Towards Non-Interactive Zero-Knowledge Proofs from CDH and LWE

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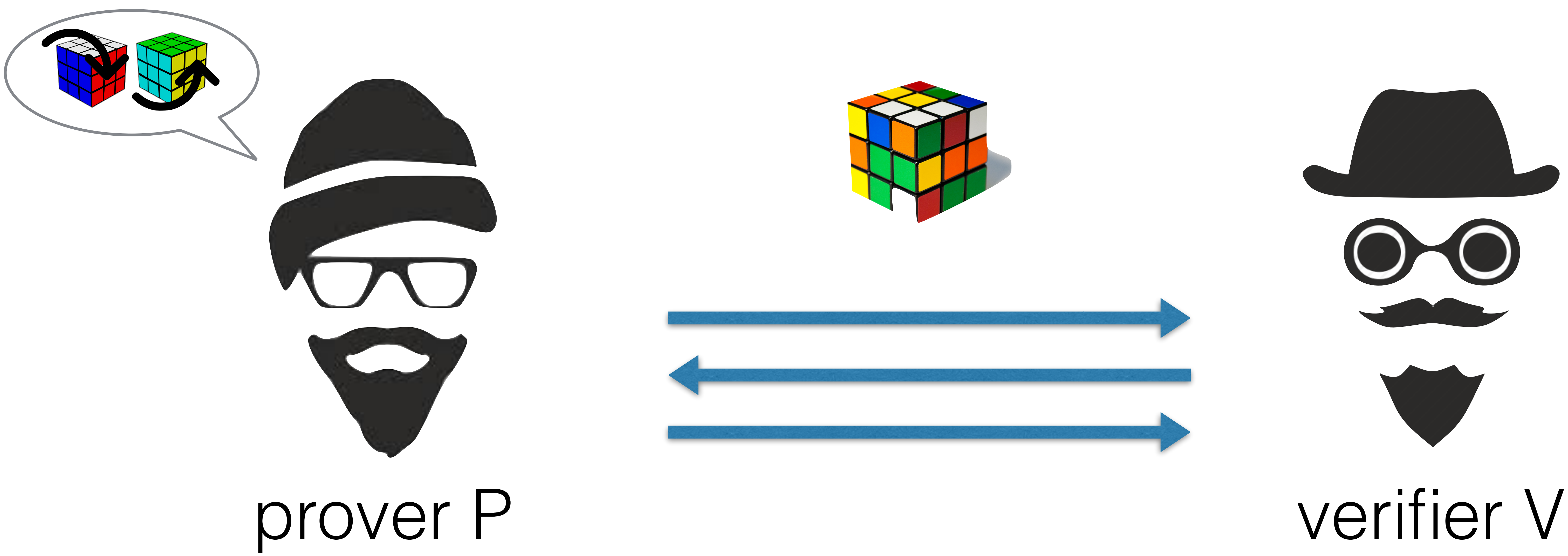
Zero-Knowledge Proof



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- Sound: if there is no solution, P cannot convince V
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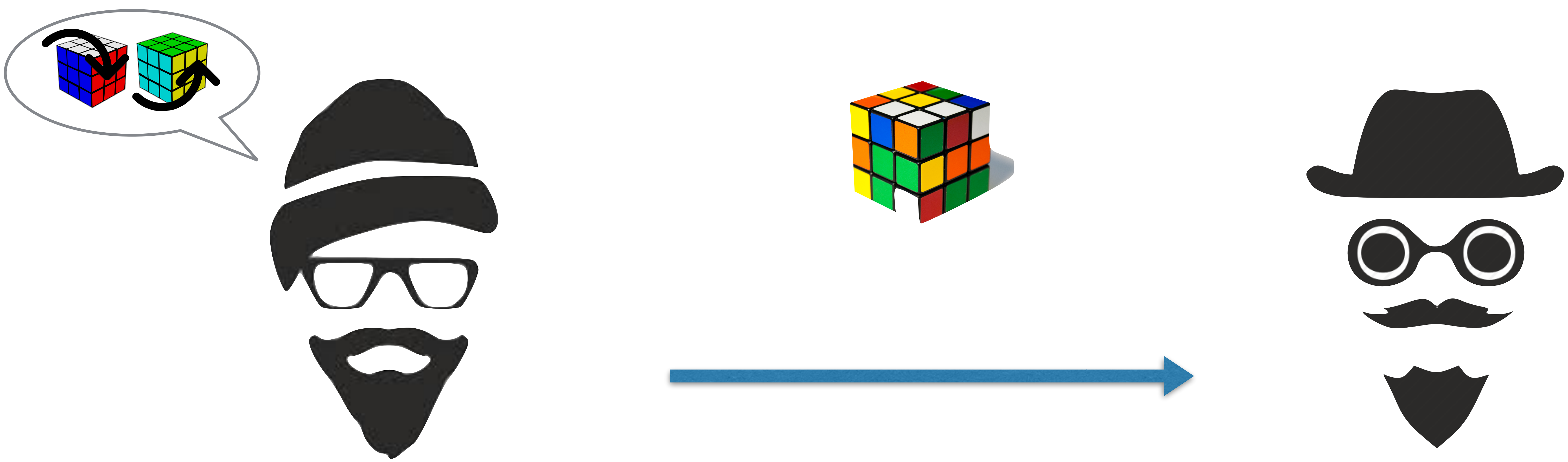
Non-Interactive Zero-Knowledge Proof



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Brief History of NIZKs

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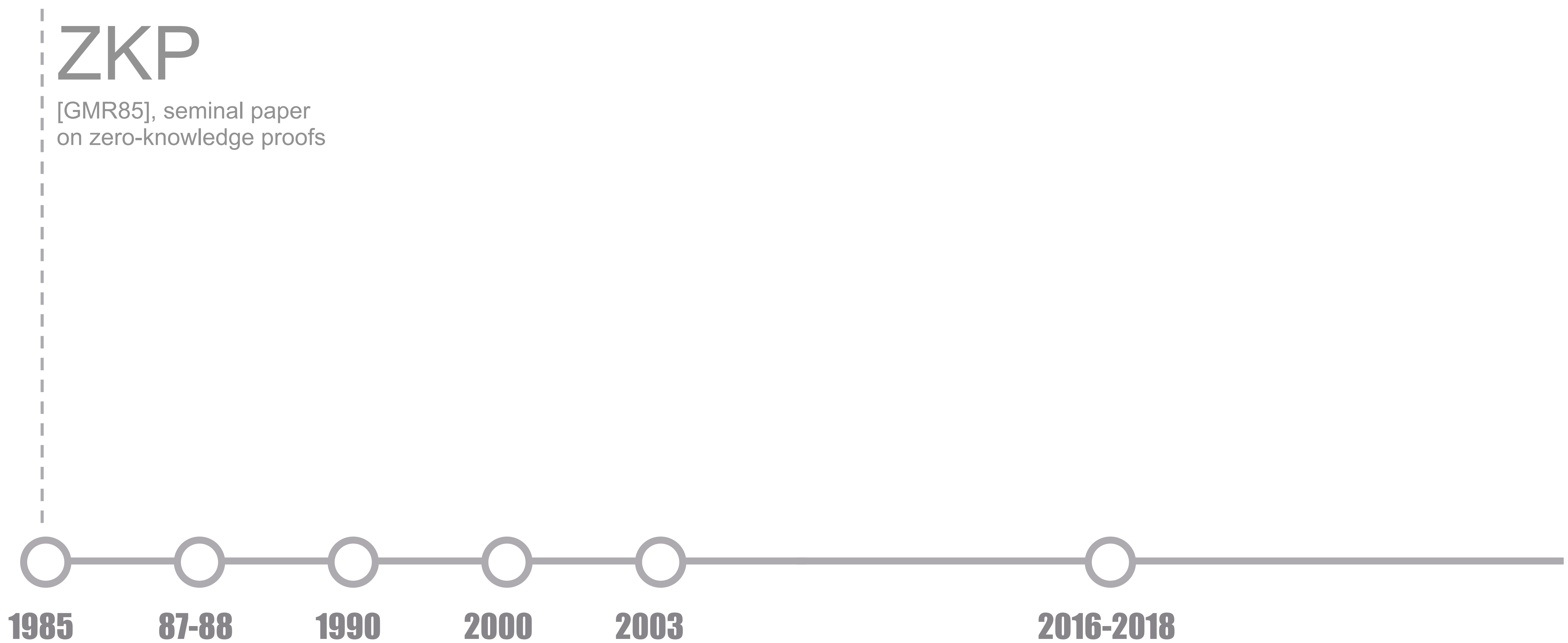

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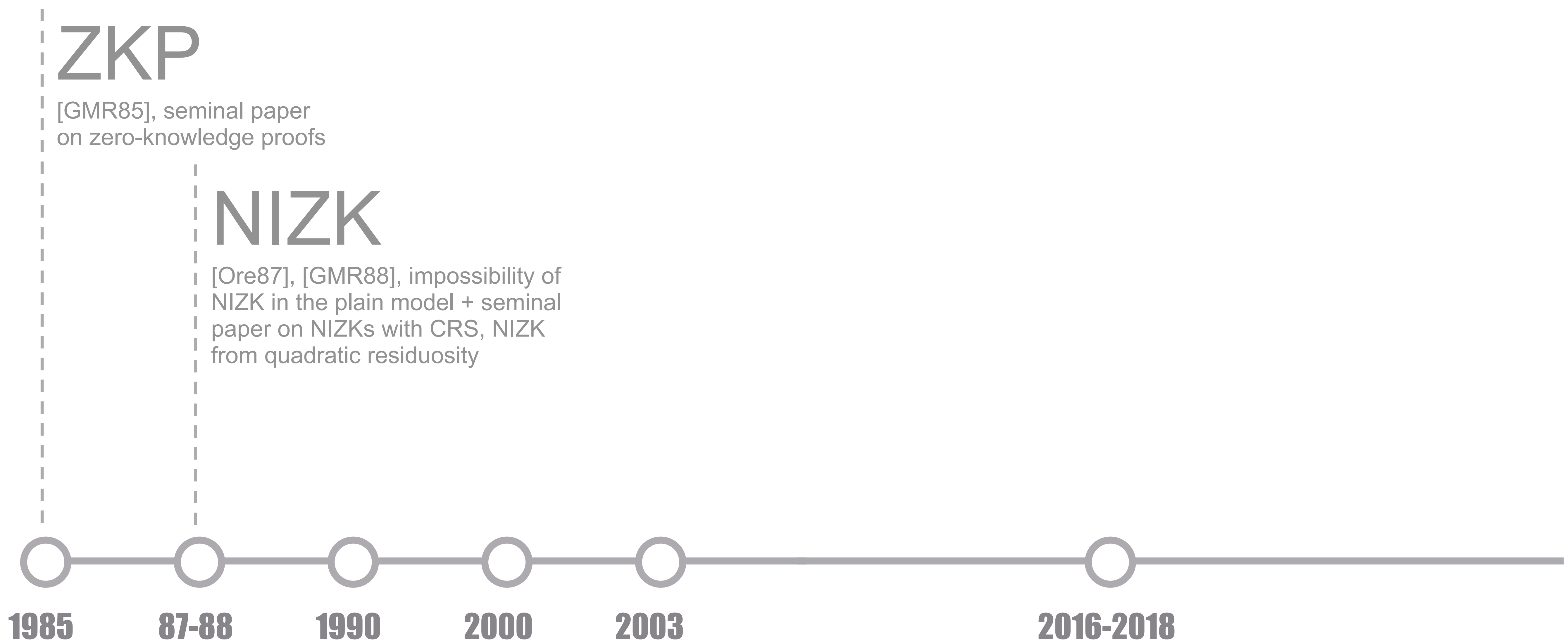

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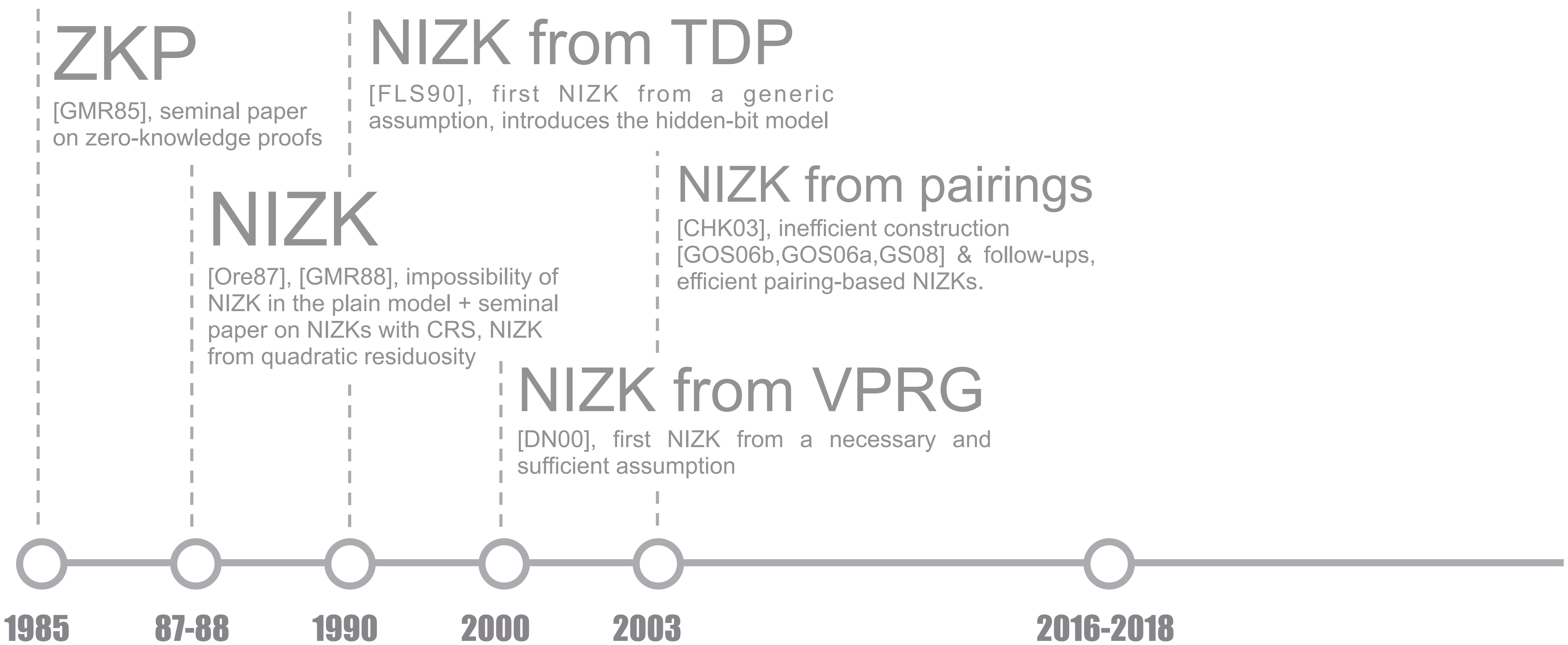

Brief History of NIZKs

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ZKP

[GMR85], seminal paper on zero-knowledge proofs

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NIZK from TDP

[FLS90], first NIZK from a generic assumption, introduces the hidden-bit model

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[DN00], first NIZK from a necessary and sufficient assumption

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[CHK03], inefficient construction [GOS06b,GOS06a,GS08] & follow-ups, efficient pairing-based NIZKs.

