

# Efficient Designated-Verifier Non-Interactive Zero-Knowledge Proofs of Knowledge

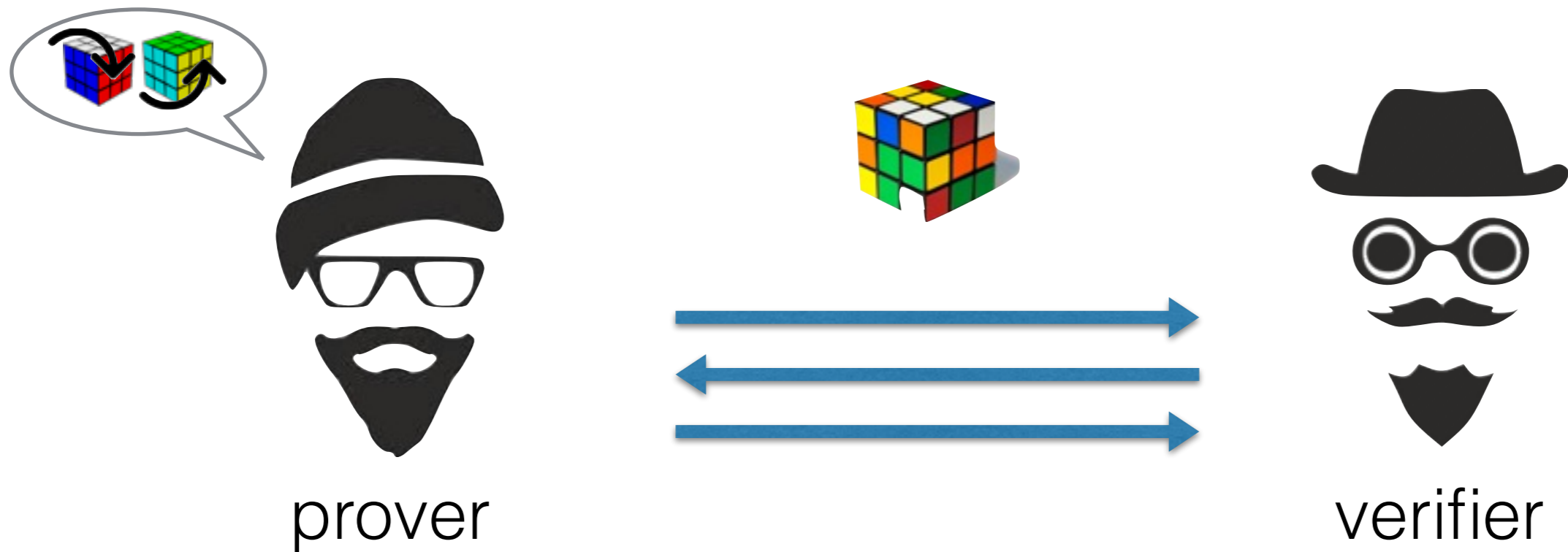
Pyrros Chaidos, *Geoffroy Couteau*



HELLENIC REPUBLIC  
National and Kapodistrian  
University of Athens

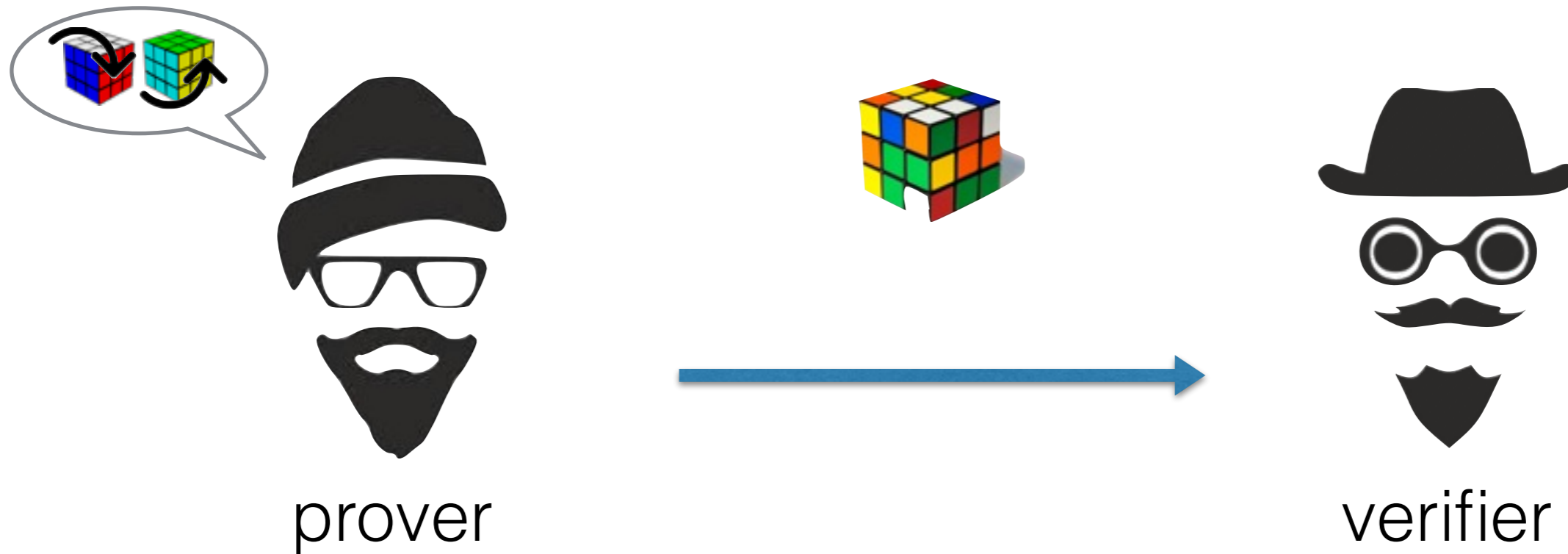


# Zero-Knowledge Proof



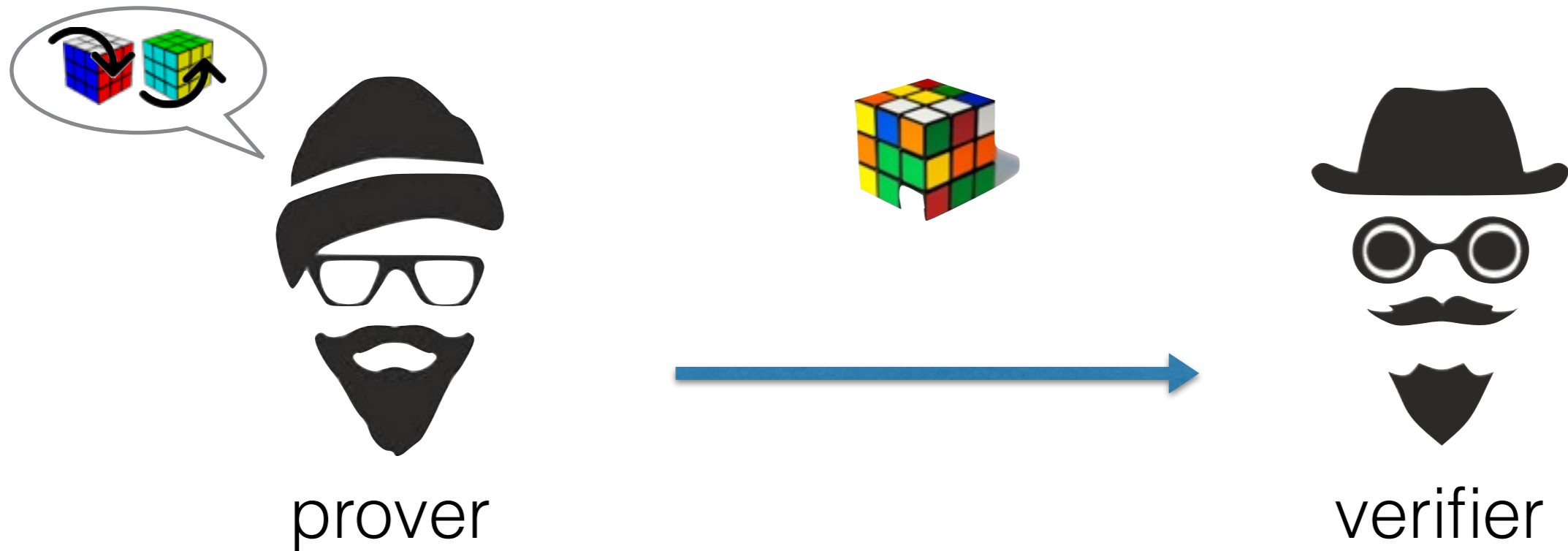
