Removing the Strong RSA Assumption from Arguments over the Integers

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Zero-Knowledge Argument

- Interactive protocol between a prover P and a verifier V;
- P knows a proof π of a statement;
- Example: I know a proof of Riemann hypothesis, but I do not want you to steal my million.

Correctness: if the proof is true, V will output "ok".

Soundness: No malicious prover P' can make V output "ok" on a wrong statement.

Zero-Knowledge: V learns nothing from the protocol, except that the statement is true.

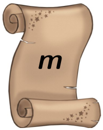
Zero-Knowledge Argument over the Integers

- Zero-knowledge proofs of relations between committed values play a fundamental role in cryptography
- We have efficient ZKA to prove algebraic relations between (finite) group elements
- Some important types of statements are not captured well by such relations (e.g.: proving that $a \geq b$)

Observation: These statements are well capture by algebraic relations over *integers* (aka Diophantine relations)

Example: $x \geq 0 \Leftrightarrow \exists (x_0, x_1, x_2, x_3) \in \mathbb{Z}^4, x = \sum_i x_i^2$

Commitment Schemes over Groups of Unknown Order



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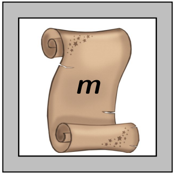

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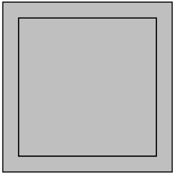

Hiding

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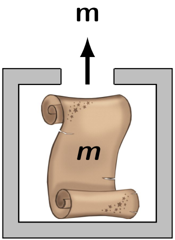

Binding

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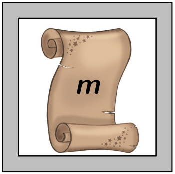

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$m \in \mathbb{G}$, $|\mathbb{G}|$ unknown

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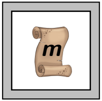

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Perfectly hiding, binding under Factorization

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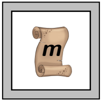

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Group Sig. | Range Proofs | Auctions | PPSS

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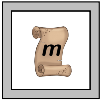

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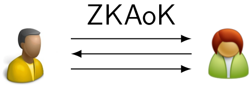

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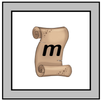

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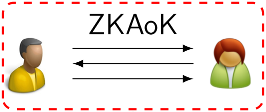

Strong-RSA

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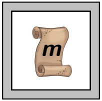

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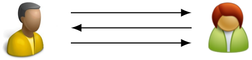

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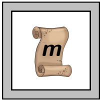

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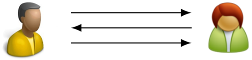

This work: RSA

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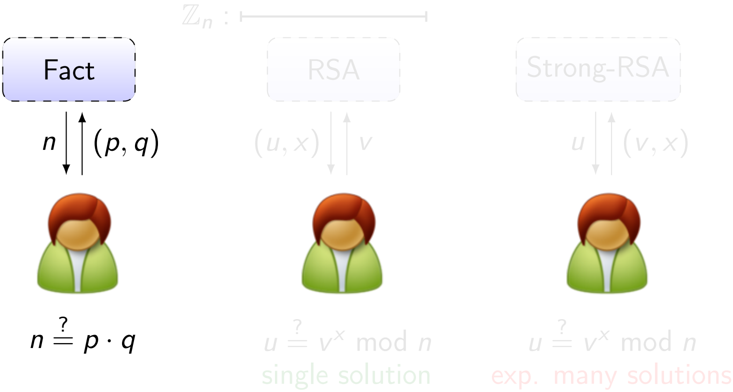
Preliminaries on RSA Groups

$\mathbb{Z}_n$, with $n = pq$, $p = 2p' + 1$, and $q = 2q' + 1$.

$$|\mathrm{QR}[n]| = \frac{(p-1)(q-1)}{4} = p'q'$$



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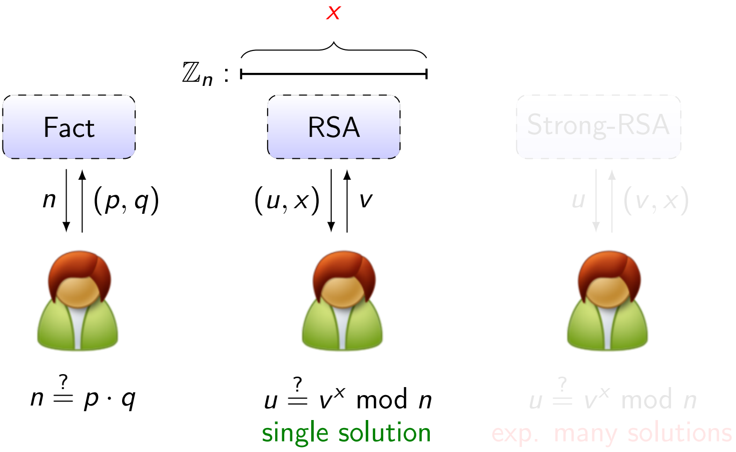