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NIZK from new assumptions

[CCR16], [KRR17], [CCRR18], [HL18], [CCH+18], [CLW18]: instantiating correlation intractable hash functions (iO, exponentially-strong KDM security, circular FHE)

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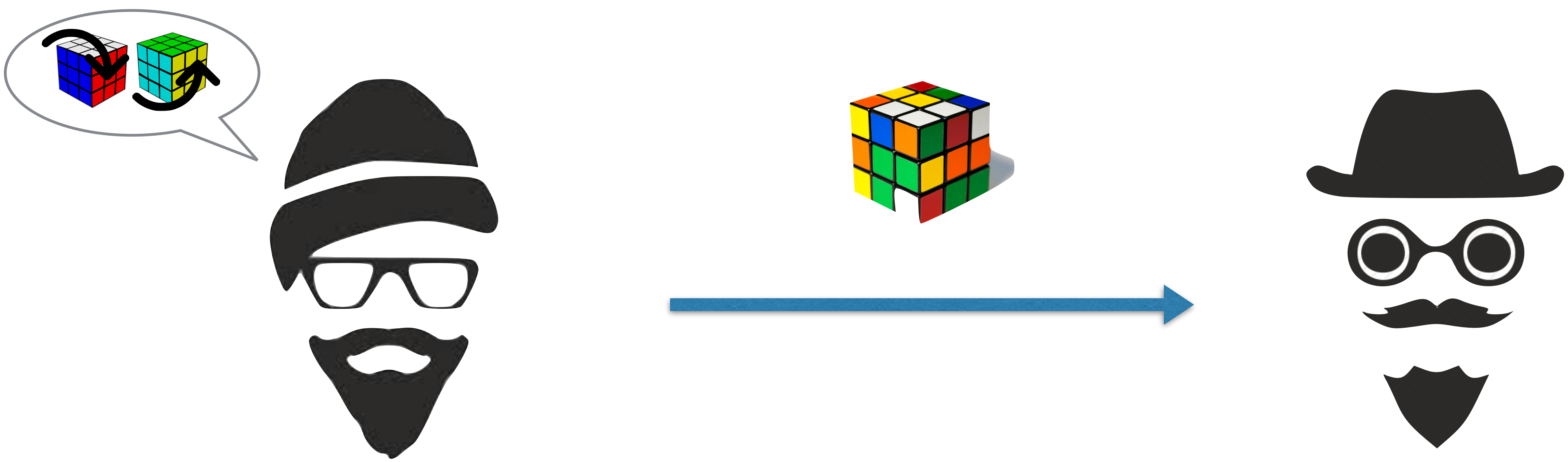
2003 – 2016-2018: Investigating relaxed notions

Designated-Verifier NIZK



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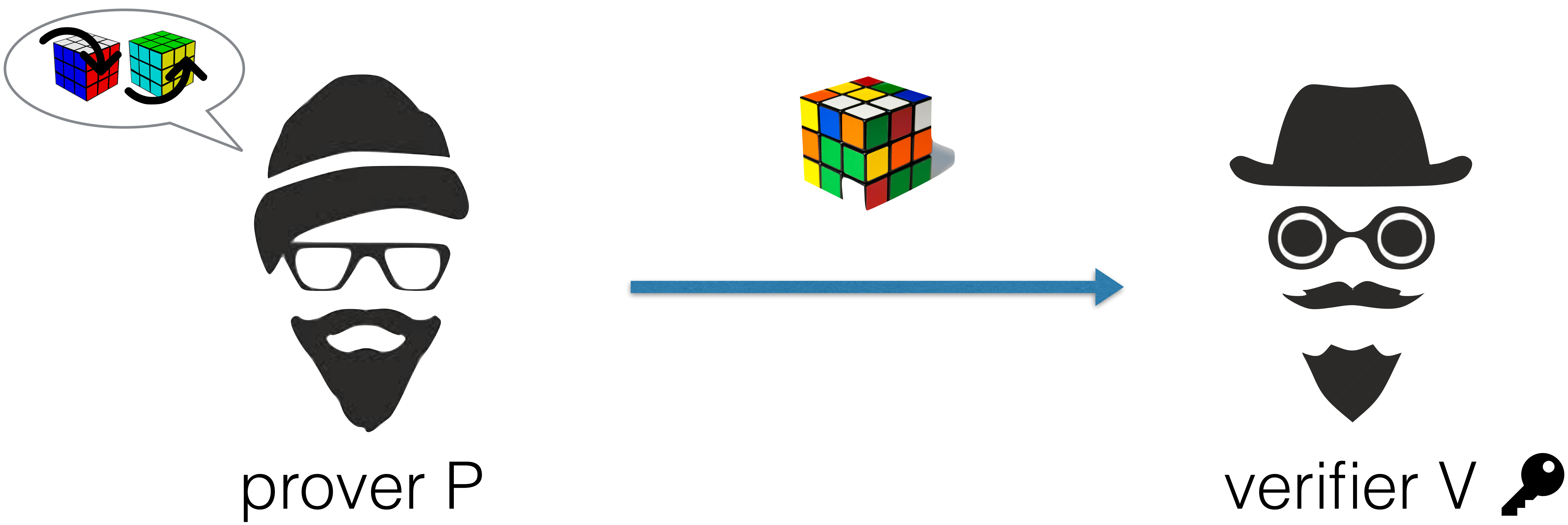

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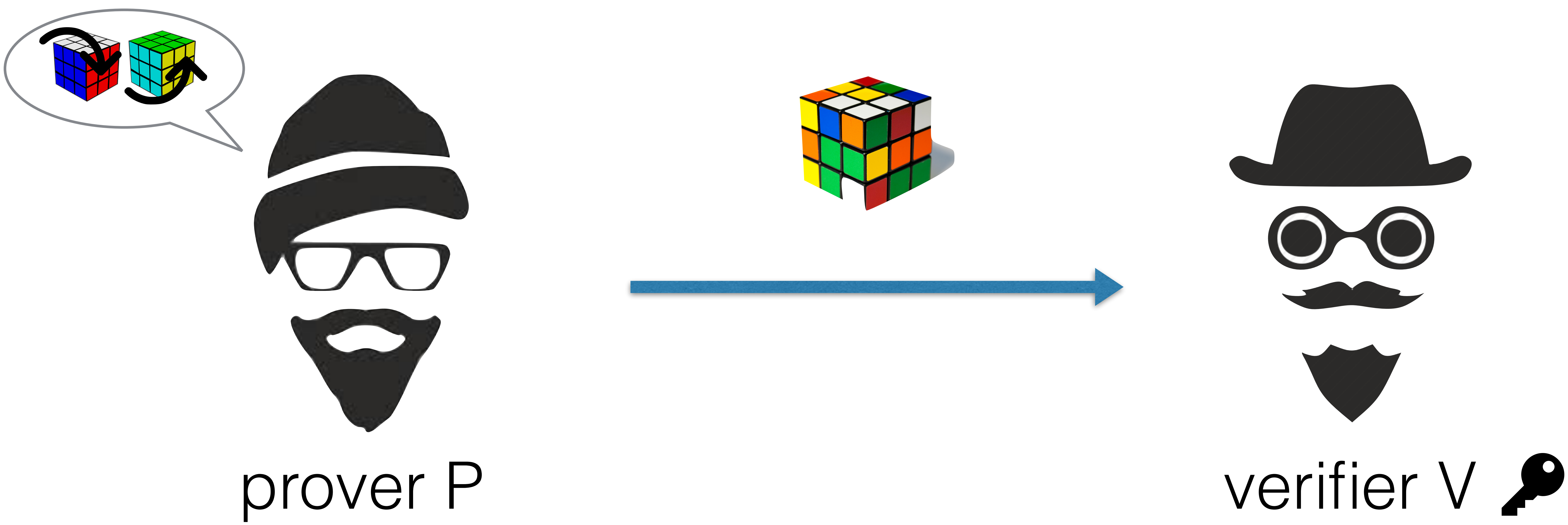

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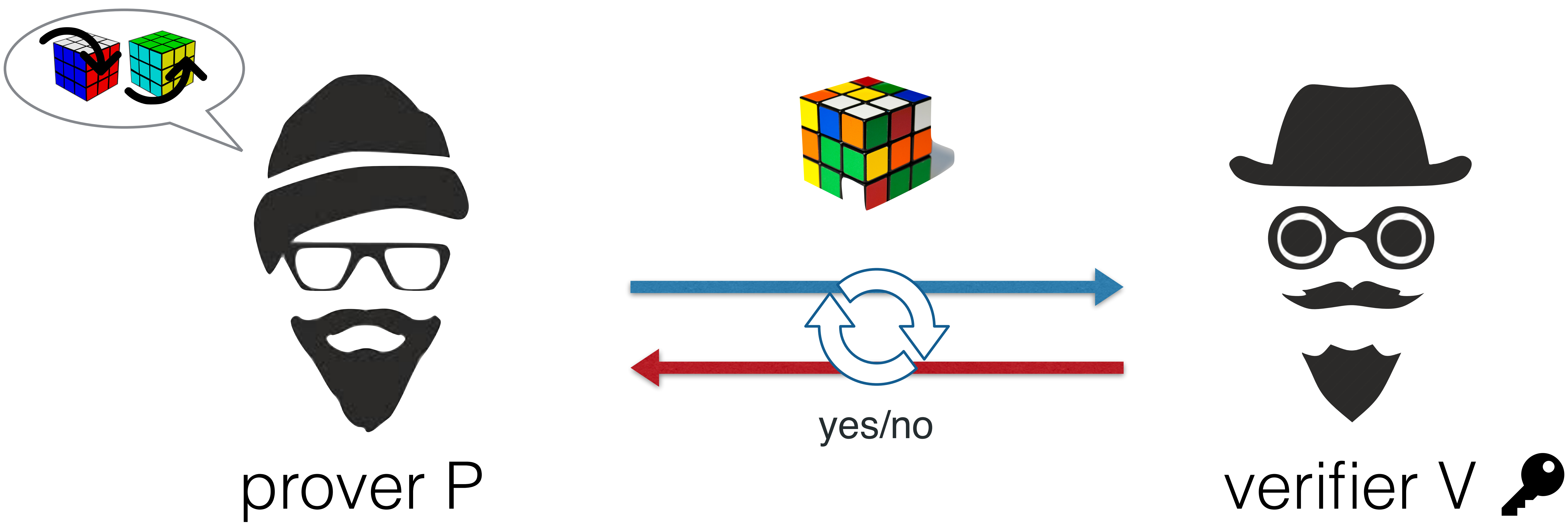

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Brief History of (DV)NIZKs

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2019

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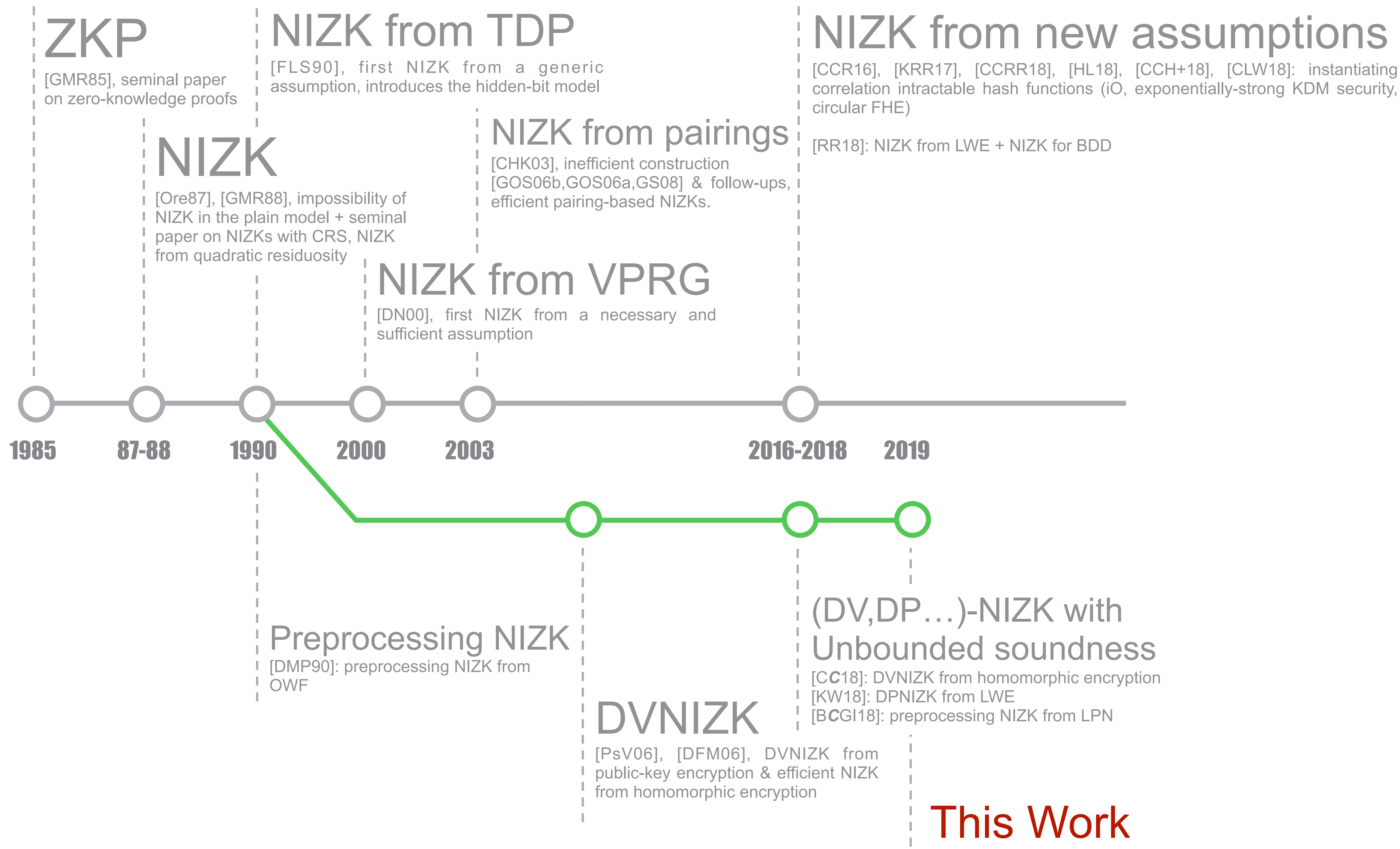

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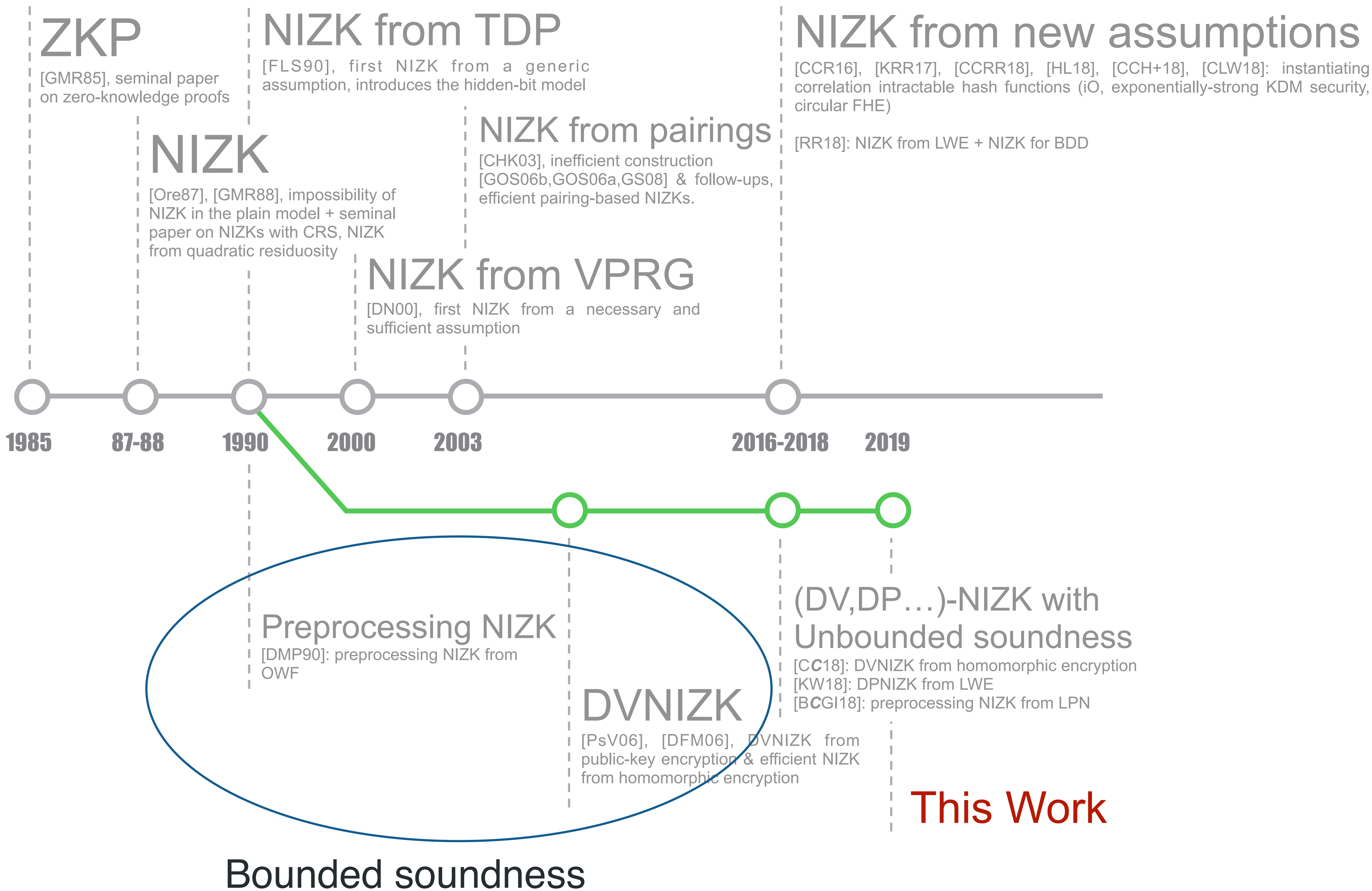

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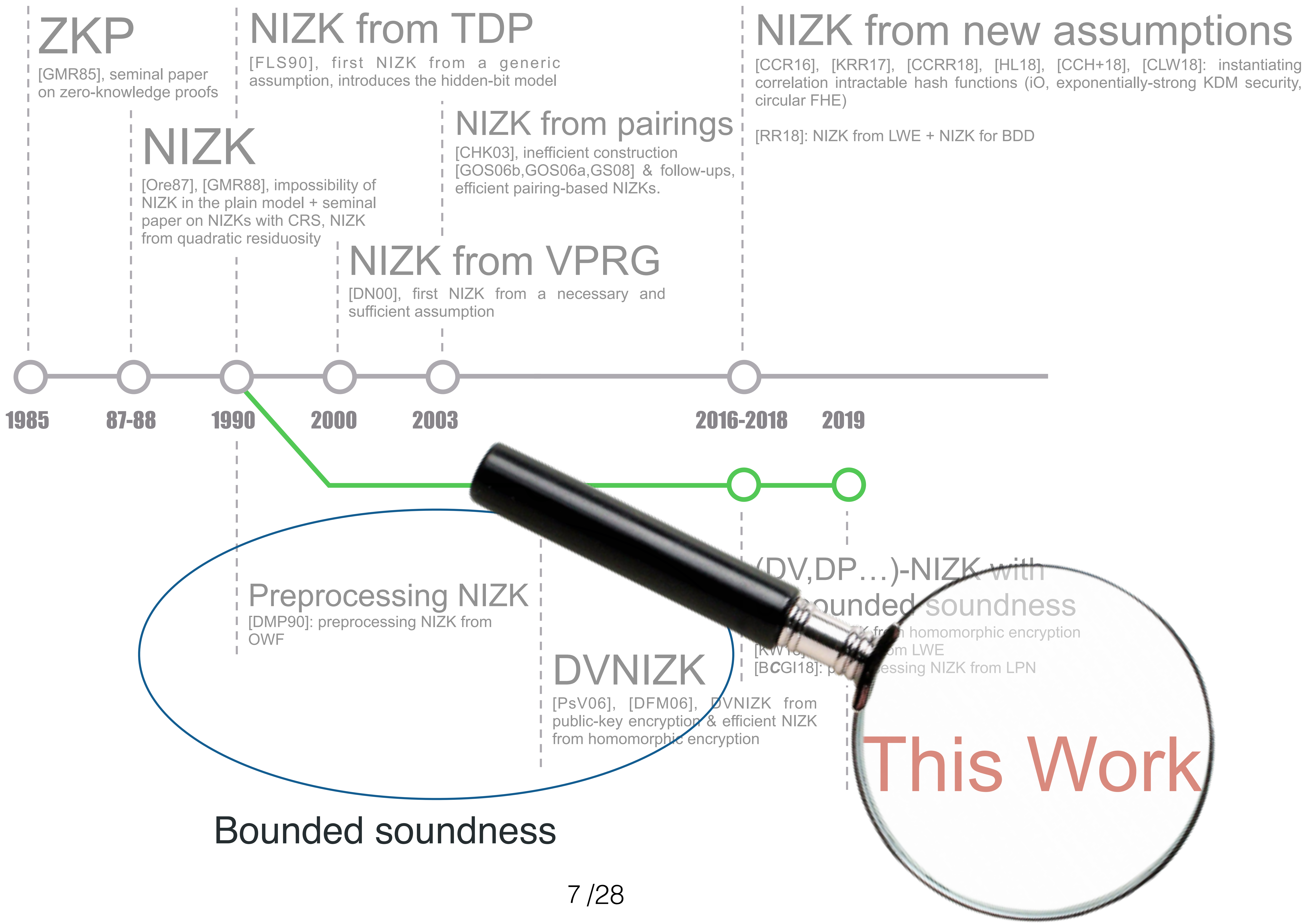

Our Contribution

We obtain two new constructions:

1) A DVNIZK for NP under the CDH assumption

First direct indication that DVNIZK with unbounded soundness are actually easier to build than standard NIZK

2) A (DV)NIZK for NP assuming LWE and the existence of a (DV)NIWI for BDD

Improving over, and considerably simplifying, the recent result of [RR18] which required a NIZK for BDD.