- Complete: if  $P$  knows a solution,  $V$  accepts
- Sound: if there is no solution,  $P$  cannot convince  $V$
- Zero-Knowledge:  $V$  does not learn the solution

# Non-Interactive Zero-Knowledge Proof



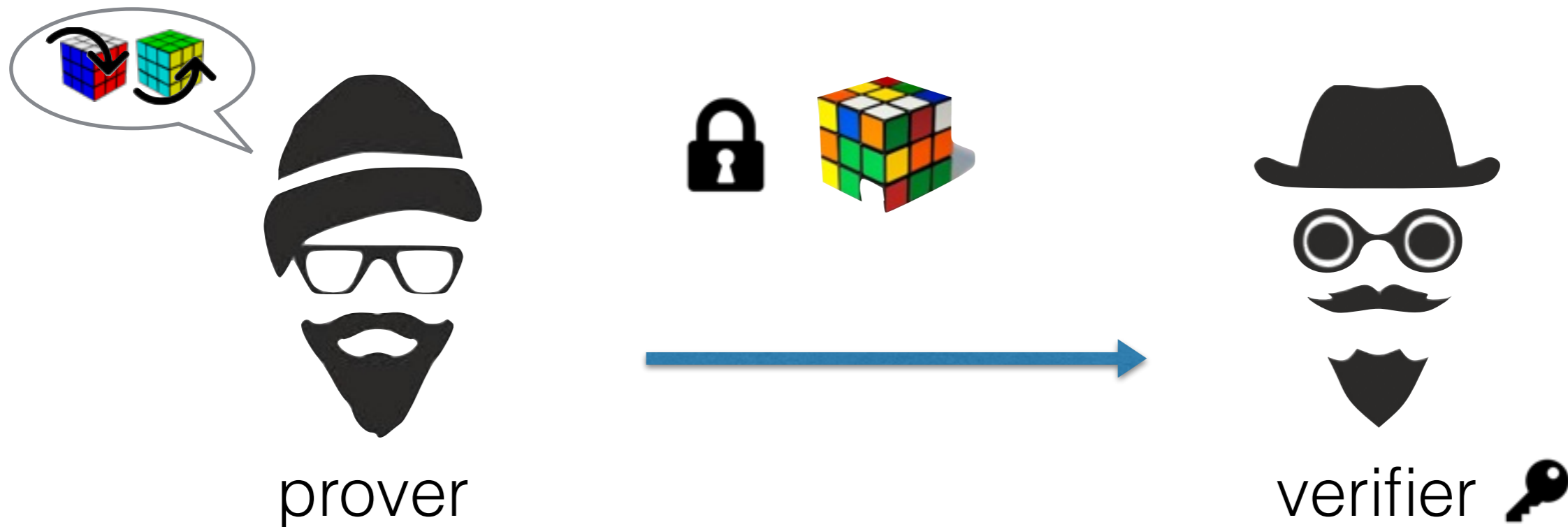
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# Non-Interactive Zero-Knowledge Proof of Knowledge



