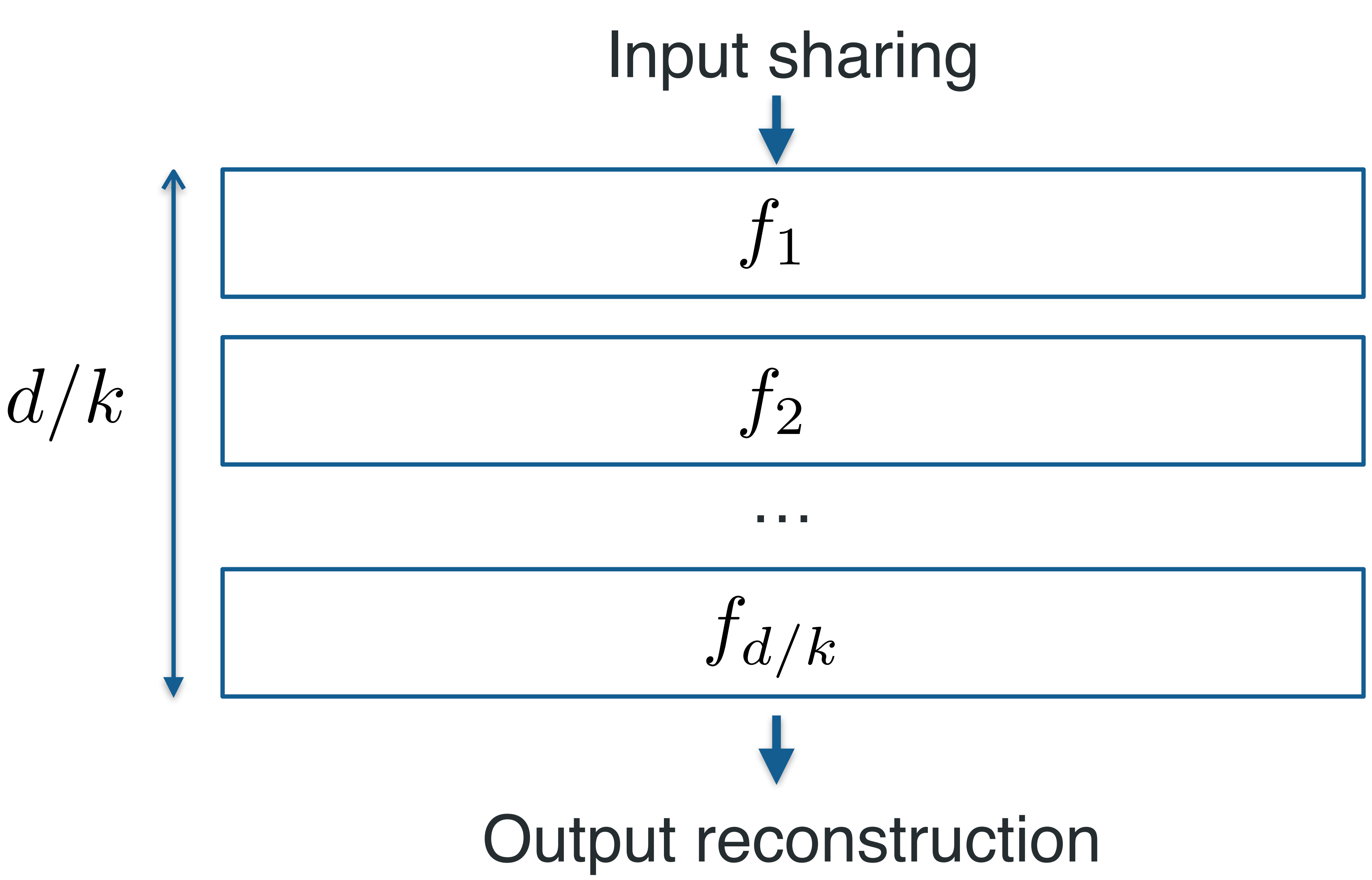

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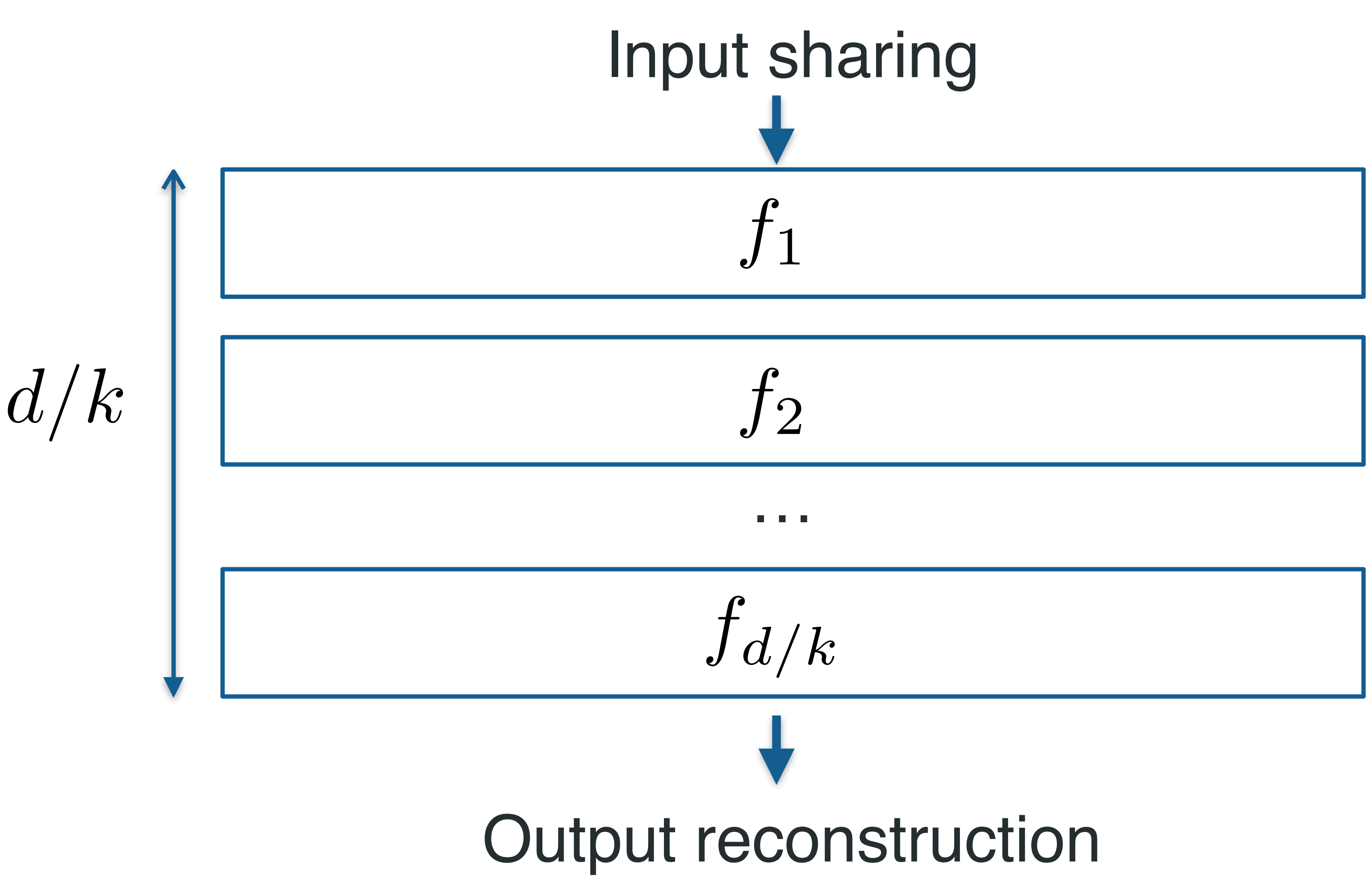


Communication: $O(w \cdot d/k) = O(s/k)$

Storage: $O(w \cdot 2^{2^k} \cdot d/k) = O(s \cdot 2^{2^k}/k)$

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$f_i$ is a $2^k$-local function with $w$ inputs and outputs

We can securely compute shares of $f_i$ with communication $O(w)$ and storage $O(w \cdot 2^{2^k})$

Input sharing

$d/k$

$f_1$

$f_2$

…

$f_{d/k}$

Output reconstruction

Communication: $O(w \cdot d/k) = O(s/k)$

Storage: $O(w \cdot 2^{2^k} \cdot d/k) = O(s \cdot 2^{2^k}/k)$

There exist a protocol to evaluate any LBC, with polynomial storage and total communication:

$$O\left(n + m + \frac{s}{\log \log s}\right)$$

# Open Questions

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- Can we get sublinear communication and linear computation?
- Can we extend the result to all circuits?

# Thanks for your attention

## Questions?

(Paper is online: ia.cr/2018/465)