Roadmap

[DN00]: $\boxed{\text{Verifiable Pseudorandom Generator}}$ + $\boxed{\text{NIZK in the hidden-bit model}}$ $\Rightarrow$ NIZK

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$\Downarrow$

Verifiable Pseudorandom Generator:
- Relaxed soundness
- Generalization to the DV setting

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⇓

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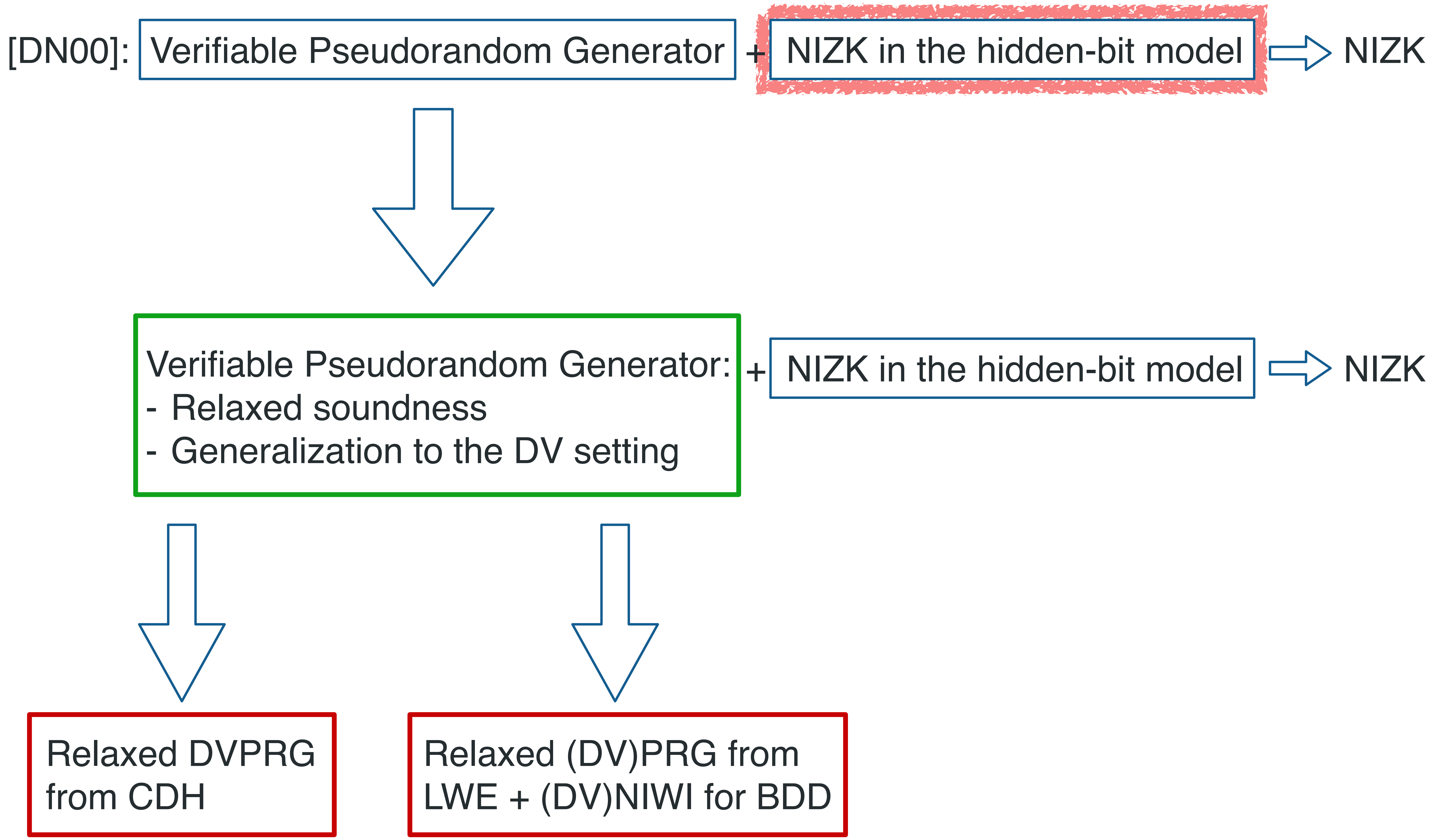
Relaxed DVPRG from CDH

Relaxed (DV)PRG from LWE + (DV)NIWI for BDD

Roadmap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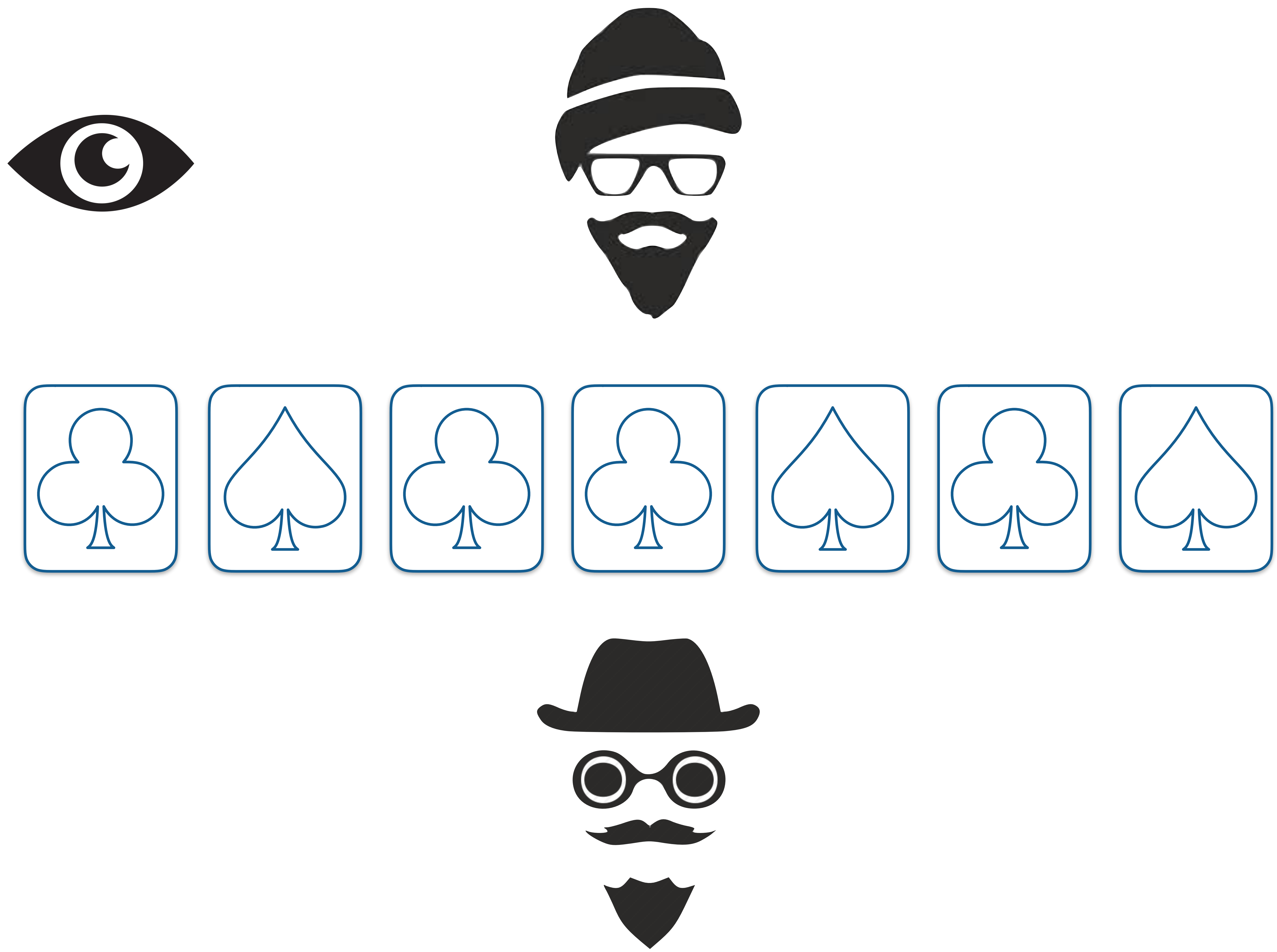
The Hidden-Bit Model



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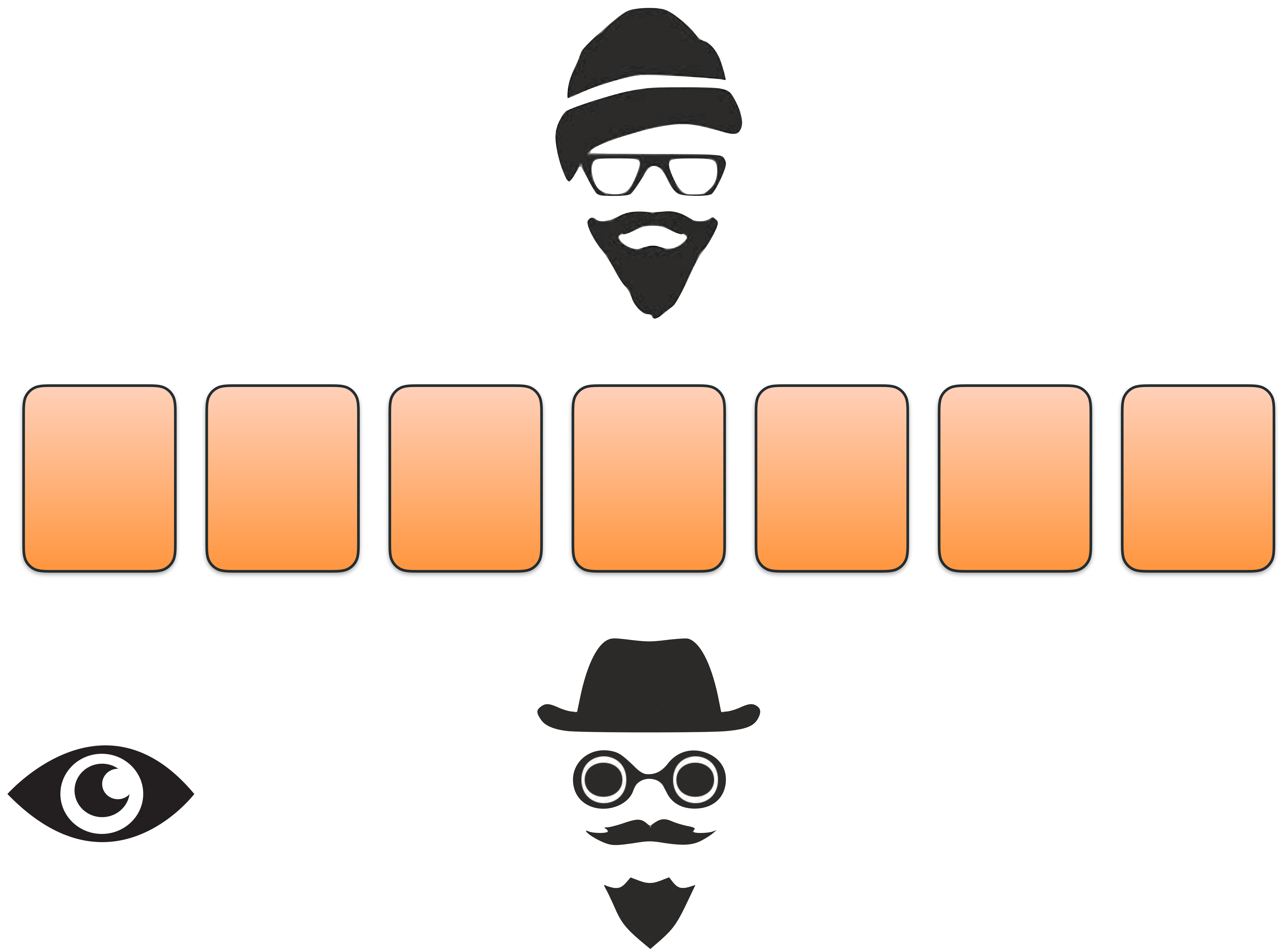

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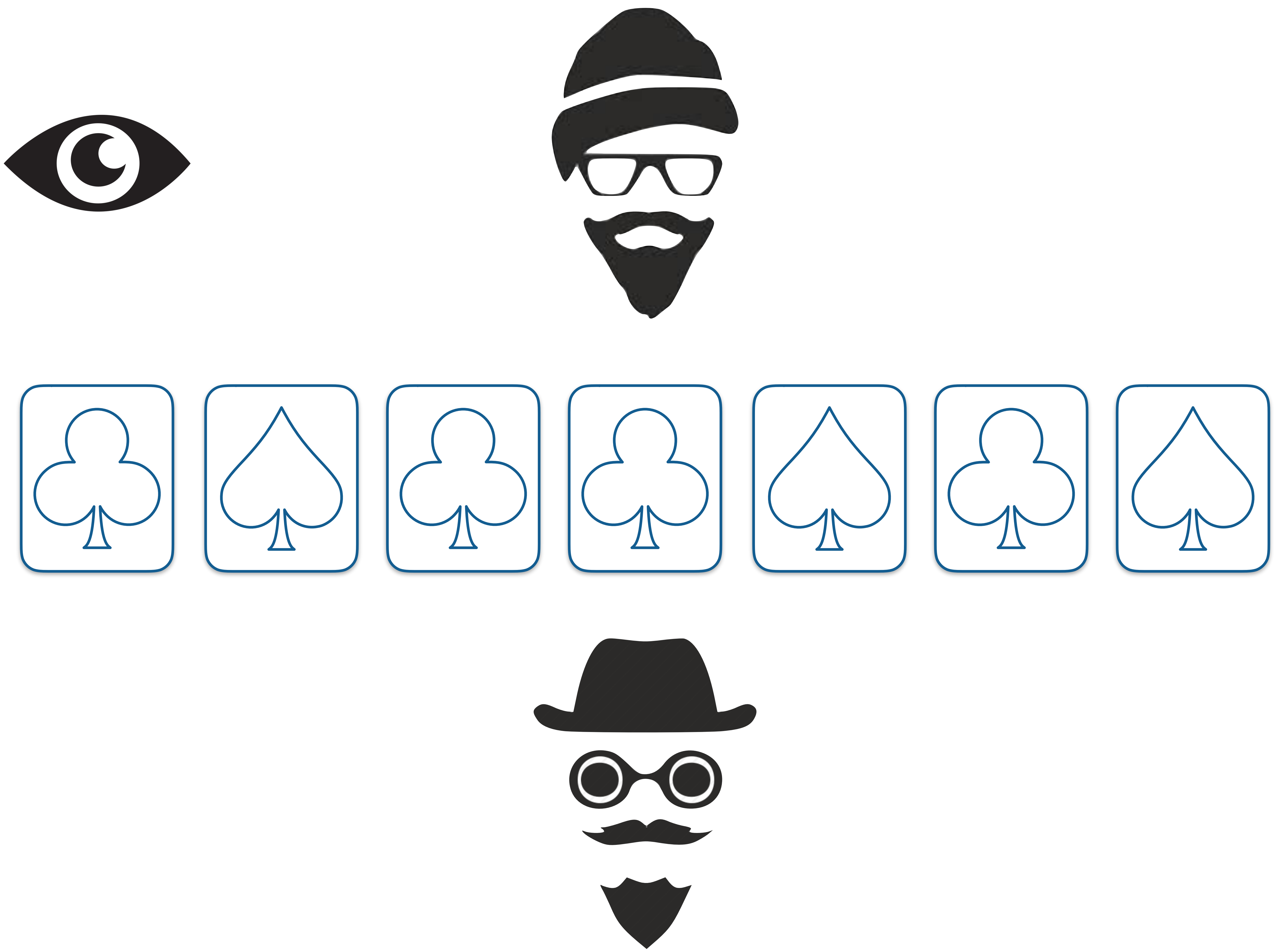