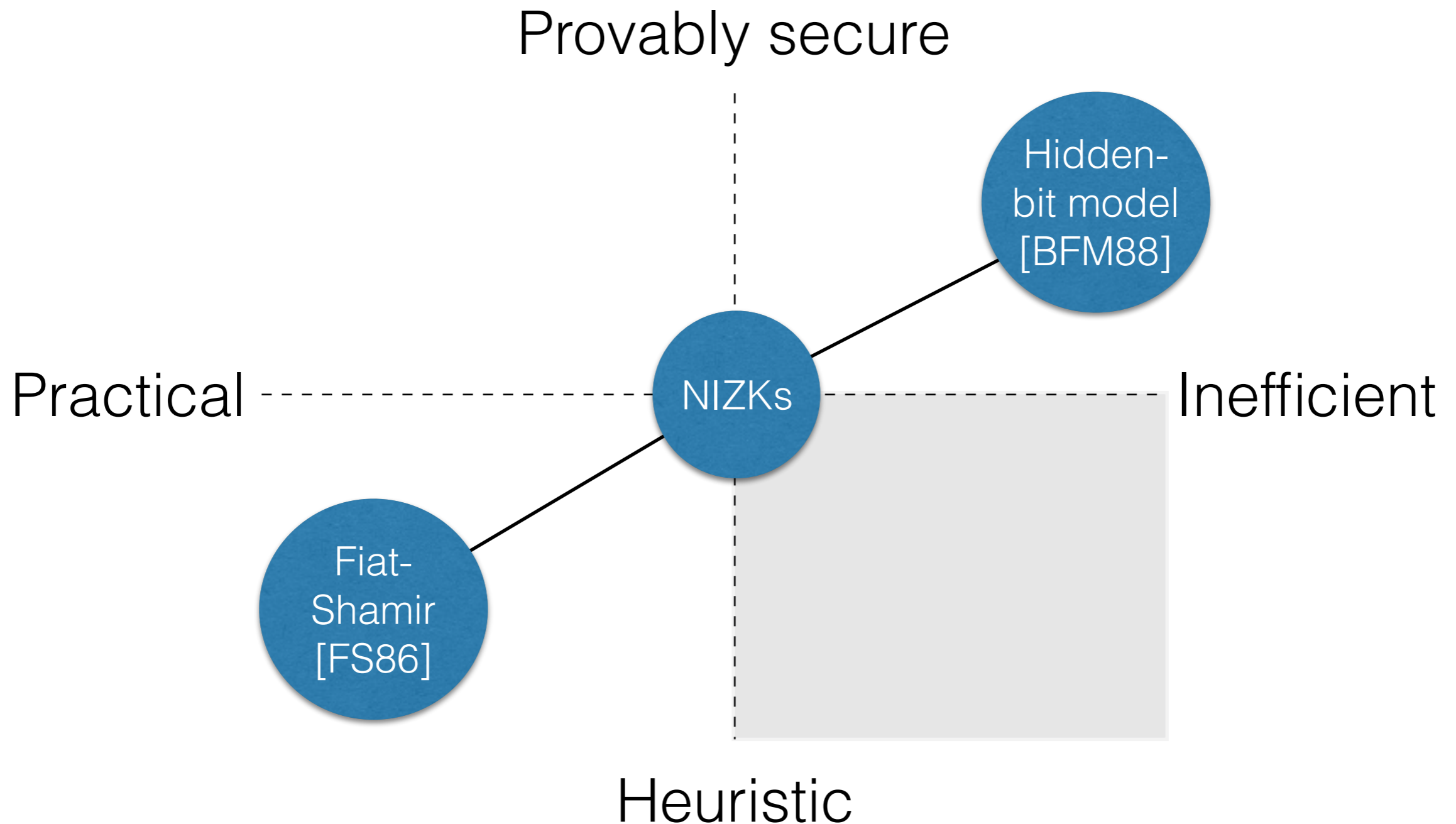
- Complete: if  $P$  knows a solution,  $V$  accepts
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# Designated-Verifier NIZKPoK