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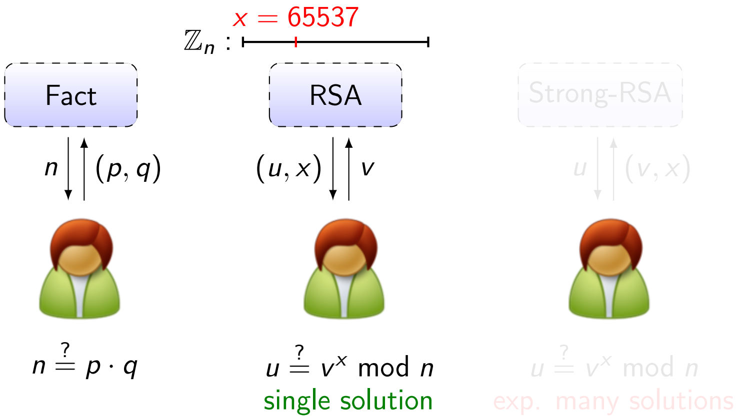

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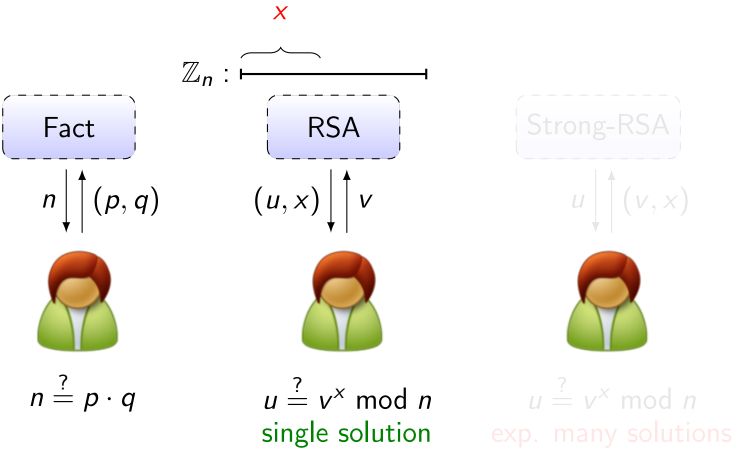

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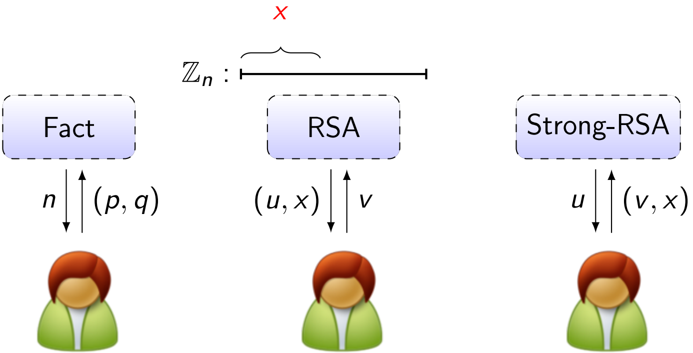


$n \stackrel{?}{=} p \cdot q$

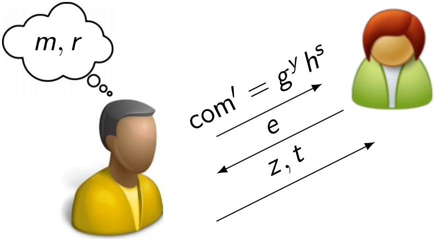
$u \stackrel{?}{=} v^x \bmod n$
single solution

$u \stackrel{?}{=} v^x \bmod n$
exp. many solutions

Zero-Knowledge Argument of Knowledge of an Opening

$n = p \cdot q$, $\langle g \rangle = \mathrm{QR}[n]$, $h^\alpha = g$

$$\mathrm{com} = g^m h^r$$



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V checks whether $\mathrm{com}^e \mathrm{com}' = g^z h^t$.

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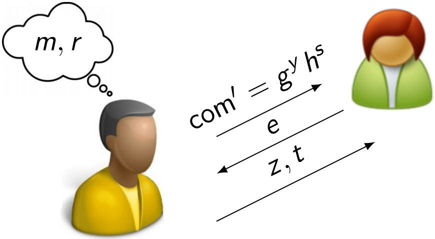

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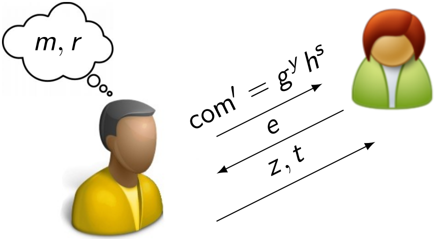

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Requires inversions over the exponents of $\mathbb{G}$!

Our Solution in a Nutshell

The analysis considers a simulator that solves a strong-RSA challenge by interacting with a malicious prover who produces an accepting proof with probability ε.

- The simulator gets a random small RSA challenge x before the proof, and perfectly hides it in his interaction with the prover;
- We study the constraints on the exponent chosen by the adversary;
- We show information-theoretically that if the exponent is larger than $O(1/\varepsilon)$, some non-trivial relation is satisfied;
- This relation allows to factor the modulus, hence the exponent must remain smaller than $O(1/\varepsilon)$;
- Therefore, the exponent chosen by the prover is equal to x with non-negligible probability $O(\varepsilon)$, contradicting RSA.

Applications, Other Contributions

Applications.

- Relations between committed values (e.g. [CM99])
- Range proofs ([Lip03])

Other Contributions.

- Can convert an FO commitment (integers) into a Gennaro commitment (modulo a small prime)
- Allows integer ZK proofs with efficient verification

Thank you for your attention

Questions?