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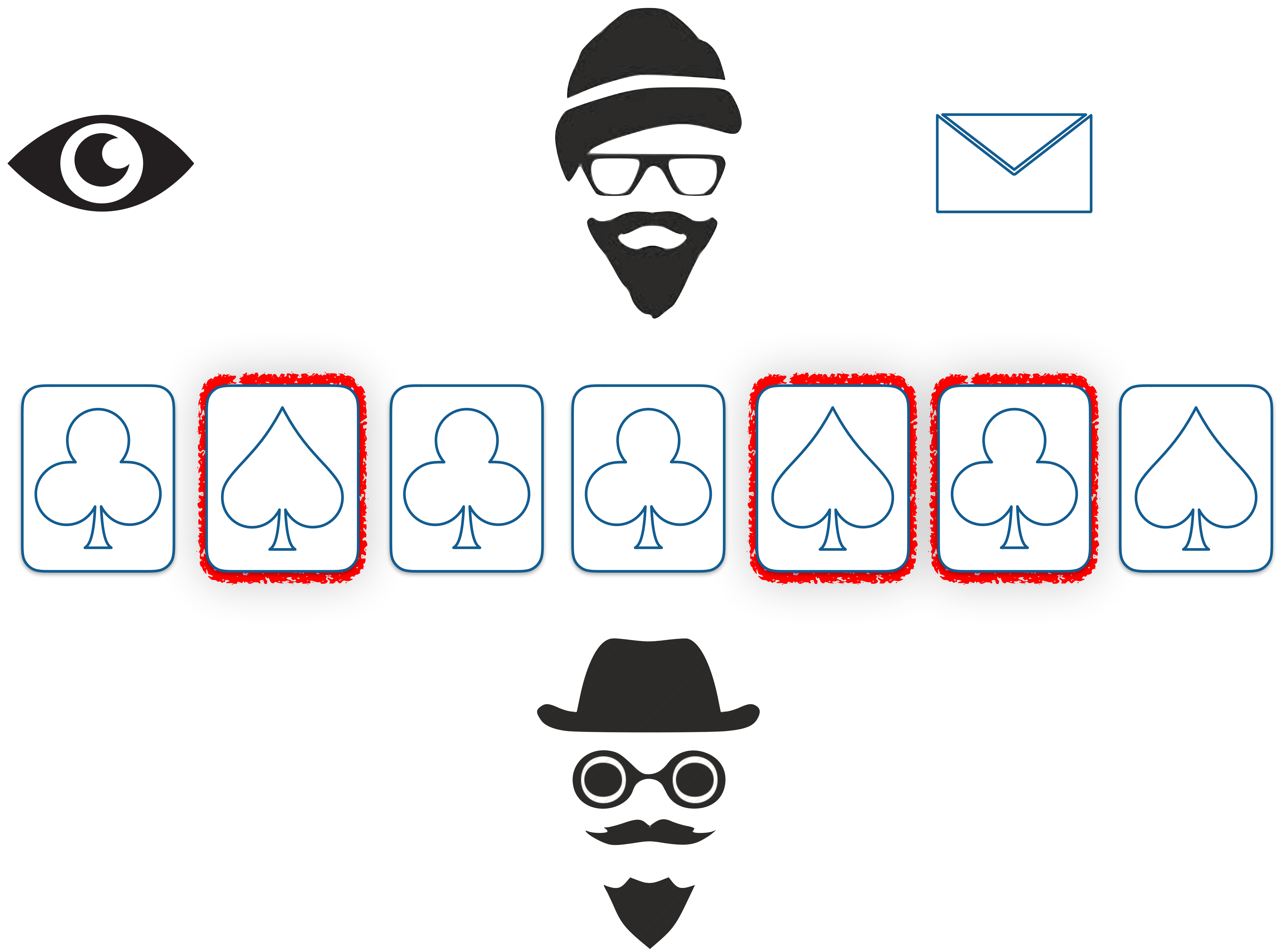

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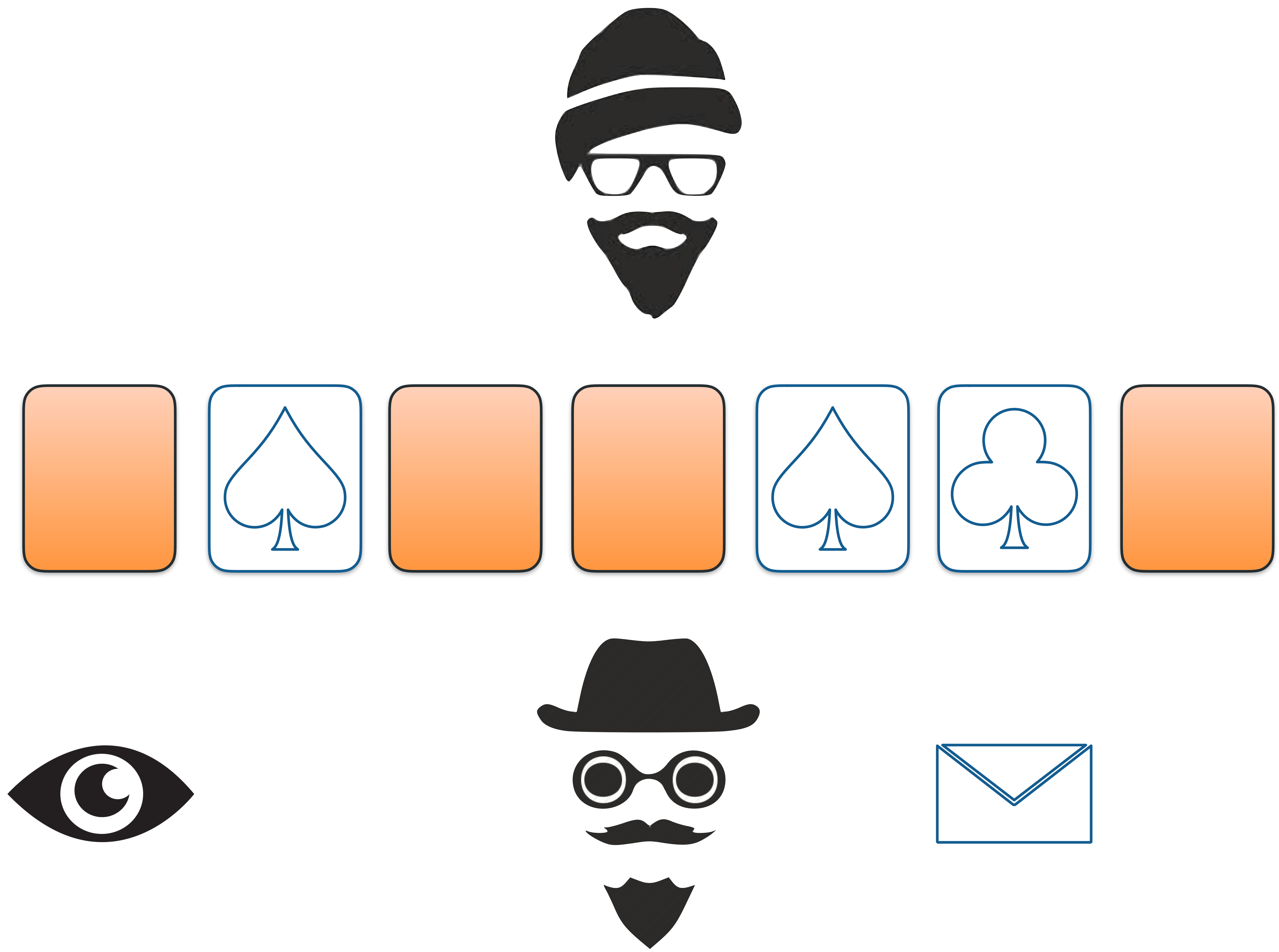

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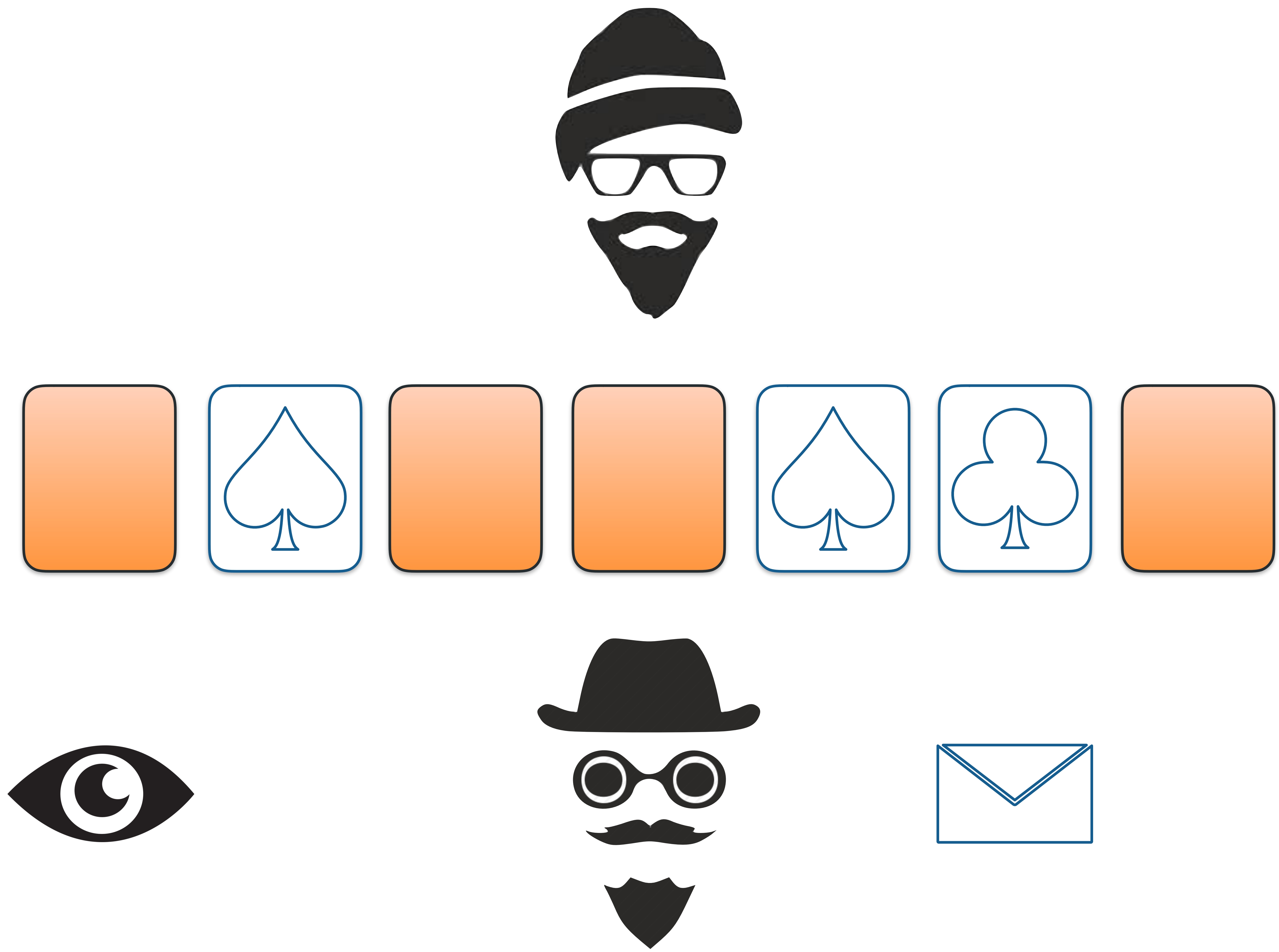
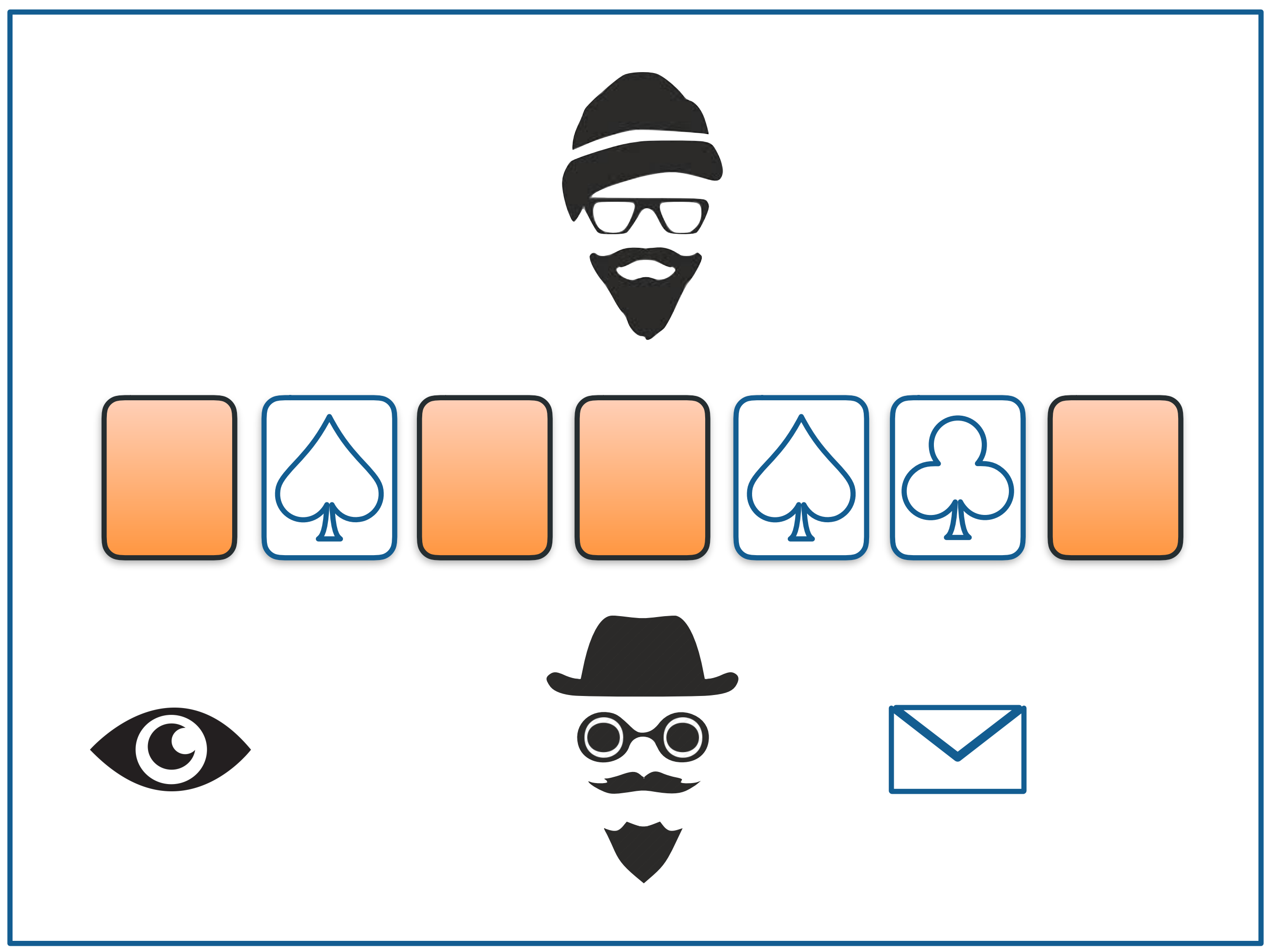
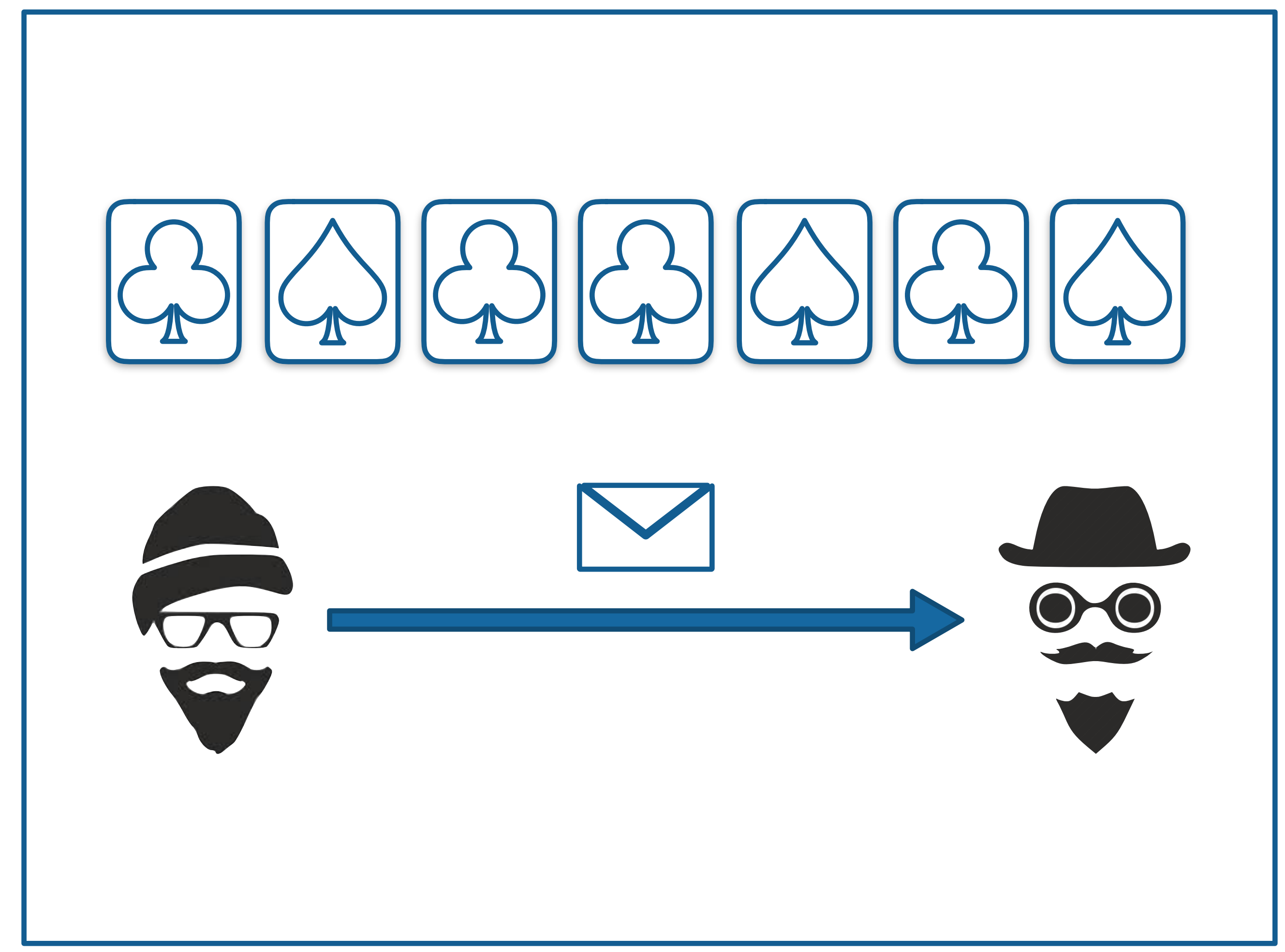