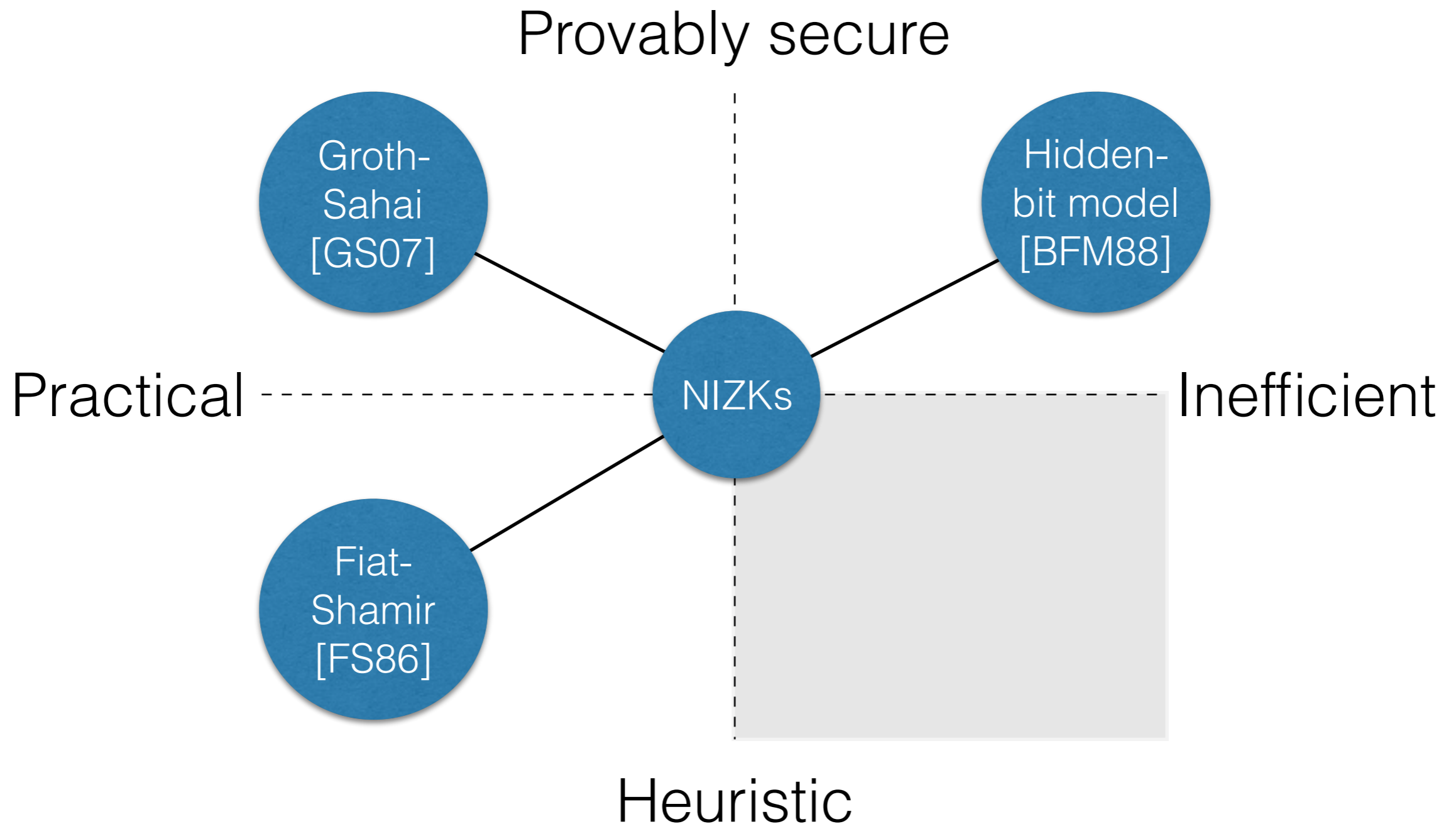


- Complete: if  $P$  knows a solution,  $V$  accepts
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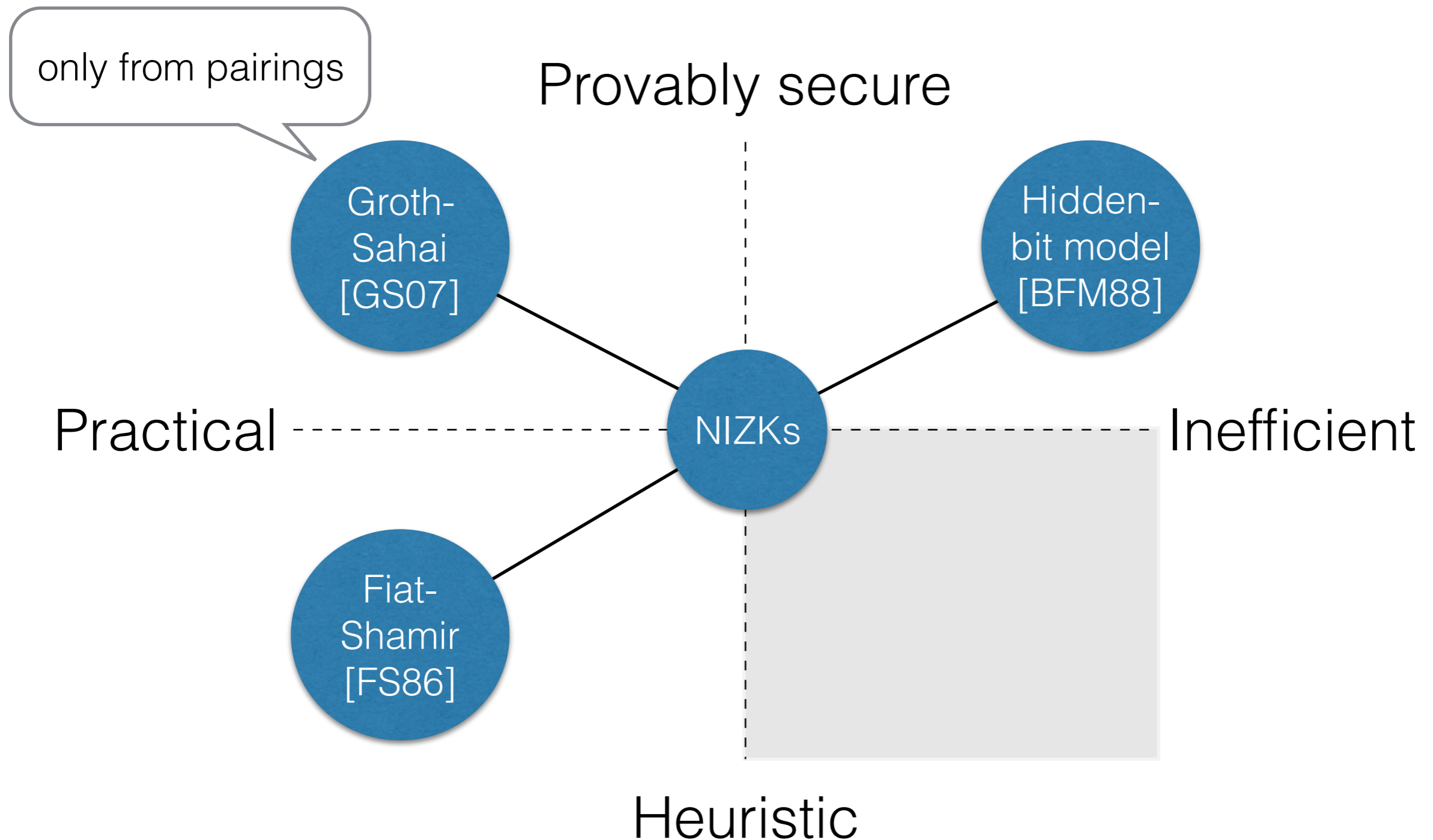
# The NIZK Landscape