[FLS90]: NIZKs for NP exist unconditionally in the HBM

Instantiating The Hidden-Bit Model

Cryptographic primitive

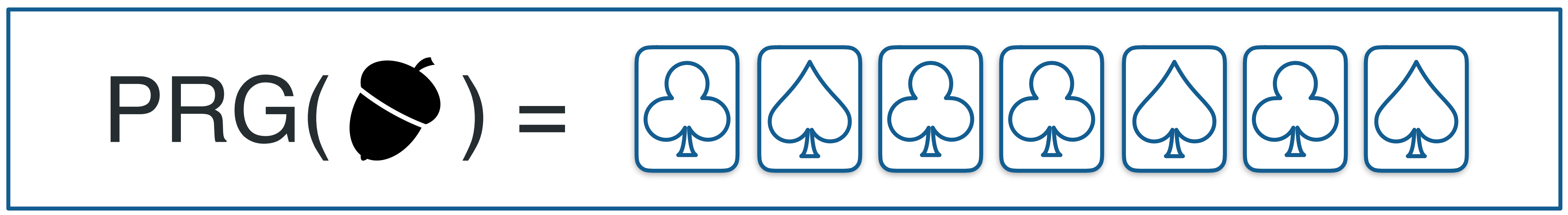


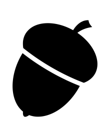
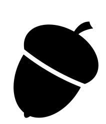


Prover's task, given the CRS:

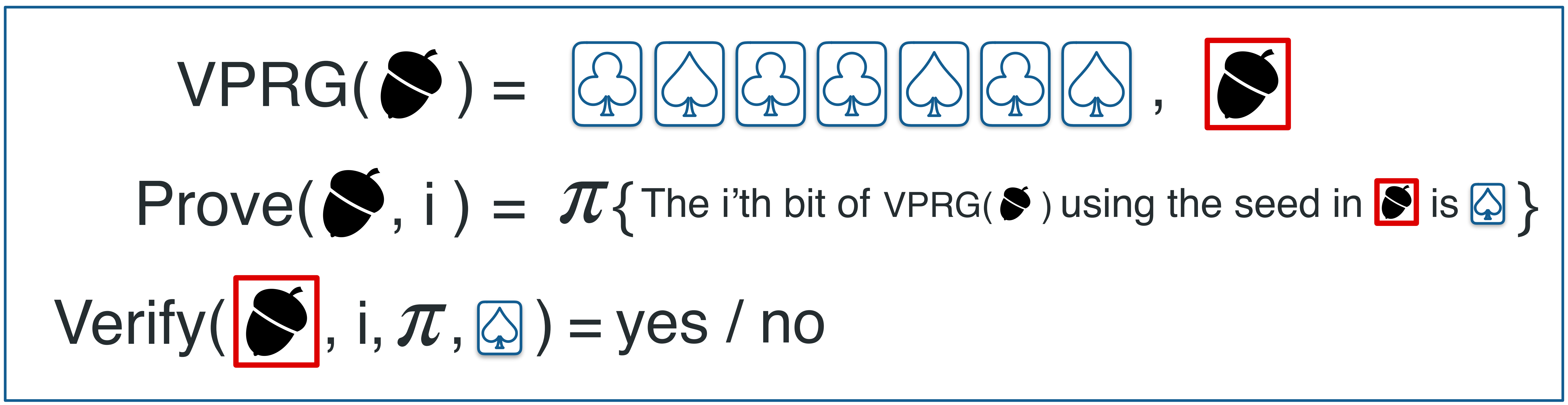
1. Produce a string which is indistinguishable from random
2. Be able to provably 'open' positions of this pseudorandom string
3. The openings should not reveal the non-opened positions

Pseudorandom Generators



-  is short
- If  is random,  cannot be distinguished from a truly random string

Verifiable Pseudorandom Generators



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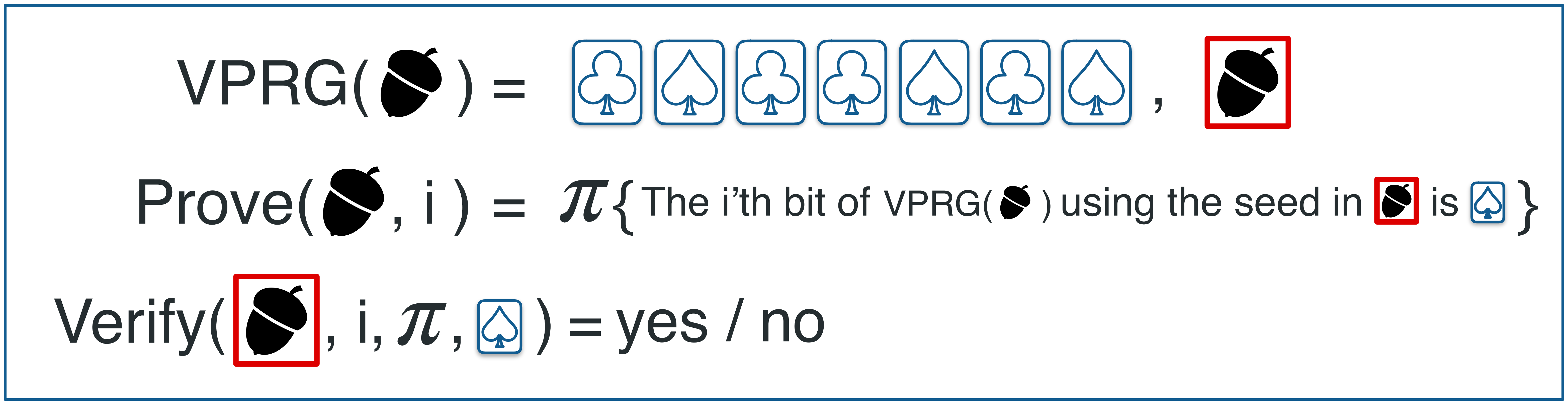

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- The proof leaks nothing more about
- The proof is sound in a strong sense

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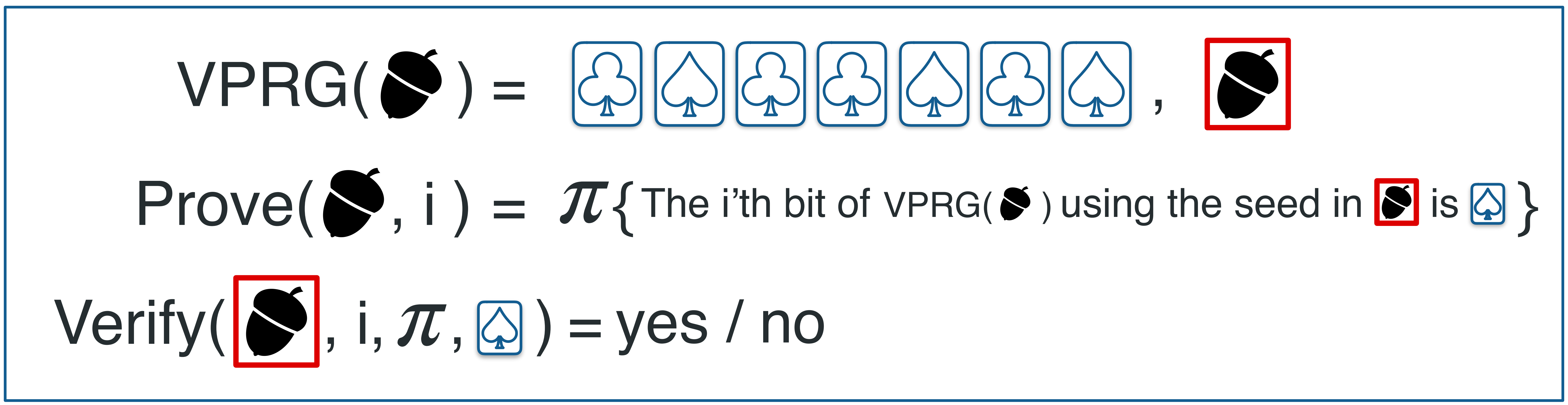

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Verifiable Pseudorandom Generators

VPRG( ) = ,

Prove(, i) = $\pi\{$The i'th bit of VPRG( ) using the seed in is $\}$

Verify(, i, π,) = yes / no

- Is short
- The proof leaks nothing more about
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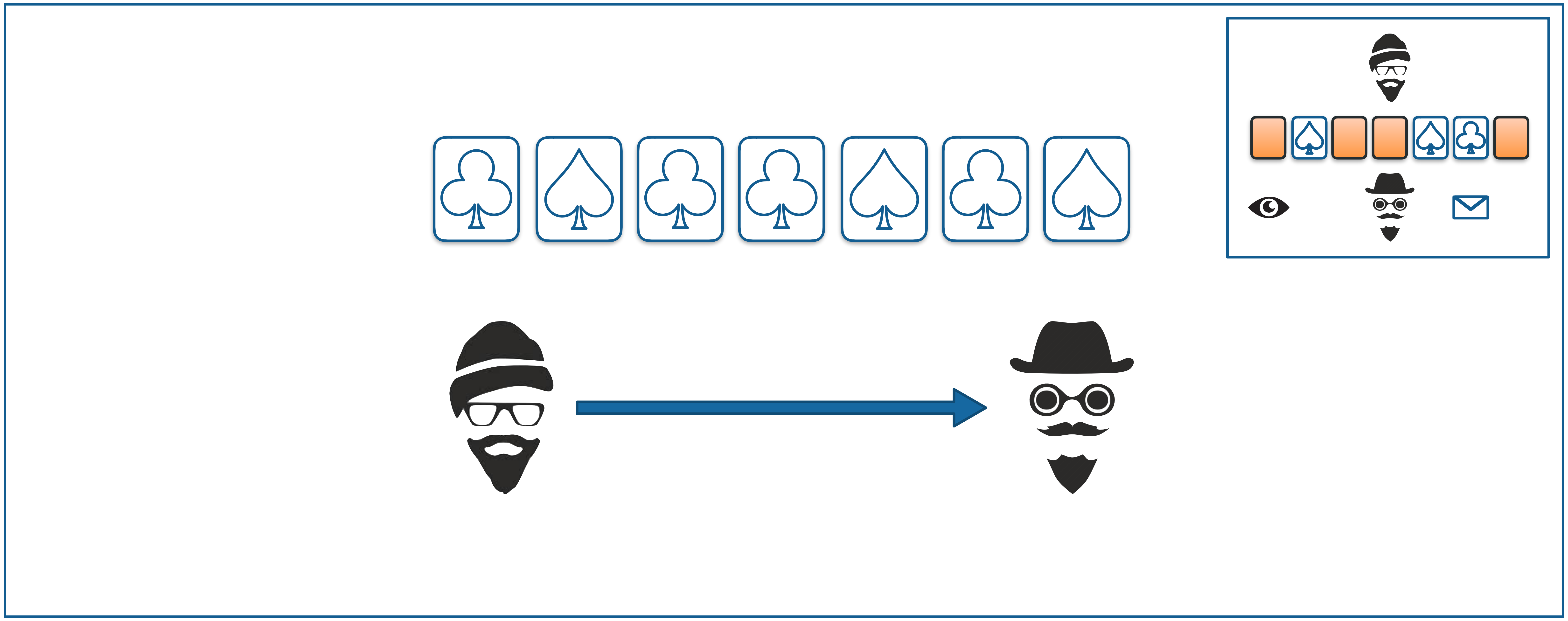
1. Every *is in the image of* VPRG(.)
2. For *every possible* , there is a *unique* associated
3. Proofs of opening to bits inconsistent with *do not exist*

Building NIZKs from VPRGs



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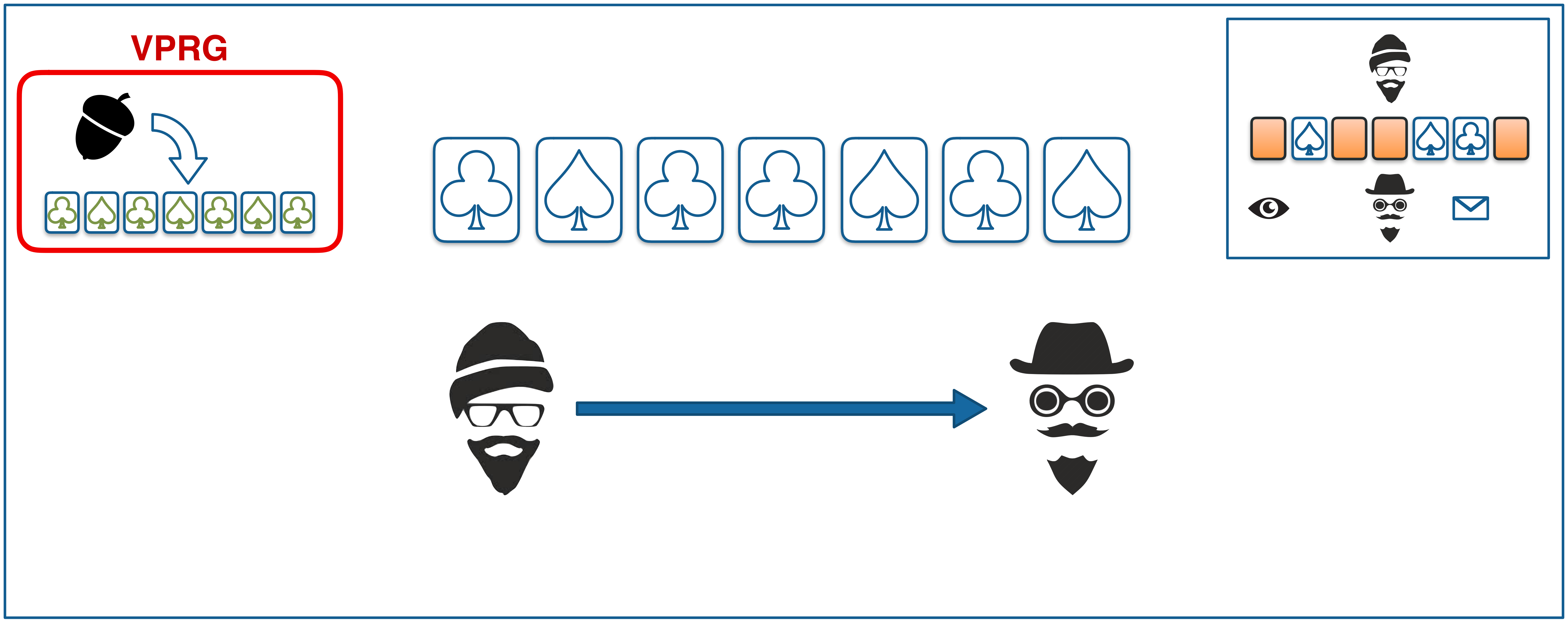

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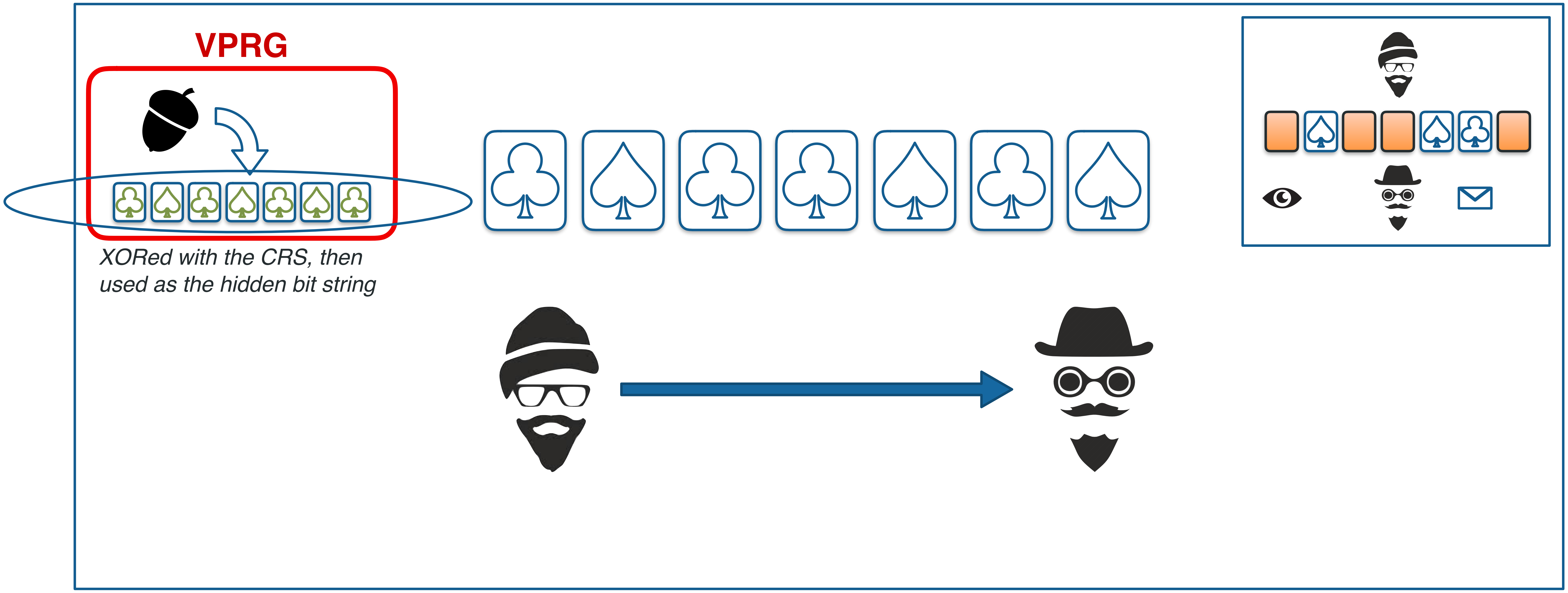

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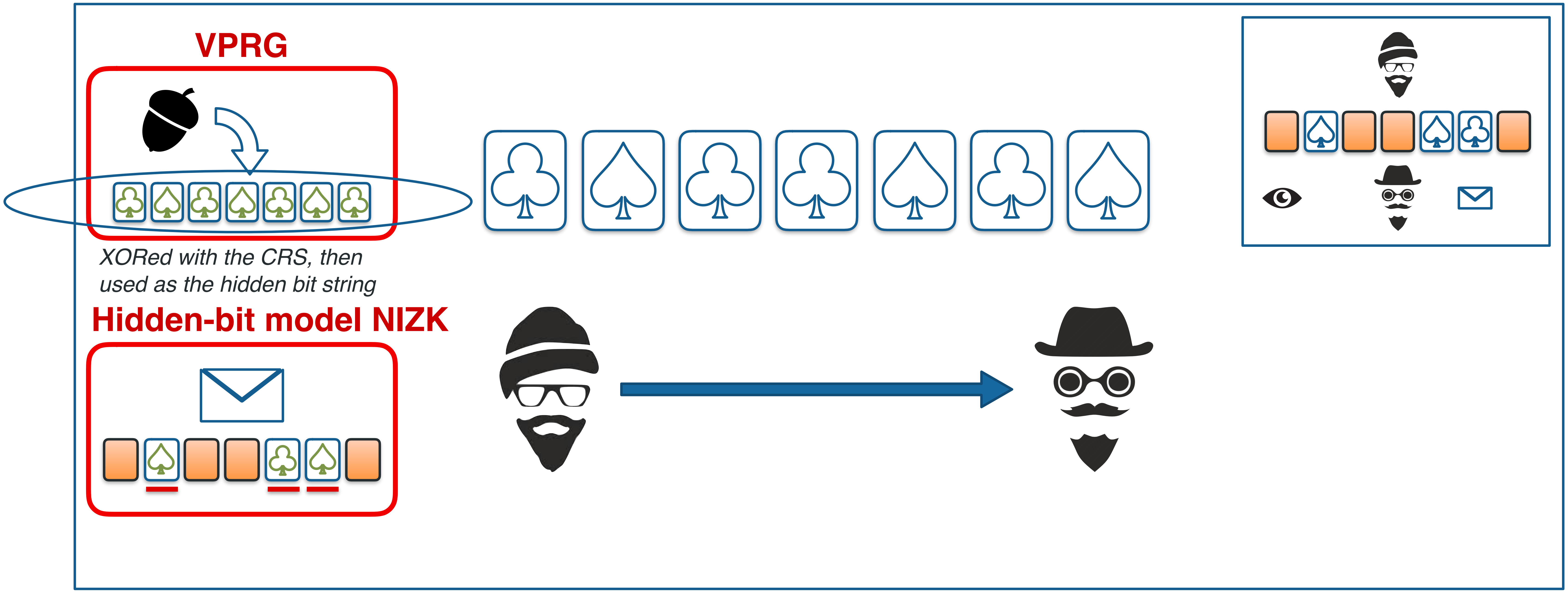

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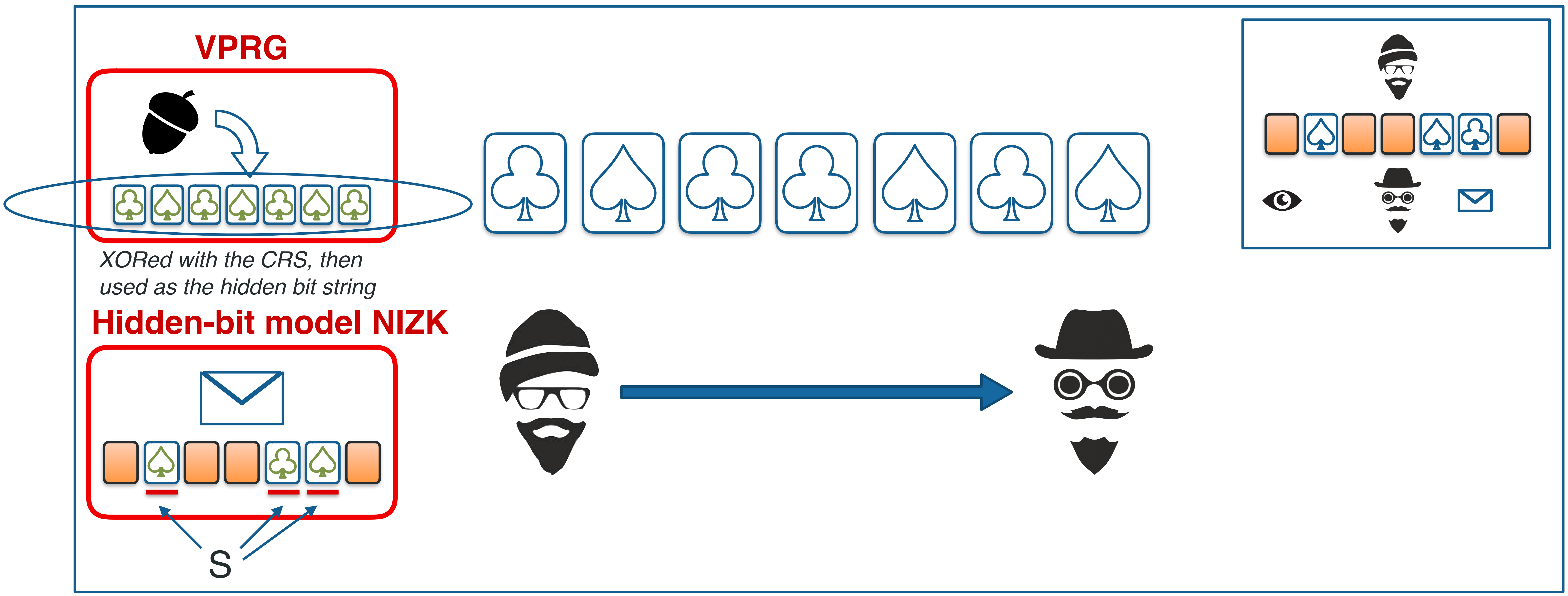

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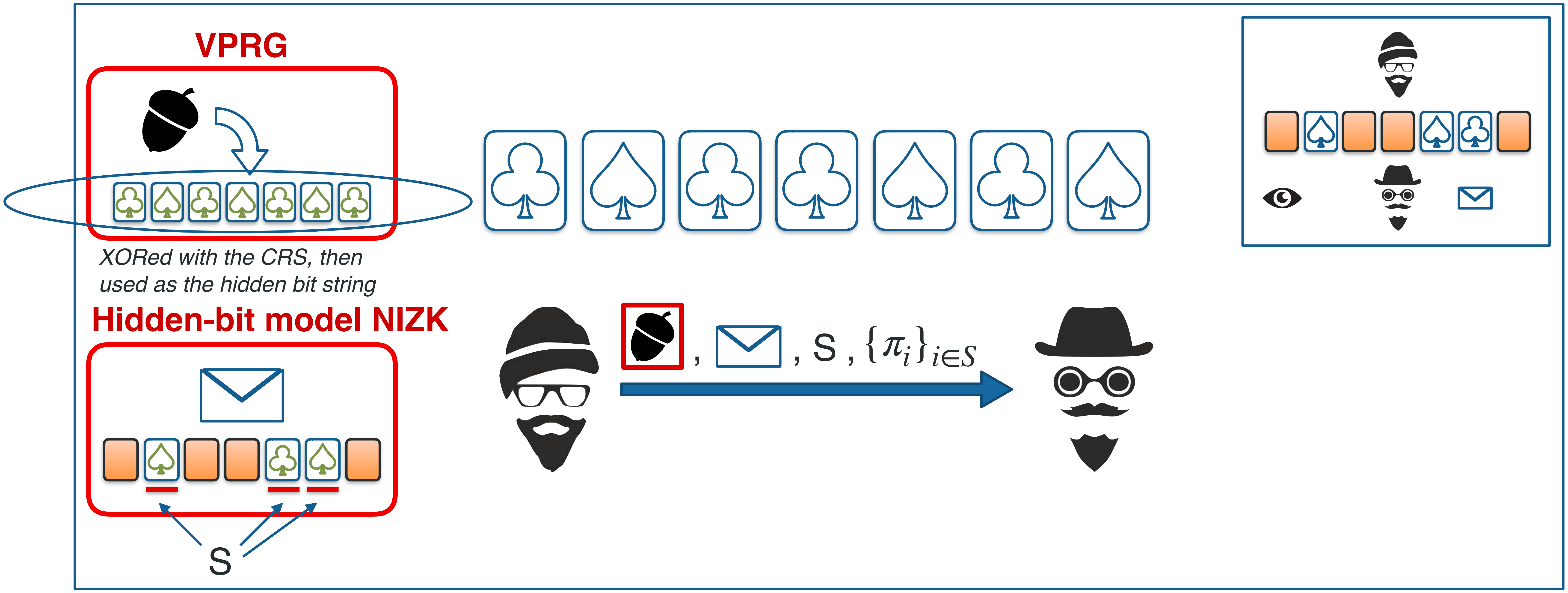

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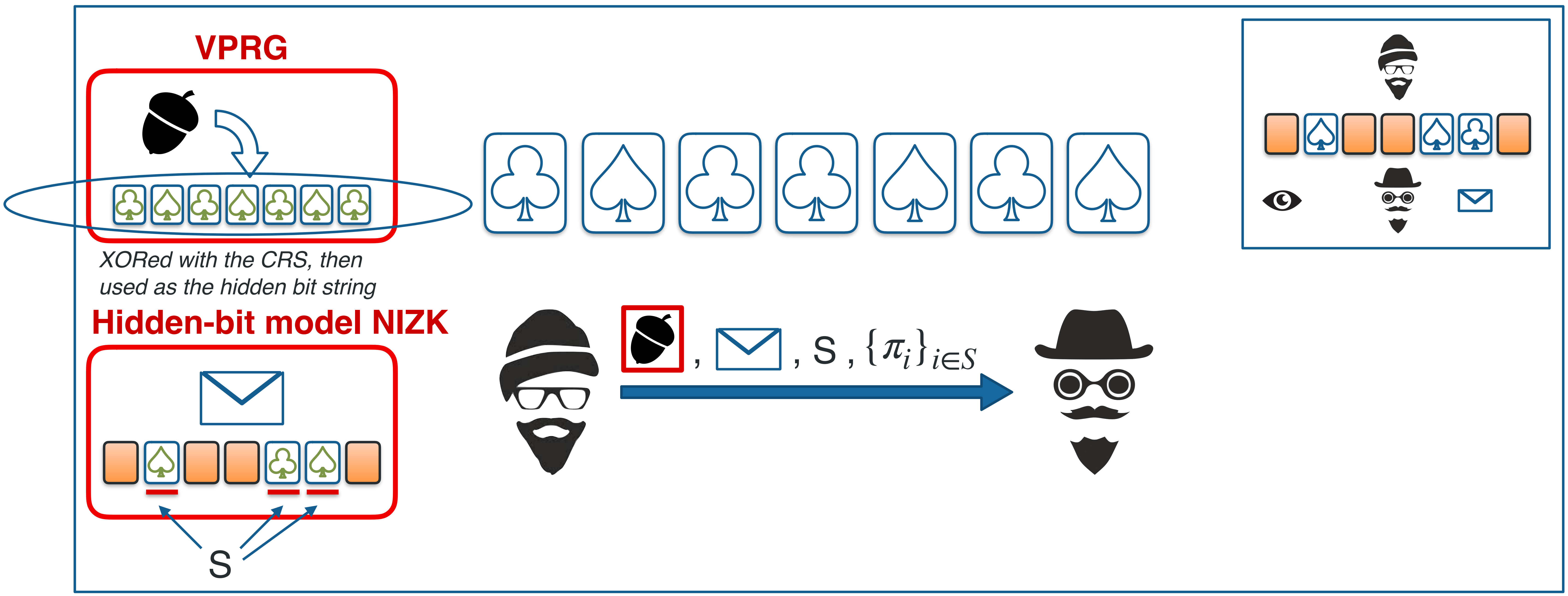

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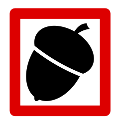
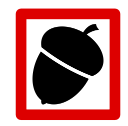
1. Every  *is in the image of* VPRG(.)
2. For *every possible*  , there is a *unique* associated ♧♤♧♧♤♧♤
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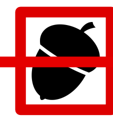
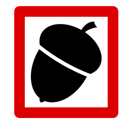
1. ~~Every  *is in the image of* VPRG(.)~~
2. For *every possible*  , there is a *unique* associated ♧♤♧♧♤♧♤
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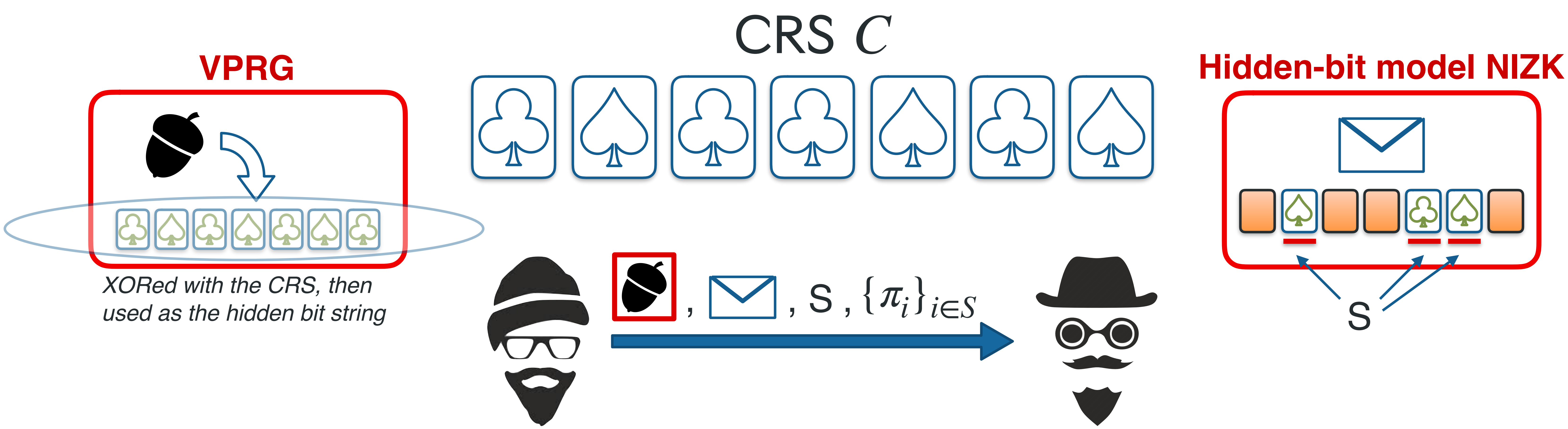

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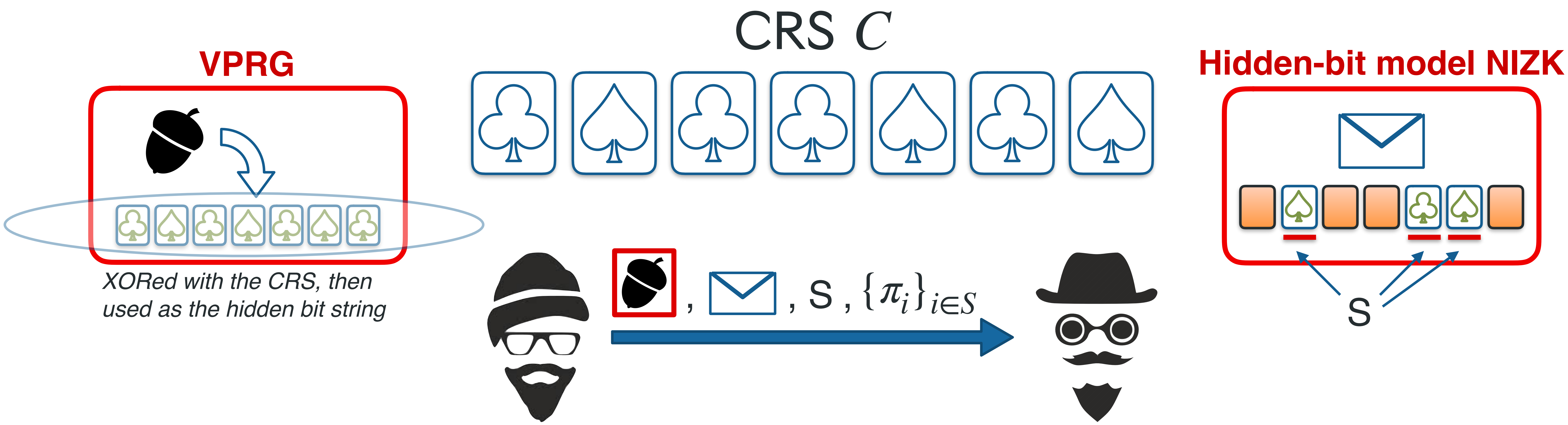
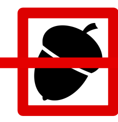
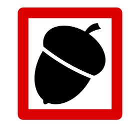


Proof Idea:

- C is 'close to a bad string' if $\exists$, Ext( ) $\oplus$ C is bad
- Proof accepted iff inconsistent opening OR the CRS is « close to a bad string » (requires (2))
- Inconsistent opening → contradiction to VPRG (3')
- Since is short, few CRS are close to a bad string.