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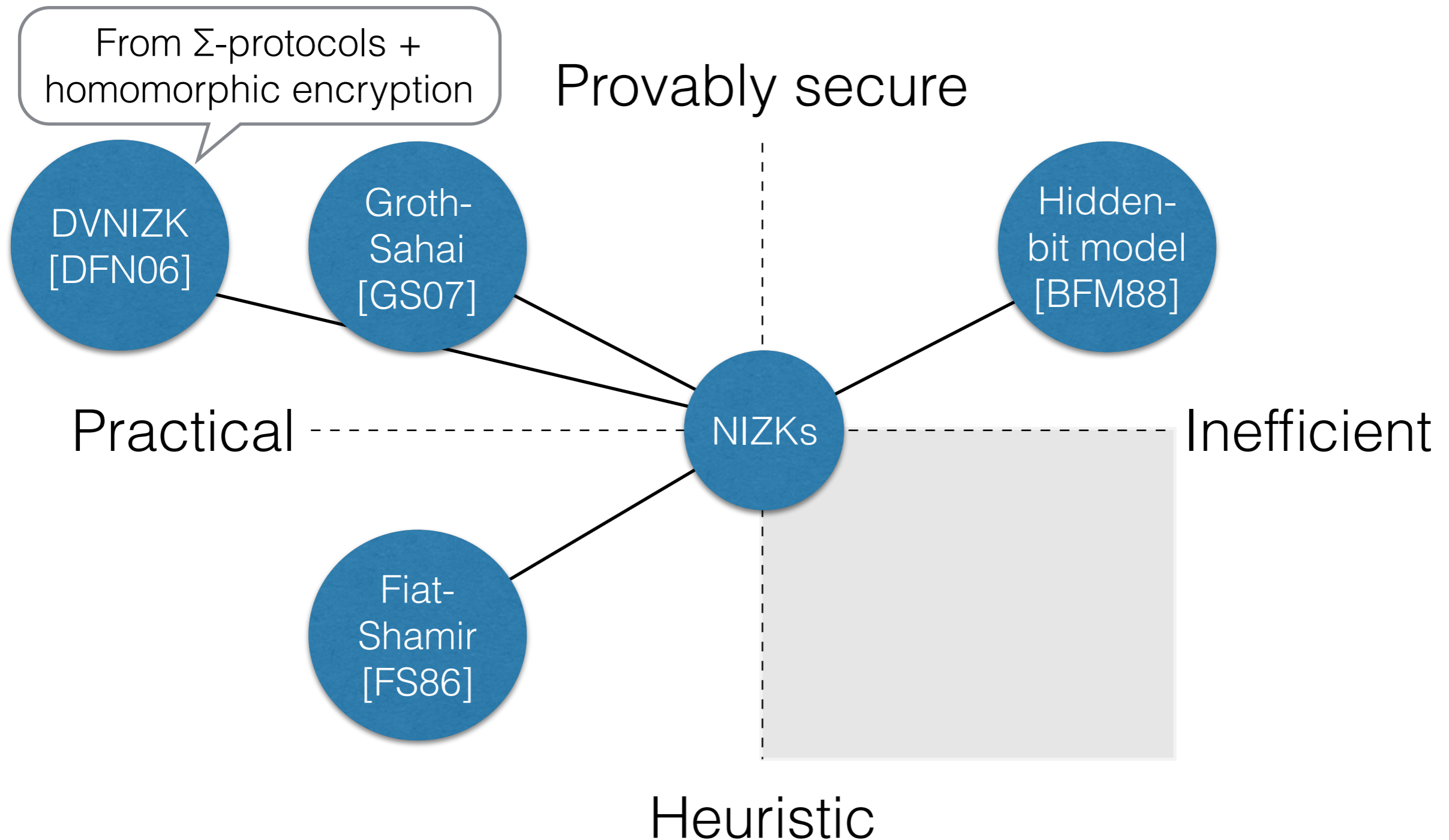


# The NIZK Landscape





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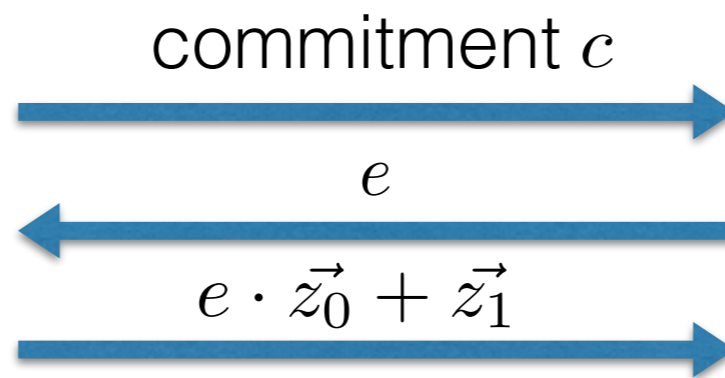


# The DFN Compiler

$\Sigma$ -protocol



prover



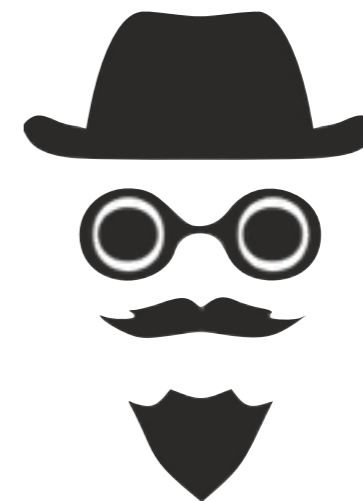
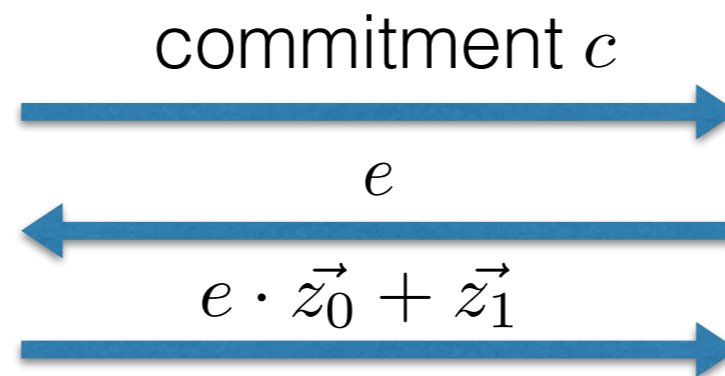
verifier

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$\Sigma$ -protocol



prover



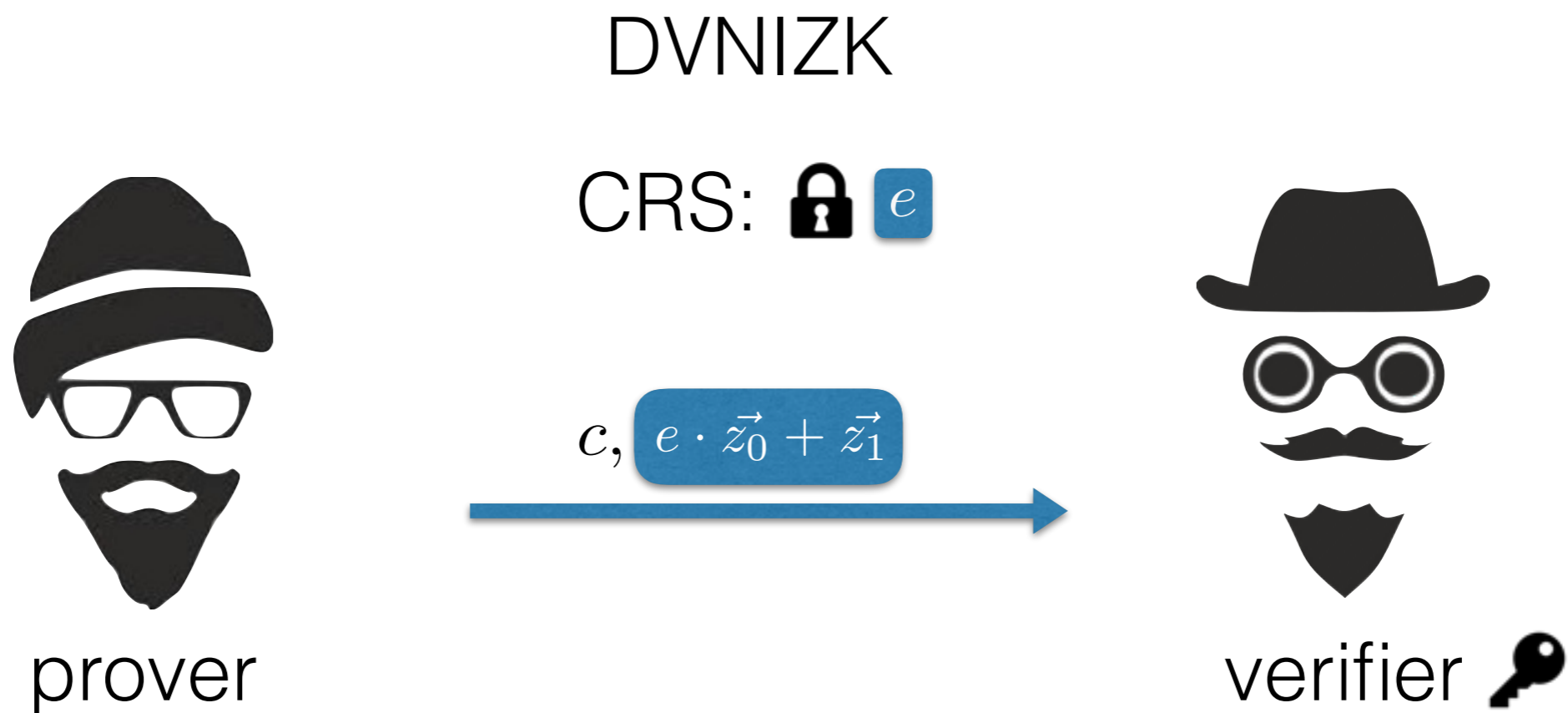
verifier

$$\text{Encrypt}(m, \text{lock}) = m$$

$$\text{Decrypt}(m, \text{key}) = m$$

$$\text{Eval}(m, \text{lock}, f) = f(m)$$

# The DFN Compiler






$$\text{Encrypt}(m, \text{lock}) = \langle m \rangle$$




$$\text{Decrypt}(\langle m \rangle, \text{key}) = m$$

$$\text{Eval}(\langle m \rangle, \text{lock}, f) = \langle f(m) \rangle$$