Relaxing VPRGs

~~1. Every~~  ~~*is in the image of* VPRG(.)~~

2. For *every possible*  , there is a *unique* associated ♧♤♧♧♤♧♤

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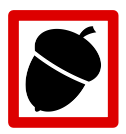
4.  *is short*

How does that help?

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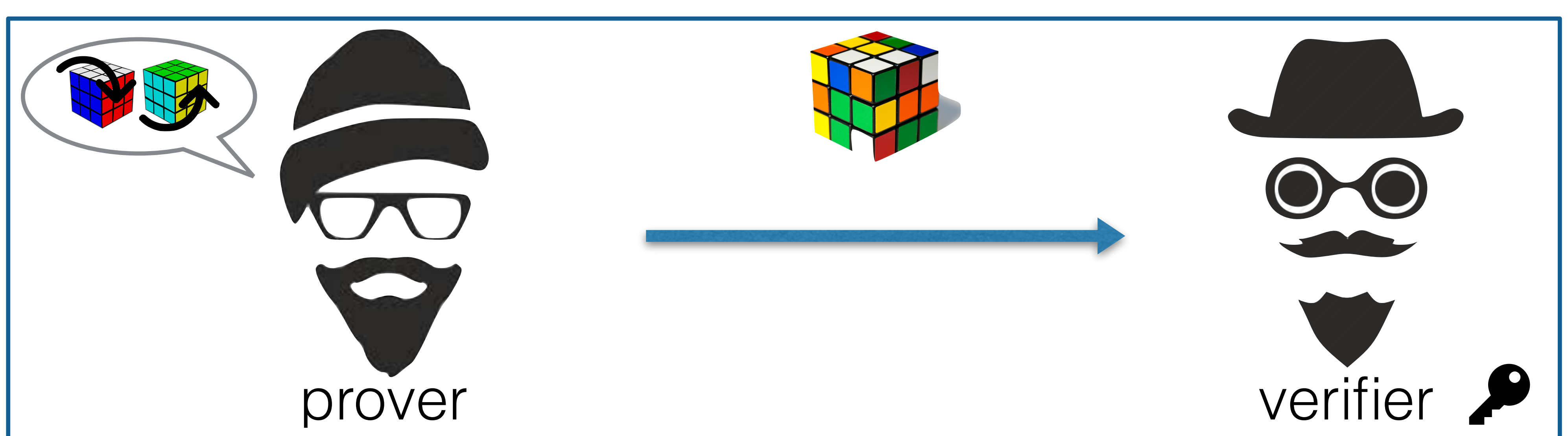
How does that help?

~~(1)~~ allows for lattice-based VPRGs

For typical LWE-based commitments, there are many invalid commitments indistinguishable from valid ones

(3') allows for designated-verifier variants

Since accepting incorrect proofs always exist in the DV setting



Instantiation 1: DVPRG from CDH

CDH over a group $\mathbb{G}$ states that given random g, g^a, g^b, it is hard to find g^{ab}

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[CKS08], gap twin-CDH: given random g, g^a, g^b, g^c, it is hard to find g^{ab}, g^{ac} *even given an oracle for the twin-DDH problem*

CDH $\Leftrightarrow$ gap twin-CDH *using some secret 'twin-DDH checking key'*

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[GL89]: explicit predicate B(.) such that given random g, g^a, g^b, g^c, it is hard to find $B(g^{ab}, g^{ac})$ with probability >> 1/2 even given an oracle for the twin-DDH problem

Equivalent to CDH

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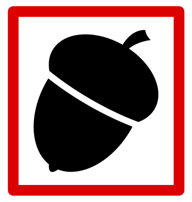
CDH over a group $\mathbb{G}$ states that given random g, g^a, g^b, it is hard to find g^{ab}

[CKS08], gap twin-CDH: given random g, g^a, g^b, g^c, it is hard to find g^{ab}, g^{ac}
even given an oracle for the twin-DDH problem

CDH $\Leftrightarrow$ gap twin-CDH *using some secret 'twin-DDH checking key'*

[GL89]: explicit predicate B(.) such that given random g, g^a, g^b, g^c, it is hard to find $B(g^{ab}, g^{ac})$ with probability >> 1/2 even given an oracle for the twin-DDH problem

Equivalent to CDH

= 

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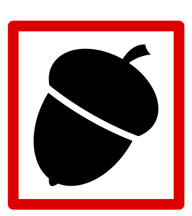
= 

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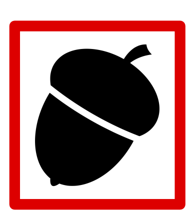
= 

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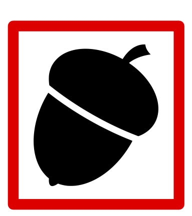
= 

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Proof: g^{ab}, g^{ac}
+ twin-DDH check

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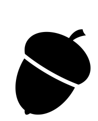
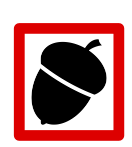
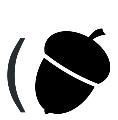
Equivalent to CDH

Public parameters: $\mathbb{G}, g, (g^{a_1}, g^{b_1}, \cdots, g^{a_n}, g^{b_n}) = (u_1, v_1, \cdots, u_n, v_n)$

Secret verification key: $(\lambda_1, \cdots, \lambda_n)$ and $(K_1, \cdots, K_n) = (a_1 + \lambda_1 b_1, \cdots, a_n + \lambda_n b_n)$

DVPRG:  $= r$,  $= g^r$, DVPRG() = $B({u_1}^r, {v_1}^r), \cdots, B({u_n}^r, {v_n}^r)$

Proof: $\pi = (u_i^r, v_i^r) = (\pi_0, \pi_1)$

Verification: check that $B(\pi_0, \pi_1) = b$ and $\pi_0^{\lambda_i} \pi_1 = (g^r)^{K_i}$

Instantiation 2: VPRG from LWE+NIWI

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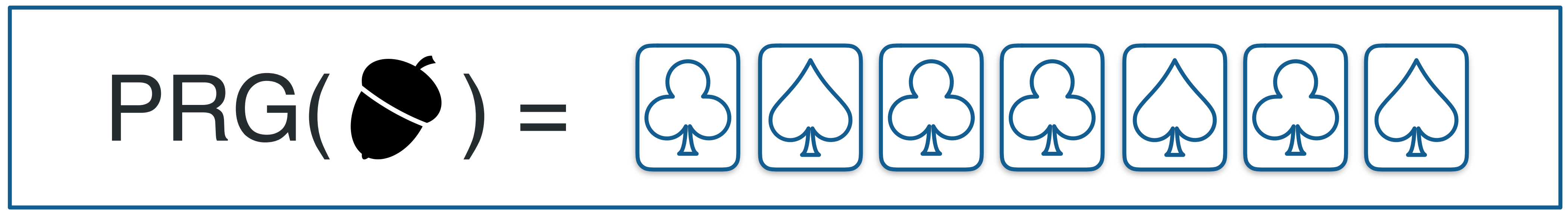

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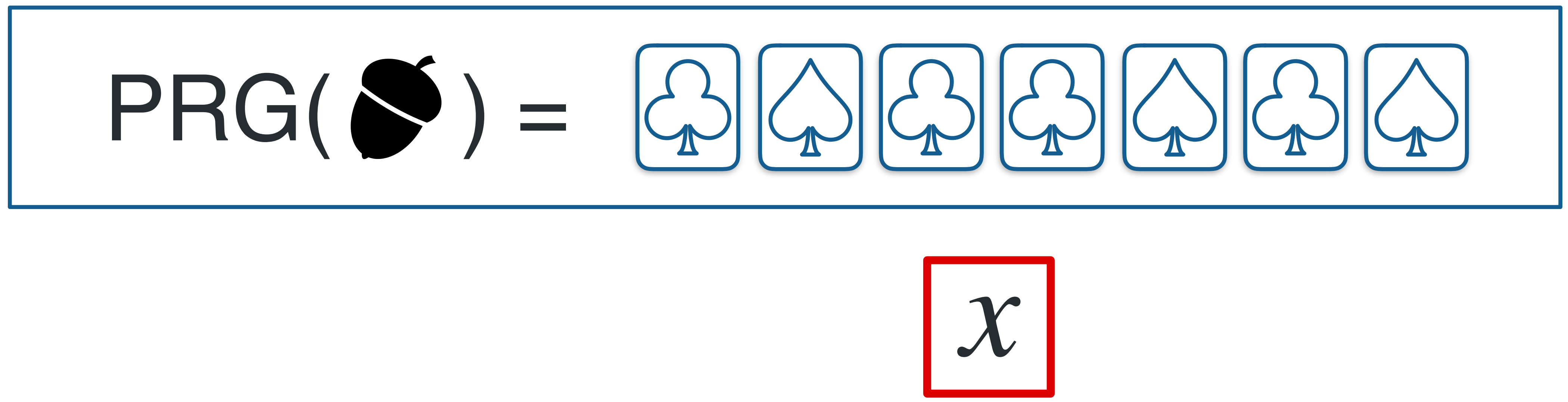

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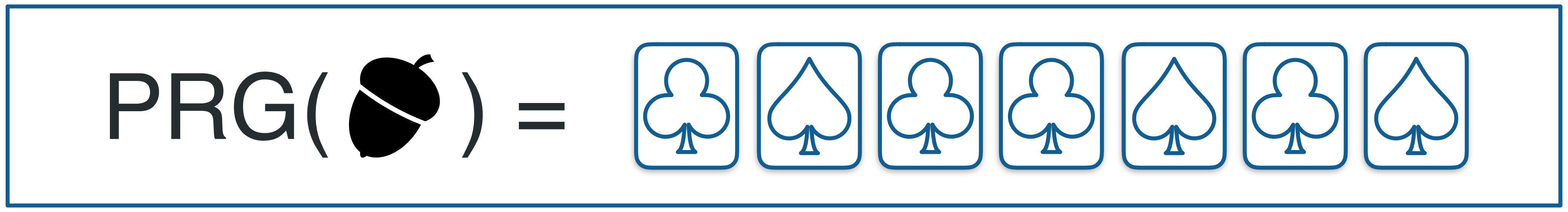
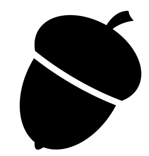
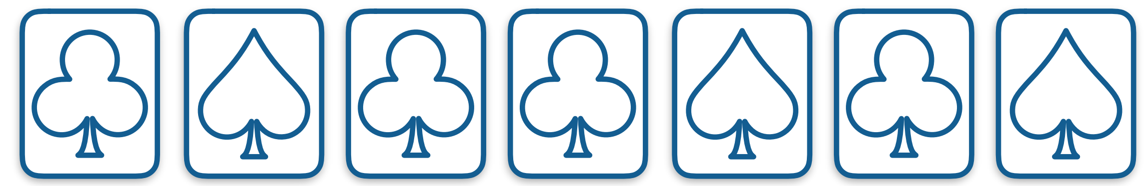


Hiding $\boxed{x}$

Instantiation 2: VPRG from LWE+NIWI

PRG() = 

Hiding $\boxed{x}$

Binding 

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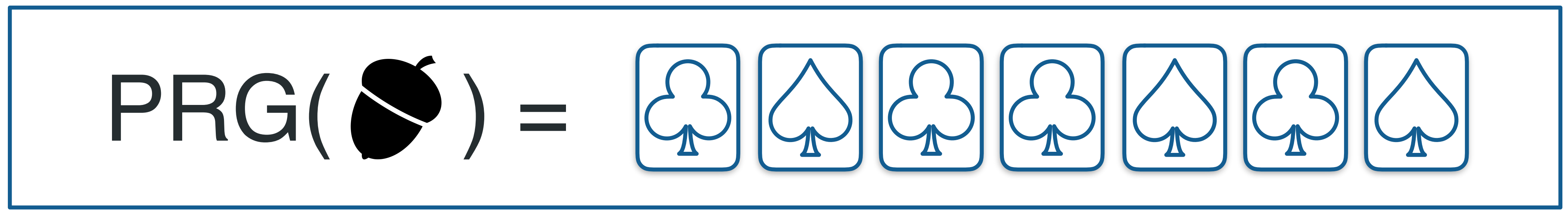
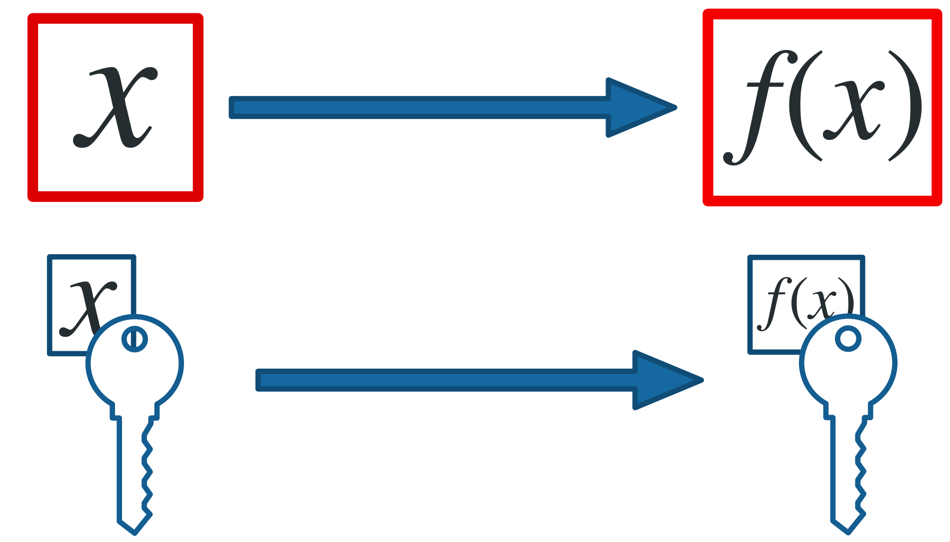


Hiding

Binding



Fully homomorphic

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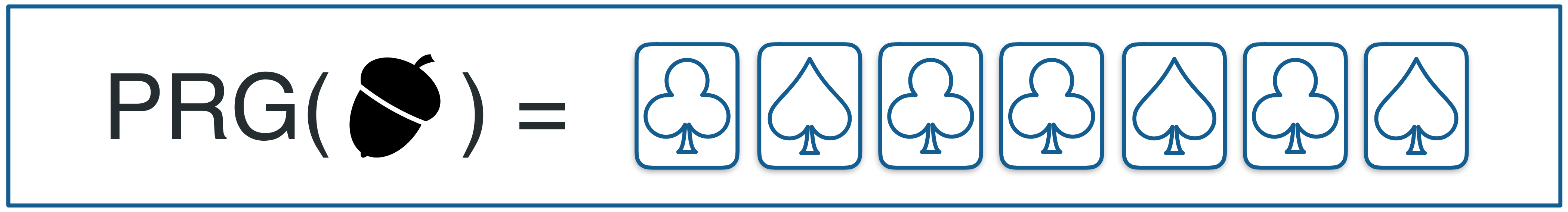
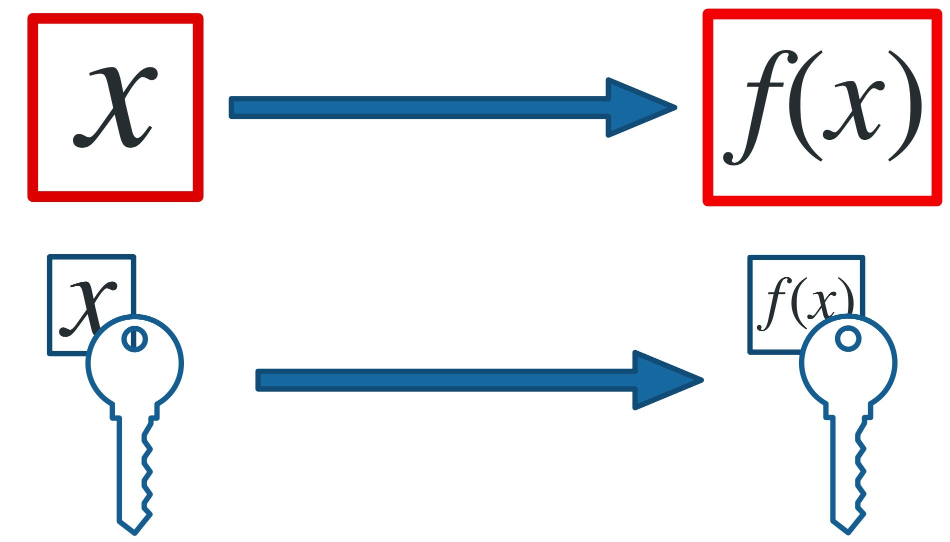