# The DFN Compiler

-  Non-standard assumption
-  Not a proof of knowledge
-  Soundness is bounded

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Non-standard assumption

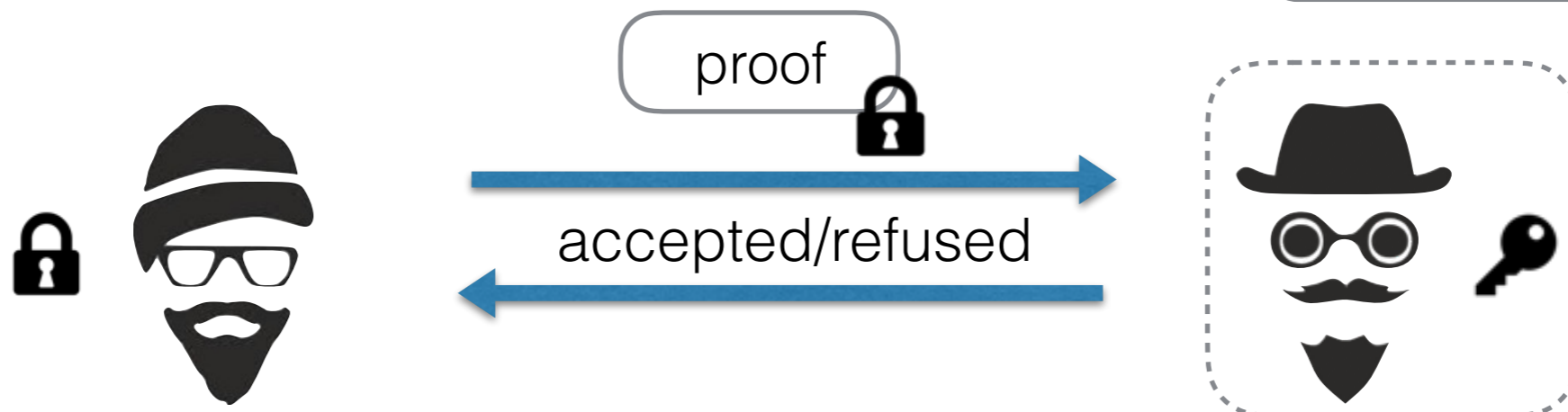


Not a proof of knowledge



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crucial in anonymous authentication



# The DFN Compiler



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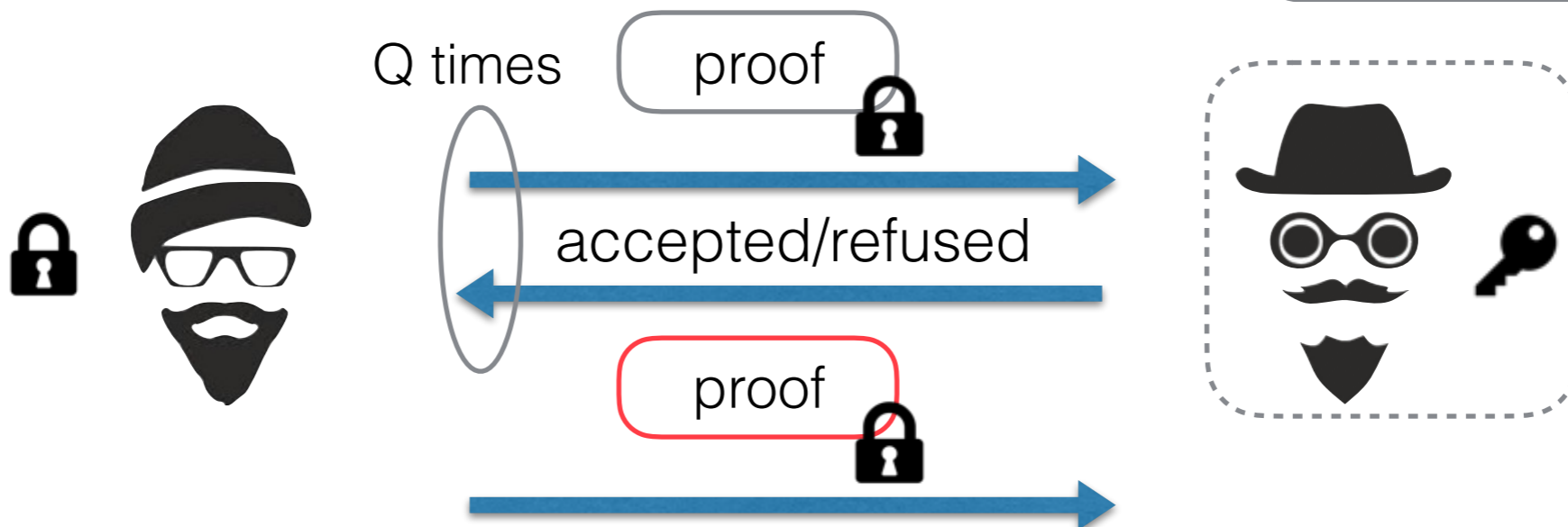


Not a proof of knowledge



Soundness is bounded

crucial in anonymous authentication



# The DFN Compiler



Non-standard assumption

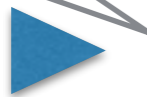


Not a proof of knowledge

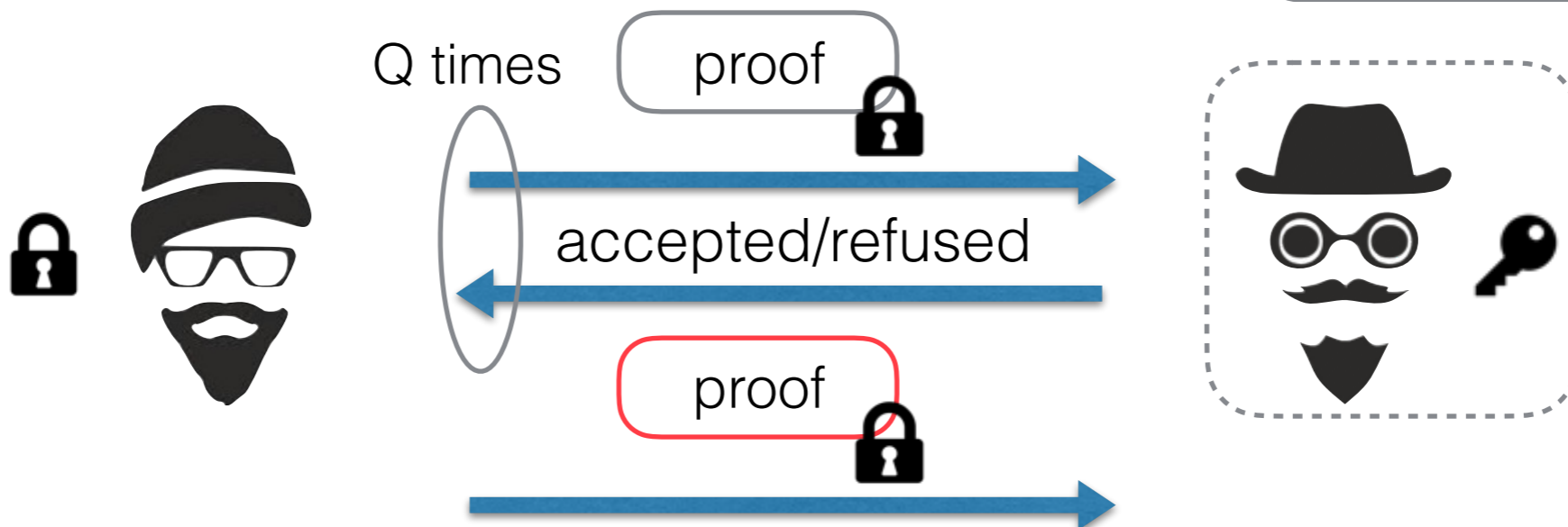


Soundness is bounded

there is an attack



crucial in anonymous authentication



# The [CG15] Compiler



Standard assumption



Not a proof of knowledge

there is an attack



Soundness is bounded

crucial in anonymous authentication

# This work



Statistical soundness



Proof of knowledge



Unbounded soundness

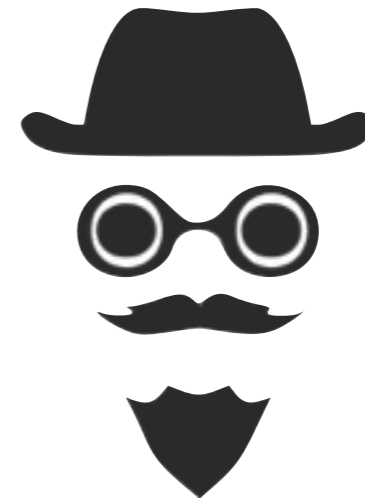
# Our Technique



prover

CRS:   $e; r_e$

$c, e \cdot \vec{z}_0 + \vec{z}_1; r$



verifier 

$\text{Eval}(e; r_e, \text{lock}, x \mapsto x \cdot \vec{z}_0 + \vec{z}_1)$

$\text{Decrypt}(e \cdot \vec{z}_0 + \vec{z}_1; r, \text{key})$

$= e \cdot \vec{z}_0 + \vec{z}_1$

# Our Technique

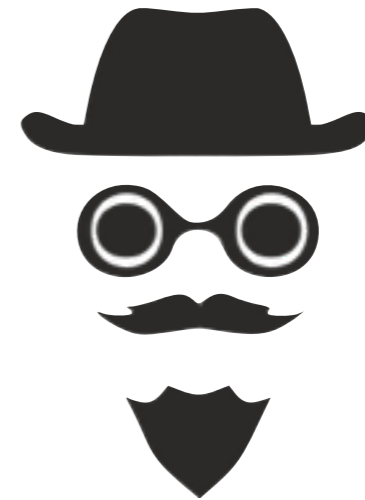
Idea: embed  $e$  as a random coin



prover

CRS:   $0; e$

$c, \vec{z}_1; -e\vec{r} \quad \vec{z}_0; \vec{r}$



verifier 

$f: (x_0, x_1) \mapsto ex_0 + x_1$

$$\text{Eval}(0; 1, \text{lock}, x \mapsto \vec{r} + \vec{z}_0) = \vec{z}_0; \vec{r}$$

$$\text{Eval}(0; e, \text{lock}, x \mapsto -x\vec{r} + \vec{z}_1) = \vec{z}_1; -e\vec{r}$$

$$\text{Eval}(\vec{z}_1; -e\vec{r} \quad \vec{z}_0; \vec{r}, \text{lock}, f)$$

$$= e \cdot \vec{z}_0 + \vec{z}_1; 0$$

need a *decodable* scheme

# Our Technique

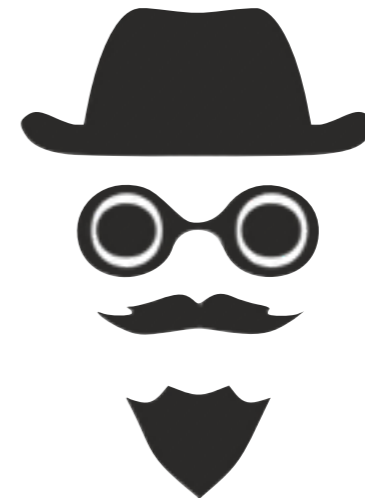
use  as extraction trapdoor



prover

CRS:   $0; e$

$c, \vec{z}_1; -e\vec{r} \quad \vec{z}_0; \vec{r}$



verifier ~~~~  $e$

$f: (x_0, x_1) \mapsto ex_0 + x_1$

$$\text{Eval}(\mathbf{0; 1}, \text{lock}, x \mapsto \vec{r} + \vec{z}_0) = \vec{z}_0; \vec{r}$$

$$\text{Eval}(\mathbf{0; e}, \text{lock}, x \mapsto -x\vec{r} + \vec{z}_1) = \vec{z}_1; -e\vec{r}$$


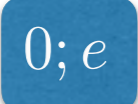
$$\text{Eval}(\vec{z}_1; -e\vec{r} \quad \vec{z}_0; \vec{r}, \text{lock}, f)$$

$$= e \cdot \vec{z}_0 + \vec{z}_1; 0$$

secret key not needed anymore!



# Why Extraction Works?

- Ext decrypts  $\vec{z}_0$  and  $\vec{z}_1$
- Knowing  does not allow P to cheat:
  - Let  $(M,R)$  be the plaintext/random coin orders
  - Pick a scheme with  $\gcd(M,R) = 1$
  -  leaks only  $e \bmod R$  (statistically)
  - Ext can extract  $e \bmod M$  from an incorrect proof

# What's in the Paper?

- We build a general, Groth-Sahai-like framework of unbounded DVNIZK proofs of knowledge, for statements over an abelian group.
- A dual variant of our framework handles statements over additively homomorphic cryptosystems.
- A third variant gives up on properties (unbounded, PoK) for efficiency, beating even Fiat-Shamir.

Thank you for your attention

Questions?