From LWE [GVW15]

- Hiding
- Binding
- Fully homomorphic



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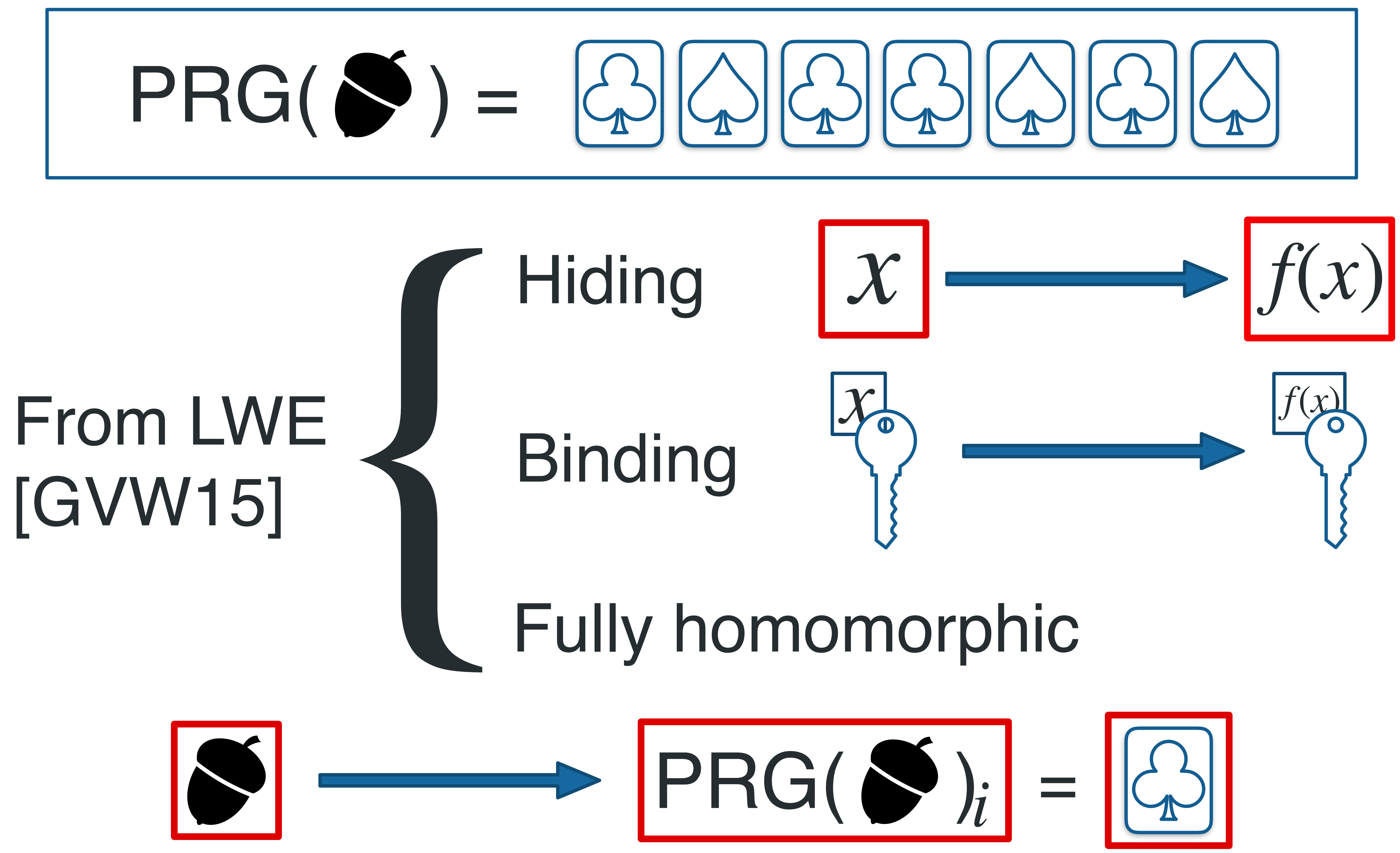

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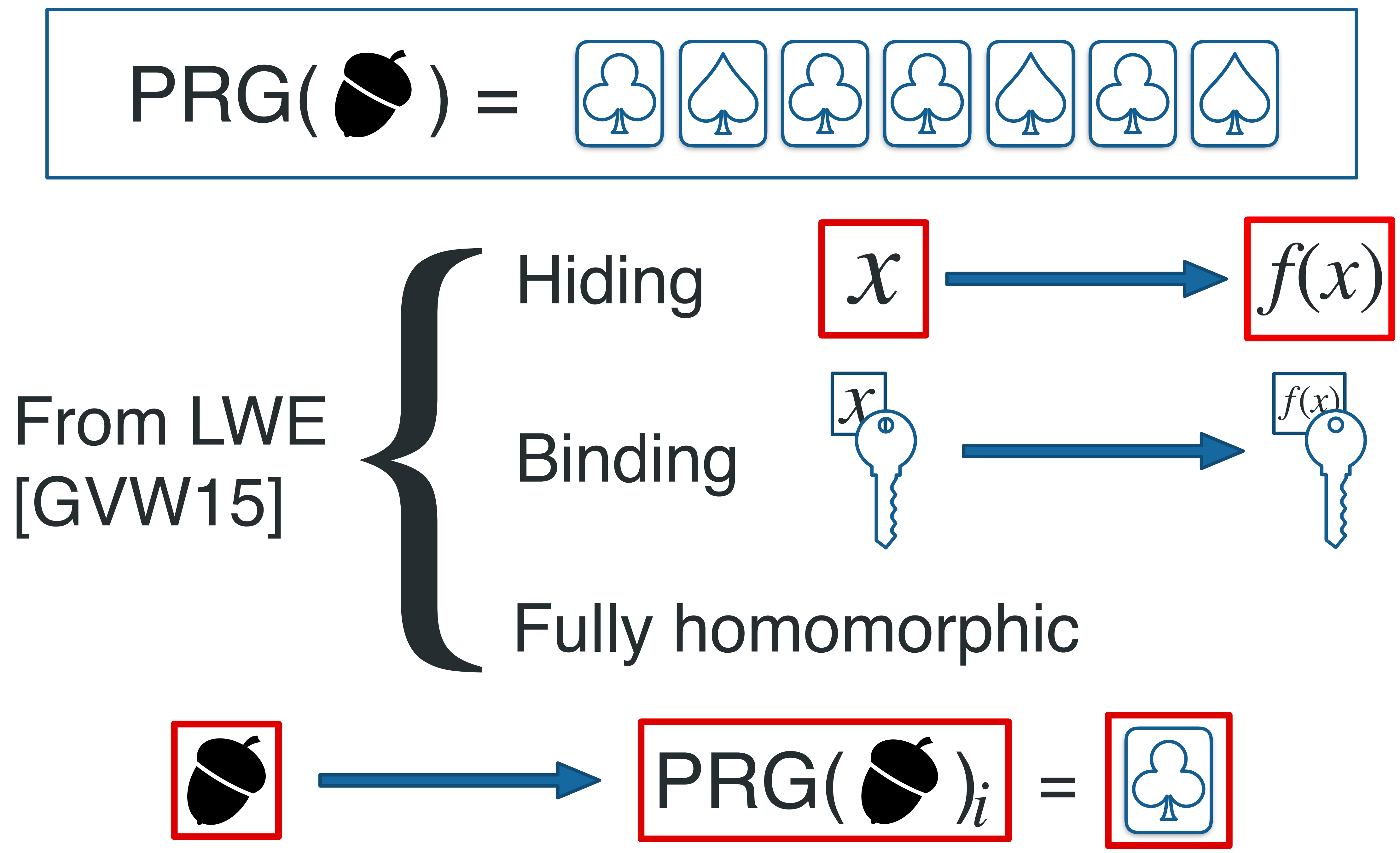

1. ~~Every is in the image of VPRG(.)~~
2. For *every possible* , there is a *unique* associated
3'. Proofs of opening to bits inconsistent with *are hard to find*
4. *is short*

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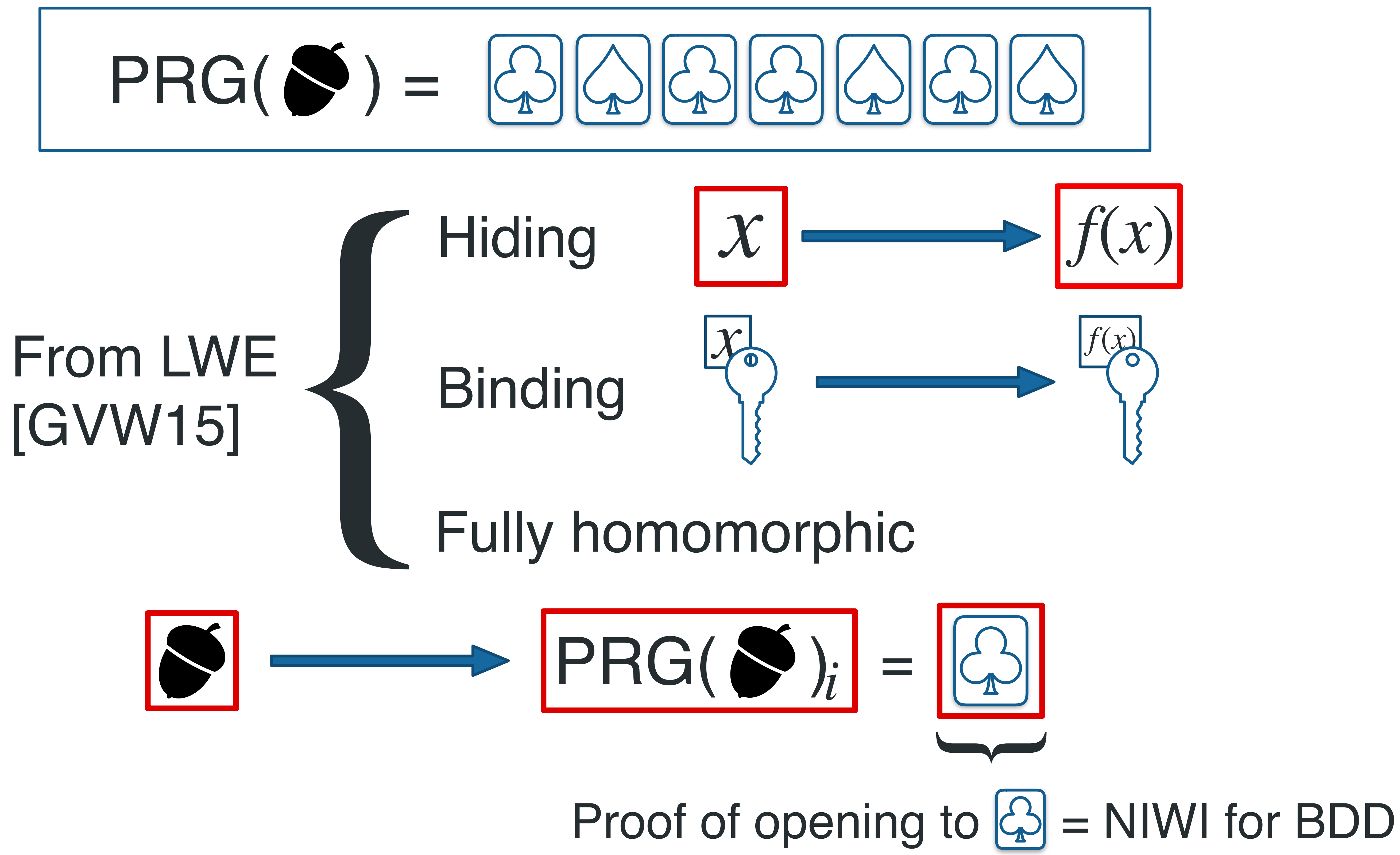

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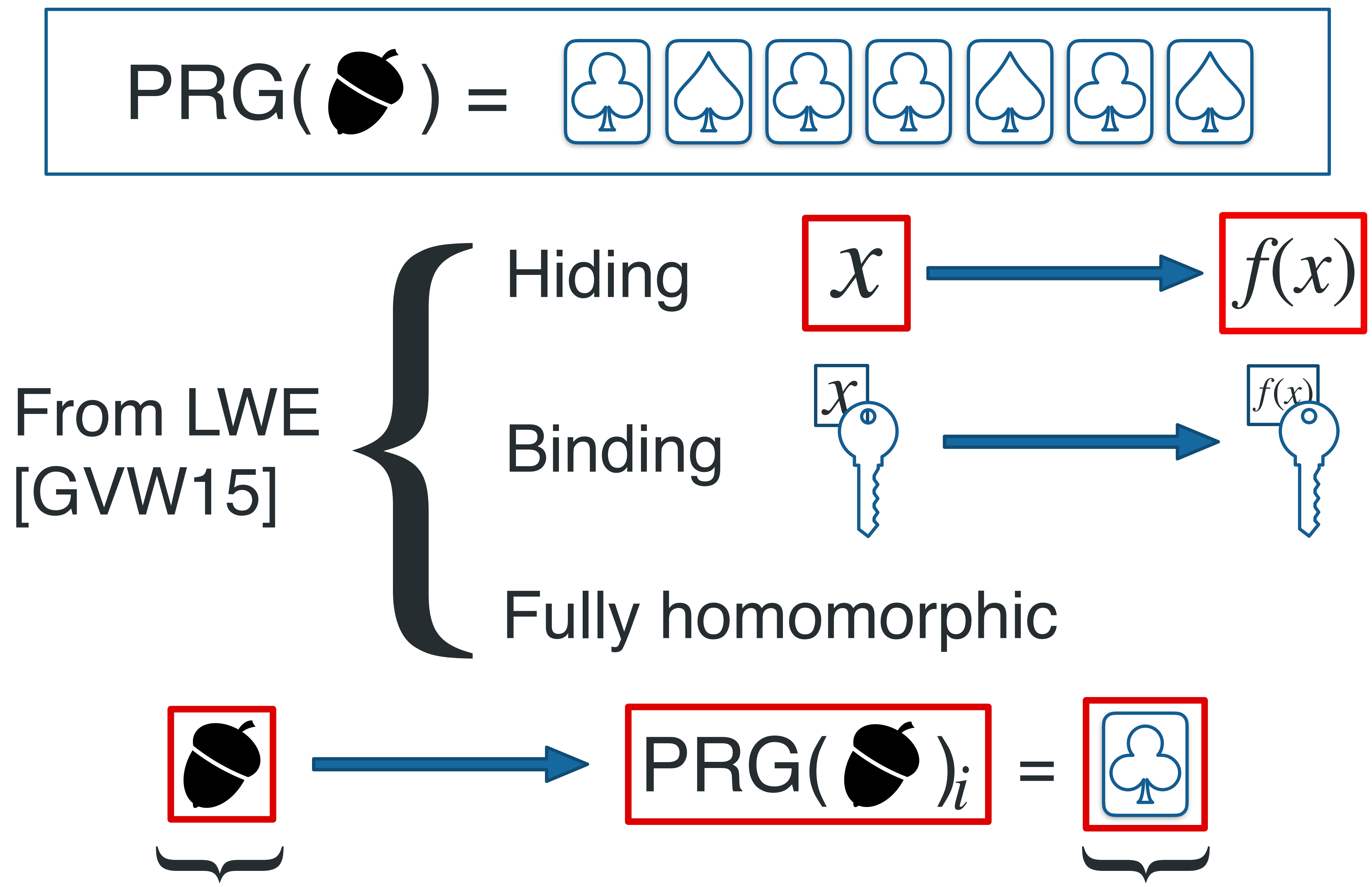

Proof of validity = NIZK for BDD. Proof of opening to = NIWI for BDD.

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Summary

We obtain two new constructions:

1) A DVNIZK for NP under the CDH assumption

First direct indication that DVNIZK with unbounded soundness are actually easier to build than standard NIZK

2) A (DV)NIZK for NP assuming LWE and the existence of a (DV)NIWI for BDD

Improving over, and considerably simplifying, the recent result of [RR18] which required a NIZK for BDD.

by relaxing [DN00]'s VPRGs, generalizing to DVPRGs, showing that it still suffices to construct (DV)NIZKs by instantiating the hidden-bit model, and providing new (D)PRGs instantiations.

Thanks for your attention

Questions?